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ELECTRICAL TRACTION

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IN TWO VOLUMES

VOL. II.

(ALTERNATING CURRENT)

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PREFACE.

SINCE the first edition of *Electrical Traction* was published in 1897, progress in this branch of electrical engineering has been extremely rapid, though not perhaps such as to satisfy its most ardent advocates. This advance has been so great that it has been found necessary, in preparing a second edition, practically to rewrite the entire book; in so doing, the extent of the ground to be covered has, in the authors' opinion, justified a considerable enlargement of the original volume.

The present book has been arranged in two parts, the first of which deals with the application of direct current electricity to traction both for tramways and for railways, and the second of which is devoted to the similar application of alternating currents. Although this forms a natural division of the subject, the installations in which direct current is employed are so much more numerous than those using alternating currents that the division is somewhat unequal. The authors have, however, preferred to adhere to this plan, rather than arrange the matter in a single volume or in two equal volumes, in the belief that in this way each volume, being practically self-contained and complete in itself, will be more acceptable to the engineering public.

In dealing with the various systems of electrical traction the authors have avoided, as far as possible, any direct advocacy of one rather than another; but have endeavoured to point out impartially the advantages and disadvantages inherent in each system. There are comparatively few engineers in a position to make a decision of such importance, and since any such decision should be based upon the consideration of all the various aspects of the case, the authors have made it their aim to present all the available information as to the apparatus obtainable, its design, efficiency and reliability, in such a way as to facilitate the work of responsible engineers.

The authors are fully conscious that the book is incomplete in many respects, and that any practical engineer who has specialised in one particular branch of electrical traction will be able to point out gaps and possibly inaccuracies in the chapters dealing with that branch. Few people, however, have a special knowledge of the whole range of traction, and it is hoped that those who have specialised may find some fresh information in those branches with which they are not so familiar. For advanced students, also, who may intend to take up this subject, the first volume, and later perhaps the second volume, may be found useful.

A word of explanation may be necessary with regard to the treatment of motor design. A great deal has been published in the past as to direct current motors, and to repeat it in this book would be to travel somewhat beyond the natural limits of the subject. The case is quite different for alternate current motors for traction purposes both polyphase and single phase, and in consequence considerable space has been devoted in the second volume to this matter, as information thereon is not readily obtainable.

In acknowledging assistance from various quarters it is necessary to mention first of all those manufacturing firms who have placed information at the disposal of the authors; of such firms Messrs Siemens-Schuckert, of Berlin, and Messrs Siemens Brothers, of London, have supplied much unpublished material and a number of original drawings, for which the authors wish to tender their thanks. Other firms have given much assistance, viz., the British Thomson-Houston Co., and the General Electric Co. of America, the British Westinghouse Co., Messrs Dick Kerr and Co., Messrs Ganz and Co., the Oerlikon Elektricitäts Gesellschaft, and the Algemeine Elektricitäts Gesellschaft.

In many cases items of information have been drawn from the technical press, acknowledgments for which have generally been made in the text; the chief source of such information has been the *Street Railway Journal*, and others are *Traction and Transmission, The Electrician*, and the *Electrical Review*. To these journals the authors wish to express their indebtedness.

The individuals to whom the authors are under an obligation are numerous. Mr A. P. Trotter, the electrical adviser to the Board of Trade, has given much information; Mr T. H. Schoepf, formerly of the British Westinghouse Co., Mr T. Stevens of the British Thomson-Houston Co., and Mr P. R. Brown of Messrs Hadfield's Steel Foundry Co., have been of assistance in many ways ; Herr A. Schmit of Messrs Siemens-Schuckert, has supplied a good deal of information with regard to Continental practice, and Mr Mittelhausen of the Bexley Heath Tramways, has put at the disposal of the authors his experience in the practical working of electric tramways. Other gentlemen who have kindly helped in various ways are Mr Dalziel, the electrical engineer of the Midland Railway, Mr Roger Smith, the electrical engineer of the Great Western Railway, Mr Huddleston of Messrs Siemens Brothers and Co., in connection with cables, and Mr A. C. Kelly of the British Westinghouse Co. Acknowledgments are due in a special way to Mr C. F. Jenkin of Messrs Siemens Brothers Dynamo Works.

In reading the manuscript and correcting the proofs the authors have received much valuable assistance from Mr R. E. Shawcross, Mr A. E. and Mr G. F. Odell, and in preparing the diagrams from Mr F. C. Summerson and Mr E. P. Hardy.

To all these gentlemen the authors tender their heartiest thanks.

ERNEST WILSON.
FRANCIS LYDALL.

LONDON,
19*th August,* 1907.

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CHAPTER 1.

THE POLYPHASE RAILWAY MOTOR.

PART 1.

GENERAL THEORY.

Design of the three-phase induction motor for traction. In dealing with alternate currents it is proposed to adopt the clock diagram method, and in so doing it is taken for granted that the method is sufficiently familiar to render preliminary explanations unnecessary.

In using the clock diagrams the following statements and conventions are necessary:

1. In any apparatus connected to a source of alternate currents, the current which is taken from the line is such as to make the internal voltage in the apparatus equal and opposite to the applied potential difference.

2. The internal voltage is the resultant of various components any one of which may be present or absent. The components are (*a*) the resistance drop; *this differs in phase from the current by* 180°; its magnitude depends on the circuit: (*b*) E.M.F. of induction self or mutual; the E.M.F. induced in a single-turn coil is equal to the rate of change of the number of lines of magnetic force which thread through that coil; this field is regarded as being *in phase with the current producing it* and the E.M.F. caused by the variation of the field as *lagging by one right angle behind the field*: (*c*) E.M.F. due to capacity: the phase of this E.M.F. is *one right angle in advance of the current which produces it.*

3. The diagram will be supposed to rotate in a *clockwise* direction.

In the following, the primary resistance is neglected, unless otherwise stated.

The diagram for a static transformer without magnetic leakage. In this diagram (see figure 1) OB represents the secondary current, OA the primary current, OC the resultant of the primary and secondary currents and therefore the magnetising current. The E.M.F. induced in both the windings is O*v* lagging 90° behind OC, and the potential difference applied to the primary is OV equal and opposite to O*v*. This is the case of a 1 : 1 transformer whose secondary is connected to a non-inductive external circuit, as shewn by the fact that OB is in phase with O*v*.

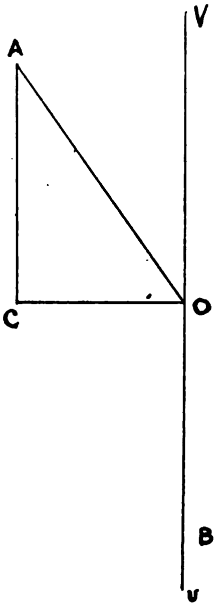

Fig. 1. Vector diagram of static transformer without magnetic leakage.

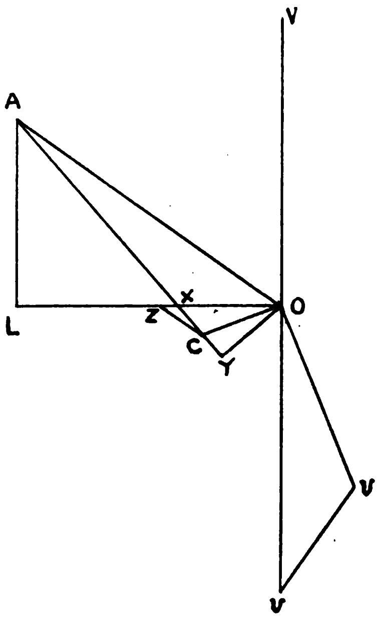

Fig. 2. Vector diagram of static transformer with magnetic leakage.

Diagram for a static transformer with magnetic leakage. In a transformer with magnetic leakage there is in addition to the main field which threads both primary and secondary a leakage field attached to each of the coils which has no inductive effect on the other coil. The main field is produced by the resultant of the primary and secondary currents, whereas the leakage fields are produced by and are in phase with their own currents.

In the diagram (see figure 2) OA represents the primary current, AC the secondary current, OY the magnetising current necessary to overcome the resistance in the secondary circuit, and CY the magnetising current necessary to overcome the back E.M.F. in the secondary circuit due to its magnetic leakage, the external secondary circuit being supposed non-inductive. The resultant of OA and AC is OC and this produces in both coils an E.M.F. $\mathrm{O}v'$ lagging 90° behind OC. The total internal voltage in the primary is compounded of $\mathrm{O}v'$ and $v'v$ the E.M.F. due to the magnetic leakage of the primary. The total voltage is therefore $\mathrm{O}v$ and the applied potential difference which is equal and opposite to $\mathrm{O}v$ is OV.

Now OY is at right angles to AC and proportional thereto,

YC is parallel to (or in phase with) AC and proportional thereto,

vv' is at right angles to and proportional to OA.

On OC as base construct the triangle OCZ similar to the triangle $\mathrm{O}v'v$; then CZ, being at right angles to vv', will be parallel to OA and the small triangle CZX will be similar to the triangle AXO. Produce OZ to L and draw AL perpendicular to OZ.

Now OZ being proportional to $\mathrm{O}v$ is fixed in magnitude and phase. Call $\mathrm{OZ} = k \times \mathrm{O}v$. Then $\mathrm{CZ} = k \times vv'$ and $\mathrm{CO} = k \times \mathrm{O}v'$; similarly $\mathrm{CY} = k \times$ leakage E.M.F. in the secondary, and $\mathrm{OY} = k \times$ resistance drop in the secondary.

It will be simpler to write

$$\frac{\mathrm{CZ}}{\mathrm{CO}} = \frac{vv'}{\mathrm{O}v'}, \text{ and therefore } \frac{\mathrm{CZ}}{\mathrm{OA}} = \frac{vv'}{\mathrm{OA}} \Big/ \frac{\mathrm{O}v'}{\mathrm{OC}},$$

or

$$\mathrm{CZ} = \lambda_1 \mathrm{OA};$$

similarly

$$\mathrm{CY} = \lambda_2 \mathrm{AC},$$

where λ_1 is the ratio of the E.M.F. per unit current induced in the primary by its own leakage field to the E.M.F. per unit current induced in the primary by the main field, there being no current in the secondary, and λ_2 is the corresponding ratio for the secondary winding.

Now

$$\text{sine } \mathrm{XAL} = \sin \mathrm{XOY} = \frac{\mathrm{XY}}{\mathrm{XO}} = \frac{\mathrm{XC} + \mathrm{CY}}{\mathrm{XO}} = \frac{\lambda_1 \mathrm{AX} + \lambda_2 \mathrm{AC}}{\mathrm{XO}}$$

$$= \frac{\lambda_1 \mathrm{AX} + \lambda_2 (1 + \lambda_1)\, \mathrm{AX}}{\mathrm{OX}} = \frac{\mathrm{AX}}{\mathrm{OX}} (\lambda_1 + \lambda_2 + \lambda_1 \lambda_2) = \frac{\mathrm{AX}}{\frac{\mathrm{OX}}{\sigma}},$$

where

$$\sigma = \lambda_1 + \lambda_2 + \lambda_1 \lambda_2.$$

This relation can be interpreted to mean that for a fixed applied voltage the locus of A lies on a circle (see figure 3)*, described on XM as diameter, where XM is a continuation of OX and is equal in length to $\dfrac{\mathrm{OX}}{\sigma}$.

---

* The authors believe that the circle diagram for induction motors was first published by M. Heyland; but other writers have claimed to have worked it out independently.

When the secondary circuit is open, it will be evident that OA coincides with OX, that is to say OX represents the open circuit current of the transformer.

The diameter of the semi-circle is, therefore, equal in length to the open circuit current divided by σ, where

$$\sigma = \lambda_1 + \lambda_2 \text{ neglecting } \lambda_1\lambda_2,$$

λ_1 and λ_2 being defined as above.

The factor σ is called the leakage factor of the transformer, and, when known, enables the performance of the transformer to be predicted throughout its entire range by means of the circle diagram, since, the position of A being determined, the magnitude and phase of the primary current are known, and the true input will be the product of the terminal voltage, the current OA and the cosine of the angle VOA.

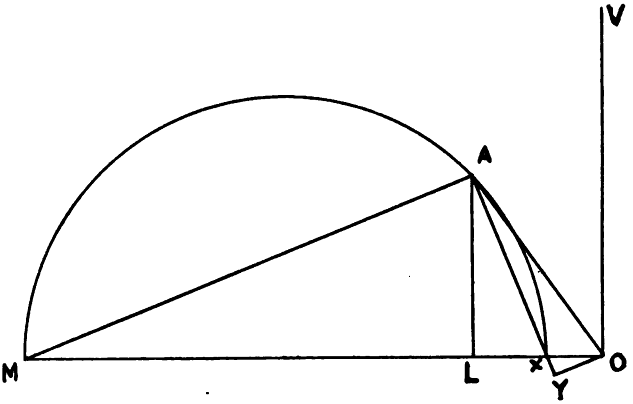

FIG. 3. Circle diagram for static transformer with magnetic leakage.

Diagram of the transformer with magnetic leakage when there is an additional E.M.F. in the secondary circuit. A diagram similar to that in figure 2 can be constructed when there is in the secondary circuit an E.M.F. in addition to those due to leakage and to resistance drop. As before let OA (figure 4) represent the primary current and let OE represent the additional E.M.F. in the secondary. If, then, the magnetising current has an extra component which will produce a field sufficient to generate in the secondary an E.M.F. equal and opposite to OE, the rest of the diagram will remain unaltered. In figure 4 OA represents the primary current; OE the additional E.M.F. in the secondary; O*e* the magnetising current required to overcome OE, lagging 90° behind OE; AC the secondary current; CY and Y*e* magnetising currents required to overcome the secondary leakage E.M.F. and the secondary resistance drop; O*v*′ the voltage in the primary due to the main field; *v′v* the primary leakage E.M.F.; and OV, equal and opposite to O*v*, the applied voltage.

Application to the induction motor*. In its simplest form an induction motor consists of a primary and a secondary winding, one of which is wound on a rotatable member. The primary winding is supplied with alternating current from an external source, and current is induced in the secondary winding. If by any means a second magnetic field is produced in phase with the secondary current and situated so that the secondary conductors lie in that field, a torque will be produced due to the interaction of the secondary current and the subsidiary magnetic field. In a polyphase induction motor this field is produced by a second phase of the supply or as a resultant of two other phases of the supply.

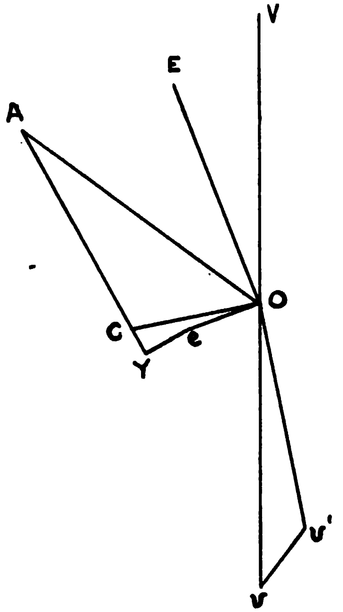

FIG. 4. Vector diagram of static transformer with magnetic leakage and an additional E.M.F. in the secondary circuit.

Thus a three-phase induction motor has three phases in its primary winding, and to maintain a continuous rotation the secondary is likewise wound with three phases. For some purposes the secondary consists of a completely short circuited winding in the shape of a squirrel cage; but such motors are not adaptable for traction purposes.

---

* The following theory of the induction motor is necessarily somewhat condensed, and assumes some slight preliminary acquaintance with such motors. Students who do not possess this knowledge should refer to an elementary text-book.

Each phase can now be treated as a transformer with an additional E.M.F. in its secondary. In this case the additional E.M.F. is proportional to the speed of the motor and to the strength of the field in which the secondary is rotating. If the motor be provided with equal windings evenly spaced the two fields will be equal in magnitude and will differ in phase by 90°; that is to say the two fields together will form a rotating field. The speed of rotation is that due to the frequency of the supply, and when the secondary is rotating at the same rate it is said to be running at the "synchronous speed." When running at any other speed the difference between that speed and the "synchronous speed" is called the slip, which may have any value positive or negative.

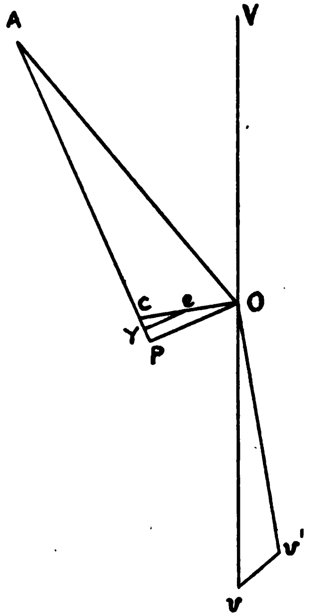

FIG. 5. Vector diagram for the polyphase induction motor.

In applying the diagram in figure 4 it must be noticed that, on account of the rotation of the secondary, the frequency of the secondary currents is not the frequency of the supply, but of the slip. Thus, if the slip be 5 per cent., and the supply frequency be 50, the secondary frequency will be only 2·5.

In figure 5 O*e* is the magnetising current necessary to overcome the E.M.F. due to the rotation of the secondary, the rest of the diagram remaining unaltered, except for the magnitude of CY.

From O draw OP perpendicular to AC; then the triangles CY*e* and CPO are similar. Now when the secondary is revolving synchronously the nett E.M.F. in the secondary circuit is nil (provided that the field is a simple rotating field), that is to say C*e* is zero. It follows, therefore, that O*e* : OC :: speed of secondary : synchronous speed, say :: *n* : N. The triangles CY*e* and CPO being similar PY : PC :: *n* : N, that is to say that since CY represents the magnetising current required to overcome the leakage E.M.F. at the frequency due to a speed N − *n*, CP represents the current that would be required at a frequency due to a speed N. Similarly OP : Y*e* :: N : N − *n*.

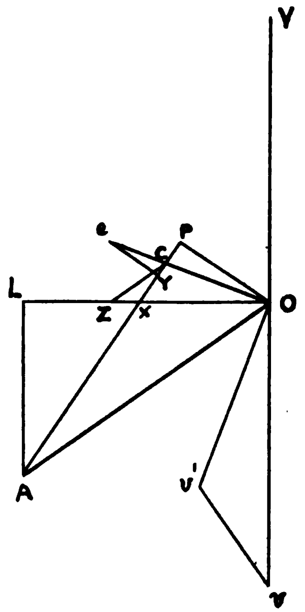

FIG. 6. Vector diagram for the polyphase induction motor when running above synchronous speed.

It follows, therefore, that the same diagram holds good for the primary and secondary currents when the motor is rotating as when it is standing at rest, provided the total resistance in the secondary when the motor is running is $\frac{N-n}{N}$ times the resistance when the motor is at rest, and carrying the same current.

Diagram for the induction motor when running above synchronism. If the speed of the motor is greater than the synchronous speed the diagram is somewhat changed. In this case (see figure 6) O*e* is greater than OC and the angle between OA and OV becomes greater than a right angle. This indicates that the machine no longer acts as a motor but as a generator, returning power to the alternating current source. The diagram for the primary and secondary currents is obtained in the same way with one exception. The leakage self-induction in the secondary circuit still makes the secondary current lag behind the E.M.F., but as the secondary is rotating at a speed which is higher than the speed of the rotating field, the lag relative to the conductors in this circuit is equivalent to a lead relative to the stationary primary circuit. In other words CY must be drawn towards A instead of away from it. With this exception the construction is precisely the same, and as before $\sigma = \lambda_1 + \lambda_2$, and the secondary resistance is $\frac{N - n}{N}$ times the resistance when the motor is at rest with the same current.

[As n in this case is greater than N, $\frac{N - n}{N}$ is a negative factor; but the resistance necessary for the same current when the motor is at rest is also a negative quantity, so that the actual resistance is a positive quantity.]

Diagram for the induction motor when running in a direction opposite to that of the field. In this case n is negative, and the resistance in the secondary circuit must be greater than the standing resistance for the same current. The diagram is precisely the same.

Influence of the distribution of the conductors on the rotating field. So far it has been assumed that the field in a three-phase motor is a uniform rotating field, that is to say that the field rotates at a uniform rate and with a constant distribution of density. In other words, it has been assumed that if the whole motor were rotated backward at synchronous speed, the field would remain stationary and its distribution in space would be constant. This assumption is only true provided the primary and secondary windings are suitably distributed.

It can be shewn that if the field is produced by three-phase sine-wave currents, the current density of which is distributed in space in accordance with a sine-law, the resulting field will fulfil the above condition. On this plan the primary would consist of three sets of windings each distributed as stated, the phases of the distribution differing by 60°. And, since the field is due to the resultant of the primary and secondary currents, the secondary windings must be on the same plan.

Such a plan is, of course, purely theoretical and quite impracticable, but the conception is valuable because it provides a method of studying other distributions by analysing them into a series of such windings to each of which the above diagrams are strictly applicable.

Analysis of the usual distribution*. The usual form of winding is a uniform distribution of the conductors of each phase in the spaces allotted to them without overlapping. Thus, in figure 7 the conductors in spaces X and X′ belong to phase 1, in Y and Y′ to phase 2, in Z and Z′ to phase 3. Each set of conductors can be analysed by Fourier's Theorem into a series of sine distribution thus

$$A_1 \sin \frac{2\pi l}{L} + A_3 \sin 3 \frac{2\pi l}{L} + A_5 \sin 5 \frac{2\pi l}{L} + \ldots,$$

where L = twice the length between the centres of X and X′.

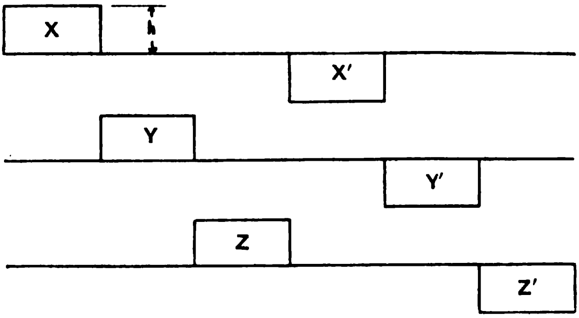

FIG. 7. Distribution of stator windings in a three-phase induction motor.

Now, if three-phase sine-wave currents be supplied to these conductors, a field will be produced which consists of

$KA_1 \sin \frac{2\pi l}{L}$ revolving uniformly at $+\frac{f}{p}$ revolutions per second,

$\frac{1}{5} KA_5 \sin 5 \frac{2\pi l}{L}$ revolving uniformly at $-\frac{1}{5}\frac{f}{p}$ revolutions per second (the negative sign indicating rotation in the opposite direction),

$\frac{1}{7} KA_7 \sin 7 \frac{2\pi l}{L}$ revolving uniformly at $+\frac{1}{7}\frac{f}{p}$ revolutions per second,

and so on

(the coefficients of $\sin 3 \frac{2\pi l}{L}$, $\sin 9 \frac{2\pi l}{L}$ and so on vanishing in the summation); where f is the frequency of the supply, p is the number of pairs of poles for which the motor is wound, and K is a constant.

The analysis, therefore, consists in replacing the uniformly distributed winding by a number of sine-distributed windings *in series*; that is to say, any current flowing in the conductors of one term of the series flows through them all.

---

* Compare Rudolf Goldschmidt, *Elektrotechnische Zeitschrift*, April 18, 1901.

By similarly analysing the secondary windings the motor is reduced to a form in which the diagrams given above are applicable.

In applying the theory it must be observed that two component windings of different pitch whether in primary or secondary have no effect on each other. Thus, if the secondary windings be analysed into

$$B_1 \sin \frac{2\pi l}{L} + B_3 \sin 3 \frac{2\pi l}{L} + \dots,$$

the component winding A_5 has no effect upon any other winding except B_5.

Starting of the three-phase induction motor. If the effect of the distribution of the windings be disregarded, an induction motor can be started with uniform torque by simply regulating the resistances which are inserted in the secondary circuits. Thus, if the speed of the motor be n and the synchronous speed N, the total resistance in each of the secondary circuits must be regulated so as to be always $\frac{N-n}{N}$ times the resistance of each secondary circuit when the motor is at rest.

Having regard, however, to the effect of the distribution of the windings, the torque must be considered as the sum of a series, each term of which represents the torque due to one of the components $B_1B_3B_5\dots$ and the rotating field of the corresponding pitch.

Thus, there will be, supposing C_1 and C_2 the primary and secondary currents,

(1) a positive torque due to C_2B_1 and the field produced by $(C_1A_1 - C_2B_1)$;

(2) a negative torque due to C_2B_5 and the field produced by $(C_1A_5 - C_2B_5)$;

(3) a positive torque due to C_2B_7 and the field produced by $(C_1A_7 - C_2B_7)$, and so on.

Torque when the motor is at rest; variation with relative position of primary and secondary. Let it be assumed that the primary and secondary currents are unaffected by the interaction of the higher component windings A_5 and B_5, A_7 and B_7. (This assumption is a close approximation to the truth.)

Then, as above, the total torque may be regarded as the algebraic sum of three separate torques. To evaluate these it may be observed that the first term is proportional to C_2B_1 and to $(C_1A_1 - C_2B_1)$. Obviously, however, the field produced by the secondary current by itself can produce no torque with the secondary conductors.

The first term may, therefore, be written, supposing the primary and secondary phase windings exactly opposite,

$$T_1 = + \lambda C_1A_1C_2B_1 \sin \theta,$$

where θ is the angle between C_1 and C_2 (angle OAC, figures 3, 4, 5, 6); in like manner $T_5 = -\frac{1}{5}\lambda C_1A_5 \times C_2B_5 \sin \theta,$

and $T_7 = +\frac{1}{7}\lambda C_1A_7 \times C_2B_7 \sin \theta.$

Now suppose the secondary turned round through an angle ψ in the direction in which the main field is rotating, then the phase of the current C_2 lags by ψ and the field produced by C_1 is also retarded in relation to the secondary conductors by the same angle ψ.

$\therefore T_1 = + \lambda C_1A_1C_2B_1 \sin (\theta + \psi - \psi)$ and is therefore unchanged.

On the other hand the phase of the field produced by C_1 in A_5 is advanced by 5ψ, and the phase of the field produced by C_1 in A_7 is retarded by 7ψ.

The total torque at an angle ψ is, therefore,

$$\lambda C_1A_1C_2B_1 \sin \theta - \lambda \tfrac{1}{5} C_1A_5C_2B_5 \sin (\theta + \psi + 5\psi)$$
$$+ \lambda \tfrac{1}{7} C_1A_7C_2B_7 \sin (\theta + \psi - 7\psi).$$

In actual windings, as will be seen later, the approximate values of A_5, A_7, B_5, B_7, are

$$\frac{A_5}{A_1} = \frac{1}{5}, \quad \frac{A_7}{A_1} = \frac{1}{7}, \quad \frac{B_5}{B_1} = \frac{1}{5}, \quad \frac{B_7}{B_1} = \frac{1}{7}.$$

Assuming these values the total torque is

$$\lambda C_1A_1C_2B_1 \{\sin \theta - \tfrac{1}{125} \sin (\theta + 6\psi) + \tfrac{1}{343} \sin (\theta - 6\psi)\}.$$

As an illustration of the effect, take the case of a motor with three slots per pole per phase on the primary and four on the secondary whose leakage factor = ·04. Let the starting resistances be so adjusted that the power factor of the primary is ·68. The value of $\sin \theta$ can be found graphically to be ·037.

The torque will be very closely

$$\lambda C_1A_1C_2B_1 \{·037 - ·014 \sin 6\psi\}.$$

This result is plotted in figure 8 for values of ψ between 0 and $\frac{\pi}{3}$, and the curve shews that between these values the torque falls to ·6 of its average value.

This effect might be, naturally, of considerable importance in electric traction, and should be kept in mind.

Torque of the motor between rest and full speed. The variation of the torque with the position of the secondary ceases to be important as soon as the motor has started. If there be dead points momentum will carry the rotating part past them.

In considering the torque when the motor is running at an intermediate speed, the total must, as before, be considered as the sum of a series. Leaving aside the effect of the secondary current in B_5 as acted upon by the field produced by A_5 in the way shewn above, a fresh conception must be introduced. A_5 and B_5 may be regarded as a separate motor whose field rotates backwards at a speed $\frac{N}{5}$, and A_7 and B_7 another motor whose field rotates forward at a speed $\frac{N}{7}$. Considered thus as three motors whose rotating parts are rigidly coupled together, it will be seen that the motor as it increases in speed departs more and more from the synchronous speed of the field due to A_5, and at first approaches and then passes and recedes from the synchronous speed of the field produced by A_7. The torque due to A_7 is therefore positive up to a speed $\frac{N}{7}$ and negative above that speed. Under certain conditions, for instance with a low resistance squirrel cage motor, this effect may be sufficient to overpower the torque due to the main field, and in that case the motor will cease to accelerate at a speed little greater than $\frac{N}{7}$. With a secondary wound with distinct phase windings the effect is not important, because the current induced in B_7 by A_7 has to flow through B_5 and B_1, these three windings being in series. And since these windings, especially B_1, are highly inductive, the current induced in B_7 is both small and a good deal out of phase with the rotating field producing it.

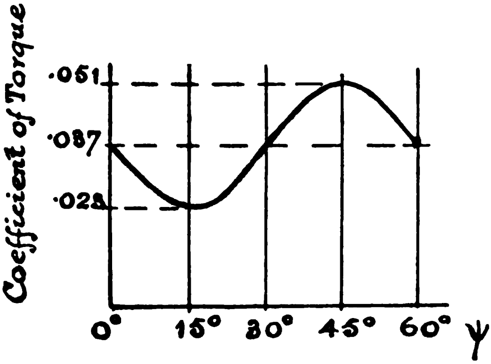

FIG. 8. Variation of starting torque for polyphase induction motor for various positions of the rotor relative to the stator.

In practice, therefore, as far as traction is concerned this effect need not be considered.

Torque of the motor at full speed as affected by the distribution of the windings. In both of the above cases, viz. when the motor is at rest and when it is running at an intermediate speed, the rotating fields of higher orders than the fundamental, whether they affect the torque or not, are the cause of extra E.M.F.s in the primary and secondary windings. When the motor is at rest, these E.M.F.s are difficult to estimate correctly; but it may be taken as very approximately true that in both these cases and also in the case when the motor is running at full speed, these E.M.F.s are proportional and at right angles to the currents producing them. Thus the E.M.F. in A_5 will be proportional to the current and to

$$\frac{1}{5}A_5 \times A_5 \times \frac{N}{5},$$

the maximum density being proportional to $\frac{1}{5}A_5$, the maximum of the winding being A_5, and the speed of the revolving field being $\frac{N}{5}$. Similarly in A_7 the E.M.F. is proportional to the current and to

$$\frac{1}{7}A_7 \times A_7 \times \frac{N}{7}.$$

Now the leakage factor $\sigma = \lambda_1 + \lambda_2$, in which λ_1 is defined as the ratio of the leakage volts to the main volts produced by a current in the primary, *i.e.* as

$$\frac{\text{leakage volts}}{\text{constant} \times A_1{}^2 N},$$

with a similar definition for λ_2.

The effect of these extra rotating fields will, therefore, be to increase

$$\lambda_1 \text{ by } \frac{(\frac{1}{5}A_5)^2}{A_1{}^2} + \frac{(\frac{1}{7}A_7)^2}{A_1{}^2},$$

and

$$\lambda_2 \text{ by } \frac{(\frac{1}{5}B_5)^2}{B_1{}^2} + \frac{(\frac{1}{7}B_7)^2}{B_1{}^2}.$$

A table is given below of the values of $\frac{A_5}{A_1}$ etc. for different methods of distribution.

The effect of overlapping phases. From the above reasoning it is obvious that any distribution which approximates more closely to the pure sine distribution will minimise the effects due to the rotating fields of higher orders. The simplest and most practicable distribution is, of course, the uniform distribution. This can, however, be improved upon by making the phases overlap as in figure 9*. The following table shews the effect of different distributions, the slots being supposed to be practically closed.

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* See British patent 5064 of 1898, also *Jour. Inst. Elec. Eng.* Part 139, Vol. 28.

This table gives the necessary relations between A_1, A_5 and A_7 and shews that the various increments that must be added to λ_1 and λ_2 are very small.

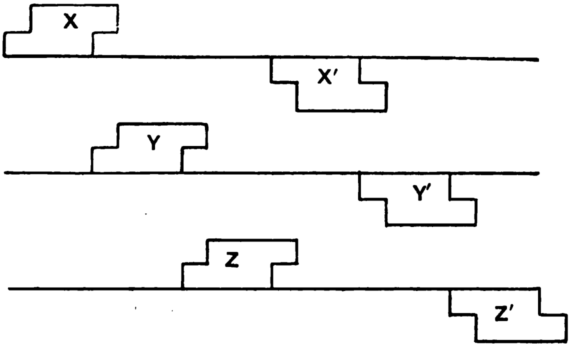

FIG. 9. Distribution of stator winding in a three-phase induction motor with staggered phases.

TABLE 1. *Distribution harmonics for a three-phase induction motor**.

Slots per pole per phase	Distribution	A_1	$5A_5$	$7A_7$	$\left\{\frac{1}{5^2}\left(\frac{A_5}{A_1}\right)^2+\frac{1}{7^2}\left(\frac{A_7}{A_1}\right)^2\right\}$
3	uniform	1·006	1·13	1·32	·28 per cent.
4	„	1	1·068	1·155	·24 „
5	„	1	1·04	1·094	·22 „
6	„	1	1·03	1·02	·21 „
infinity	„	1	1·00	1·00	·2 „
4	3 full, 2 half	·996	·854	·704	·14 „
5	4 full, 2 half	·998	·906	·814	·16 „
5	3 full, 4 half	·984	·523	·113	·04 „
6	5 full, 2 half	·998	·937	·873	·17 „
6	4 full, 4 half	·988	·664	·366	·075 „

The coefficients are related to the actual windings in the following way. Let the density of the winding where it is uniform be h conductors per unit length of the periphery; then the maximum densities of the component windings are

$$\frac{2h}{\pi}A_1;\quad \frac{2h}{\pi}A_5;\quad \frac{2h}{\pi}A_7.$$

---

* The coefficients in the table only apply to motors in which the slots are nearly or completely closed.

The combined effect of three sine-wave currents differing in phase by $\frac{\pi}{3}$, flowing in the windings of a three-phase induction motor, can be calculated by addition; the coefficients of the resulting series are

$$\frac{3}{2}\frac{2h}{\pi}\mathrm{A}_1\mathrm{C};\quad \frac{3}{2}\frac{2h}{\pi}\mathrm{A}_5\mathrm{C};\quad \frac{3}{2}\frac{2h}{\pi}\mathrm{A}_7\mathrm{C},$$

where C is the maximum current per phase,

or $$3\sqrt{2}\frac{h}{\pi}\mathrm{A}_1\mathrm{C}_0;\quad 3\sqrt{2}\frac{h}{\pi}\mathrm{A}_5\mathrm{C}_0;\quad 3\sqrt{2}\frac{h}{\pi}\mathrm{A}_7\mathrm{C}_0,$$

where C_0 is the R.M.S. current per phase.

Cascade connection of polyphase induction motors. From the foregoing it will be seen that at any other speed than what may be called "full speed" it is necessary to insert resistances in the external circuits of the secondary windings. In practice the "full speed" of motors with wound secondaries (*i.e.* excluding squirrel cage motors) is of the order of 5 per cent. below synchronous speed.

This fact means that it is only near synchronism that the efficiency of the motor is not greatly reduced below its normal value by the losses in the external resistances. In electric traction this is, naturally, a great drawback, more particularly in that class of traction in which the trains are constantly starting and stopping. In such cases the motors are working at their maximum efficiency for only a very small fraction of the total time, and, moreover, the average efficiency from start to full speed is below 50 per cent.

A system somewhat resembling the series parallel method in direct current working has been used or proposed in several cases of three-phase railways, notably on the Valtellina Railway in the north of Italy, equipped by Messrs Ganz & Co.

This method consists in employing two motors as a single unit, the primary of the first or "main" motor being connected to the line, its secondary being connected to the primary of the second or "auxiliary" motor, while the secondary winding of the "auxiliary" motor is in circuit with adjustable resistances. This is called the "cascade" connection.

Synchronous speed of motors in cascade. The frequency of the current supplied to the auxiliary motor will depend on the speed of the main motor. Suppose the frequency of the main supply to be f. Let the number of poles in the main motor be p and in the auxiliary motor p'.

Then the synchronous speed of the main motor is $f\Big/\frac{p}{2}$. At a speed of N revolutions per second (the motors being supposed to be rigidly coupled together) the frequency of the supply to the auxiliary motor is

$$f - \mathrm{N} \cdot \frac{p}{2}.$$

The synchronous speed of the auxiliary motor with this frequency is

$$\frac{f - \mathrm{N}\frac{p}{2}}{\frac{p'}{2}}$$

which must be equal to N for the synchronous speed of the combination.

Therefore $$\frac{f - \mathrm{N}\frac{p}{2}}{\frac{p'}{2}} = \mathrm{N}, \text{ or } f = \mathrm{N}\left(\frac{p}{2} + \frac{p'}{2}\right);$$

i.e. the synchronous speed of the combination is the same as that of a motor which has $p + p'$ poles.

For example, if the two motors have four poles each, and the frequency of the main supply is 25, the synchronous speed of each motor is 750 revs. per min., and of the two in cascade ½ × 750 or 375.

Diagram for induction motors in cascade at a speed below the synchronous speed of the combination. In a general way it may be said that the diagram for the auxiliary motor is that already given for the induction motor, for although the voltage of the supply decreases as the motors accelerate, the frequency decreases at practically the same rate, so that the value of the magnetising current remains unaltered, and therefore, also, the diameter of the semicircle. Strictly speaking, however, the voltage of the supply is not independent of the currents in the main motor; the total drop in the main motor must be considered.

For any definite value of the currents the voltage applied to the auxiliary motor will be that due to the rotating field minus the drop in the secondary of the main motor. The voltage due to the rotating field decreases uniformly as the speed rises, and the leakage component of the drop decreases similarly, being proportional to the frequency; the resistance component, however, remains constant.

For practical purposes it will be sufficiently accurate to disregard this variation for speeds below that of cascade synchronism. That is to say, it will be assumed that provided the auxiliary motor is so regulated as to keep the current constant while the speed rises, the voltage at its terminals will vary in direct proportion to the frequency.

For any particular value of the current, then, the diagram for the "main" motor will differ from the standard diagram in that the secondary current is not in phase with the nett voltage in the secondary, as is the case when there are non-inductive external resistances, but lags by an angle ϕ_0 where $\cos\phi_0$ is the power factor of the auxiliary motor for that particular current.

A convenient graphical construction for cascade motors can be obtained as follows. (The construction is slightly approximate, as it assumes that the resistance drop in the secondary of the main motor is in phase with the voltage applied to the auxiliary motor.)

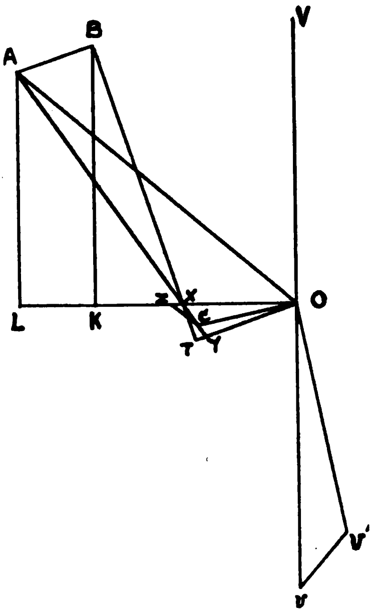

FIG. 10. Vector diagram for two polyphase induction motors connected in cascade.

In figure 10 the diagram is constructed similar to that in figure 2, but with OY making an angle $\frac{\pi}{2} + \phi_0$ with YA. Through X draw BXT perpendicular to OY, *i.e.* making an angle ϕ_0 with AXY, and draw AB perpendicular to BX, and BK perpendicular to OX.

Then as before $\sin \mathsf{XBK} = \sin \mathsf{XOT} = \frac{\mathsf{XT}}{\mathsf{XO}}$;

now $\mathsf{XT} : \mathsf{XB} :: \mathsf{XY} : \mathsf{XA}$.

$$\therefore \sin \mathsf{XBK} = \frac{\mathsf{XB}\,\frac{\mathsf{XY}}{\mathsf{XA}}}{\mathsf{XO}} = \frac{\mathsf{XB}}{\mathsf{XO}} \cdot \frac{\mathsf{XC}+\mathsf{CY}}{\mathsf{XA}} = \frac{\mathsf{XB}}{\mathsf{XO}}(\lambda_1 + \lambda_2 + \lambda_1\lambda_2).$$

That is to say, that locus of B is the characteristic semicircle for the main motor.

To find the primary current of the main motor, therefore, construct the ordinary circle diagram on $\text{XM} = \frac{\text{XO}}{\sigma}$ as diameter, and set off XB (figure 11) equal in length to C cos $\phi_0 (1 - \lambda_1)$ where C is the secondary current. Join BM and set off along it BA = C sin $\phi_0 (1 - \lambda_1)$. Then OA will represent in magnitude and phase the primary current.

This method holds good for the two motors at rest, and it will be fairly obvious that, within the limits of error imposed by disregarding the variability of the drop as the speed rises, it holds equally good when the motors are running, provided, as already mentioned, that the auxiliary motor is regulated so as to keep the current constant.

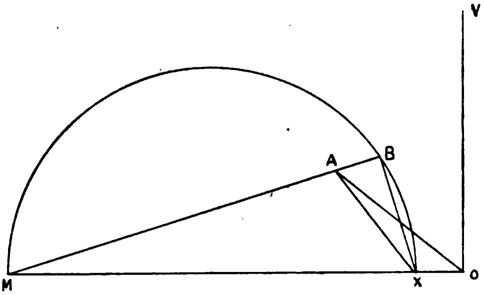

FIG. 11. Circle diagram for polyphase induction motors connected in cascade.

Diagram for two motors in cascade above the synchronous speed of the combination. When the speed is between the synchronous speed of cascade working and the synchronous speed of direct working, both motors are working as generators and supplying energy instead of absorbing it.

In this case as the auxiliary motor is working above synchronism, the angle ϕ_0 is something less than two right angles. The same method, however, is applicable, the only difference being that cos ϕ_0 is now a negative quantity, and BX is drawn below XM instead of above it (see figure 12). As before, the same construction holds good for speeds considerably above that of cascade synchronism, provided that the resistance drops in the secondary of the main motor and the primary of the auxiliary motor are neglected; that is to say, for a speed n which is greater than the cascade synchronous speed N, the total secondary resistance of the auxiliary motor must be

$$\frac{(\text{N} - n)\frac{p}{p'} - n}{\text{N}}$$

times the resistance when the motors are at rest with the same current.

In this case, however, greater attention must be paid to the neglected resistance drops, as they assume more and more importance as the speed approaches the synchronous speed of the main motor.

Two motors in cascade above the synchronous speed of the main motor. When the speed of the combination exceeds the synchronous speed of the main motor both machines are again acting as motors. This condition, however, is of no practical value, as there is no second cascade synchronous speed, that is to say, the combination is not efficient.

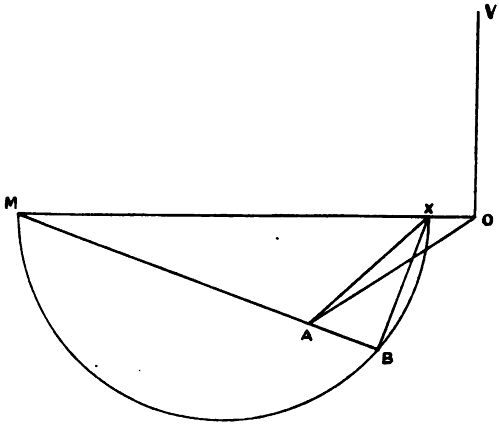

FIG. 12. Circle diagram for polyphase motors in cascade running above cascade synchronous speed.

Importance of the power factor of the auxiliary motor. From the above it will be evident, and it will be brought out more clearly in designing a motor for cascade running, that the power factor of the auxiliary motor is of very great importance. For satisfactory working it is essential that this power factor should be kept as high as possible.

Before applying the above principles to the calculation of an actual motor, a few preliminary considerations are necessary.

In the first place it must be remarked that throughout the above general discussion it has been assumed that the primary and secondary windings are similar, or in other words, that the ratio of transformation is unity. In practice this is very seldom the case, and proper allowance must be made for this when calculations are entered upon.

Effect of primary resistance. So far no account has been taken of the effect of the drop in volts due to the primary resistance. This effect in any practical case is small and the correction is easily applied to the diagrams. The correction, when made, leaves the power factor practically unaltered over a fairly wide range of working; but it must be taken into account in calculating the total drop in the primary and also in working out the efficiency.

Calculation of the speed. In figure 5, OP is proportional to the resistance drop in the secondary at standstill. In the circle described on XM as diameter $\mathrm{OP} = \sigma \mathrm{MA}$. Consequently MA is proportional to the resistance drop at standstill. It has been shewn already that the currents are precisely the same when the motor is running with a total secondary resistance R_0 as when it is at rest with a total secondary resistance $\mathrm{R} + \mathrm{R}_0$ where

$$\frac{\mathrm{R}_0}{\mathrm{R} + \mathrm{R}_0} = \frac{\mathrm{N} - n}{\mathrm{N}}. \qquad \text{(See p. 7.)}$$

If then a point A_0 be found on the circle such that OA_0 represents the current at standstill with total secondary resistance R_0, the slip corresponding to the current OA when the motor is running with the same secondary resistance is

$$\text{slip} = \frac{\mathrm{N} - n}{\mathrm{N}} = \frac{\mathrm{MA}_0}{\frac{\mathrm{XA}_0}{1 - \lambda_1}} \div \frac{\mathrm{MA}}{\frac{\mathrm{XA}}{1 - \lambda_1}} = \frac{\mu_2}{\sigma} \times \frac{\frac{\mathrm{XA}}{1 - \lambda_1}}{\mathrm{MA}},$$

where μ_2 is a ratio similar to λ_2 in terms of the secondary resistance instead of the leakage self-induction.

Calculation of the internal E.M.F. of the motor. Neglecting the voltage drop due to the primary resistance, the E.M.F. in each of the primary windings when the secondary circuits are open is $(1 + \lambda_1)$ times the E.M.F. generated in the component winding A_1.

Suppose first that the reluctance of the iron in the magnetic circuit is negligible; then the induction in the air gap will be proportional to the ampere turns and inversely proportional to the air gap length.

Now the three currents in the three windings are equivalent to $\frac{3}{2}$ times the current in a single winding revolving at synchronous speed (see p. 15). If, therefore, the maximum current per phase be $\mathrm{C}\sqrt{2}$ amperes, the total ampere conductors per pole due to the three windings will be

$$\frac{3}{2}\mathrm{C}\sqrt{2}\int_0^l \frac{2h}{\pi}\mathrm{A}_1 \sin\frac{\pi x}{l}\,dx$$

$$= \mathrm{C}\sqrt{2}\,.\,\frac{3h}{\pi}\,.\,2\mathrm{A}_1\frac{l}{\pi} = \mathrm{C}\sqrt{2}\,\frac{6h\mathrm{A}_1}{\pi^2}\,l,$$

where l = pole pitch.

The maximum induction will be

$$\frac{4\pi}{10 \times 2\delta} \times \mathrm{C}\sqrt{2} \cdot \frac{6h\mathrm{A}_1}{\pi^2}\, l = \Delta \text{ say}, = \frac{3{\cdot}4\mathrm{C}h\mathrm{A}_1 r}{p\delta},$$

in which r is the radius of the stator bore, δ the radial depth of the air gap, and p the number of poles.

The field due to the currents in A_1 will be distributed according to a sine law, the maximum being Δ. This field revolves synchronously, and by its rotation produces a back E.M.F. in each of the primary windings. The maximum back E.M.F. will occur when the axes of the field and of the winding coincide.

The actual value will be

$$\mathrm{V} \times b \times 10^{-8} \int_0^l \frac{2h}{\pi} \mathrm{A}_1 \sin\frac{\pi x}{l} \times \Delta \sin\frac{\pi x}{l}\, dx \text{ per pole},$$

where V = speed of rotation,

and b = breadth of air gap in an axial direction.

This equals $\mathrm{V} \,.\, b \,.\, 10^{-8} \,.\, \frac{2h}{\pi} \mathrm{A}_1 \frac{l}{2} \Delta$ per pole.

If there are p poles, r being the radius of the air gap and f the frequency,

$$\mathrm{V} = \frac{2\pi r f}{\frac{p}{2}} \text{ and } l = \frac{2\pi r}{p}.$$

The maximum E.M.F. per winding will therefore be

$$\frac{2\pi r f}{\frac{p}{2}} \times b \times 10^{-8} \times \frac{2h}{\pi} \times \mathrm{A}_1 \times \frac{2\pi r}{2p} \times \Delta \times p$$

$$= \frac{8\pi r^2 f b}{p} h\mathrm{A}_1 \Delta \times 10^{-8};$$

the R.M.S. value of this is obtained by dividing by $\sqrt{2}$ and the volts between terminals with a star connected winding by multiplying by $\sqrt{3}$. The final product multiplied by $(1 + \lambda_1)$ is equal to the terminal voltage E,

i.e. $$\mathrm{E} = (1 + \lambda_1) \sqrt{\frac{3}{2}}\, \frac{8\pi r^2 f b}{p} h\mathrm{A}_1 \Delta \times 10^{-8},$$

or $$(1 + \lambda_1) \sqrt{\frac{3}{2}}\, 4 r f h \mathrm{A}_1 \Delta \times 10^{-8}$$

multiplied by the area of the gap per pole*.

---

* If wide open slots are used, this expression should be multiplied by the effective gap area, just as in calculating the saturation curves for direct current machines.

Influence of saturation in the magnetic circuit. In three-phase induction motors which are used for continuous service saturation can very often be neglected, more particularly when the motors are not very small. When such motors are used for purposes of electric traction, if the service is intermittent, as, for instance, in the case of trains frequently starting and stopping, it is possible to keep the maximum induction in the teeth at a higher value than in the case of continuously running motors.

The maximum value of the induction in the air gap will not, therefore, depend only on the length of the air gap. The effect of the saturation will be to flatten the curve of induction at its maximum. This curve will no longer be a sine curve, and as it revolves uniformly it will no longer produce in each winding a sine wave E.M.F.

It can easily be proved that this departure from a sine wave of induction can be expressed almost exactly as the introduction of another sine wave of one-third the wave length; in other words, if Δ represent the induction

$$\Delta = \Delta_1 \sin \theta - \Delta_3 \sin 3\theta.$$

As this rotates it will induce in each winding an E.M.F. of the same frequency as that of the terminal voltage, and also an E.M.F. of three times this frequency. The three primary windings, however, are not independent in the case of a star connection but are interlinked. The terminal voltage is equal to the voltage induced in two windings in series, making due allowance for phase difference, and the triple frequency E.M.F. in any one winding is neutralised by a similar E.M.F. in either of the other windings.

Consequently the total induced voltage

$$= (1 + \lambda_1) \sqrt{\frac{3}{2}} \frac{8\pi r^2 f b}{p} h \mathsf{A}_1 \Delta_1.$$

Now Δ_1 is greater than the maximum Δ as obtained from the saturation curve by an amount which depends on the degree of saturation. This amount can be determined by analysing the wave form of the field produced by currents in A_1. This is done arithmetically from the saturation curve in precisely the same way as Fourier's analysis is carried out mathematically.

It may be taken for granted that for large traction motors, where the air gap is from 1·5 mm. to 2·0 mm. all round the rotor, Δ_1 is equal to Δ within ½ per cent., provided the ampere turns for the stator and rotor teeth at the position of maximum induction are not more than 50 per cent. of the ampere turns for the air gap at the same place.

Iron losses. The diagrams already evolved take no account of losses due to hysteresis and eddy currents in the iron of the magnetic circuit. Assuming that this loss is a constant quantity, which is unfortunately not true, it can be expressed as so many watts, or so many amperes per phase multiplied by the terminal voltage. Set off this current as an extension of OV (see figure 13), say OH. Then HA will represent the primary current instead of OA.

The estimation of the iron loss in any particular case is a matter of considerable difficulty. It is well known that the loss increases rapidly as the motor is loaded, and this increase can be attributed to several causes, among which will be the stray fields due to the primary and secondary currents. The high periodicity rotating fields will also be responsible for an increase depending on the primary and secondary currents. The rotating fields due to B_5 and B_7 will have an effect on the iron loss in the primary, and the fields due to A_5 and A_7 will have an effect on the iron loss in the secondary. The frequency of the alternations in the secondary iron due to fields produced by A_5 and A_7, when the secondary is rotating at synchronous speed, is six times the frequency of the supply. On the other hand, the primary loss due to the stray field produced by A_1 is very much greater than the secondary loss due to the stray field produced by B_1 since the frequency of the latter field is very low.

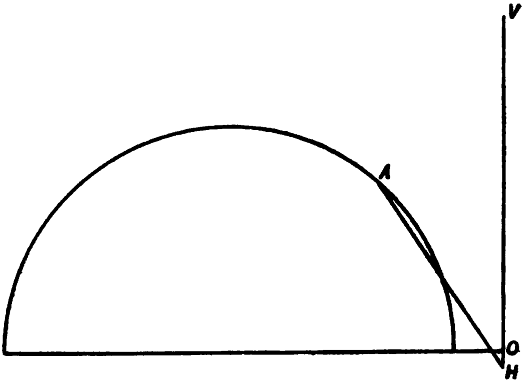

FIG. 13. Circle diagram of polyphase induction motor with iron loss.

A rough idea can be obtained of the importance of the secondary iron loss due to A_5 and A_7. Suppose the primary current to be, say, six times the magnetising current, then the maximum inductions will be

$$6 \times \frac{\frac{1}{5}A_5}{A_1} \text{ and } 6 \times \frac{\frac{1}{7}A_7}{A_1} \text{ respectively,}$$

of the maximum induction in the main field. If then the iron loss be supposed to be proportional to the 1·5th power of the maximum induction and to the frequency, the additional loss due to this cause will be at synchronous speed

$$6 \times \left(6 \times \frac{\frac{1}{5}A_5}{A_1}\right)^{1{\cdot}5} + 6 \times \left(6 \times \frac{\frac{1}{7}A_7}{A_1}\right)^{1{\cdot}5}$$

times the iron loss in the secondary when at rest with no current.

This expression, assuming

$$A_5 = \tfrac{1}{5}A_1 \text{ and } A_7 = \tfrac{1}{7}A_1,$$

becomes $$6 \times \left(\tfrac{6}{25}\right)^{1{\cdot}5} + 6 \times \left(\tfrac{6}{49}\right)^{1{\cdot}5},$$

or $$= {\cdot}66 + {\cdot}44 = 1{\cdot}1.$$

These two fields will, therefore, produce a loss greater than the open circuit iron loss in the secondary at rest.

Similarly the primary iron loss will be approximately increased by

$$5 \times \left(6 \times \frac{\frac{1}{5}B_5}{A_1}\right)^{1{\cdot}5} + 7\left(6 \times \frac{\frac{1}{7}B_7}{A_1}\right)^{1{\cdot}5},$$

the frequencies being five times and seven times the fundamental frequency respectively, or, assuming

$$B_5 = A_5,\ B_7 = A_7 \text{ and } B_1 = A_1,$$

$${\cdot}55 + {\cdot}51 = 1{\cdot}06.$$

Very roughly, therefore, the rotating fields of high periodicity when the primary or secondary current is six times the magnetising current suffice to treble the iron loss of the motor running light.

The additional loss due to primary stray field is very difficult to estimate as it will depend very largely on the construction of the motor. Some allowance, however, must be made for it.

If the above method of estimating the iron loss be correct the additional iron loss on load will be considerably reduced by winding the primary and secondary circuits with overlapping phases, as shewn in table 1 on page 14, in which the various distribution harmonics are given.

The leakage factor. The question of leakage in an induction motor is of great importance, more particularly in traction motors. A small leakage factor implies a high value for the maximum power factor, and a large overload capacity. For intermittent railway service the latter is essential, and the successful designing of a three-phase traction motor depends very largely on the leakage factor being kept small. Still more important is this factor when motors are connected in cascade, for the power factor of the combination is much lower than the power factor of either motor.

As illustrating the importance of the leakage factor the following points may be mentioned:

(1) The maximum power factor $= \frac{1}{1+2\sigma}$.

(This relation can easily be proved for the case when the primary current is tangential to the characteristic circle.)

(2) The maximum value of the true watts that can be supplied to the motor is proportional to the radius of the semicircle, the true watts varying with AL (figure 3), *i.e.* with the primary current multiplied by $\cos\phi$.

Much has been written on this subject, and many formulae evolved which give good results for commercial motors.

For instance, Behrend* suggests

$$\sigma = \mathrm{C}\frac{\delta}{\tau},$$

where C is constant, δ the radial length of the air gap in centimetres, and τ the pole pitch in centimetres, C varying from 10 to 15 according to circumstances.

Behn-Eschenburg† has evolved a more complete formula, viz.

$$\sigma = \frac{3}{\mathrm{N}^2} + \frac{\delta}{\mathrm{X} \times \mathrm{N} \times \tau} + \frac{6\delta}{b},$$

where N is the mean between the slots per pole on the stator and the slots per pole on the rotor,

X is the width of the opening at the top of the slot,

τ the pole pitch in cms.

δ the radial length of the air gap,

and b the net axial length of the laminations.

The latter formula is certainly more comprehensive than the former and is based upon a large number of tests. It is, however, necessary to examine the class of motor on which the tests were carried out, before applying the results to traction motors.

The distinguishing characteristic of a three-phase traction motor is its relatively long air gap. For instance, during the Arbitration on the Electrification of the Inner Circle Herr Blathy in his evidence on behalf of Messrs Ganz & Co. stated that each motor would have a 2 mm. air gap, and that the maximum power factor would be 94 per cent. The latter statement implies a leakage factor of ·032. Taking a motor whose axial dimension is 13 inches or 33 cms. the last term in Behn Eschenburg's formula becomes

$$\frac{6 \times \cdot 2}{33} = \cdot 036.$$

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* *The Induction Motor*, Behrend, p. 36.

† *Proc. Inst. Elec. Eng.* January 15, 1904, or *Electrician*, vol. 52.

This term relates solely to the leakage at the ends of the motor round the end windings of the coils, and it is to be noticed that in the motors tested end windings are bent down over one another against the end plates. This plan obviously conduces to leakage as it provides ample space for the leakage fields of the primary and secondary circuits, and a better arrangement is undoubtedly that in which both the primary and secondary coils form a "barrel" winding at the ends, as is customary in the armatures of direct current machines.

Beyond the somewhat scanty information about the Ganz motors mentioned above there is very little that is available*, and the designer is thrown upon his own resources.

The authors suggest that the simplest way is to calculate from the dimensions of the machine and of the slots the leakage flux surrounding each limb of a coil in the same way as is done in calculating the reactance voltage in the theory of commutation. An example of the calculation is given in the application of the above general principles to the design of a railway motor.

It can be said, however, that it is important to keep the slots open (not necessarily completely so), to allow as many slots per pole per phase as is practicable, and to keep the number of poles low, or in other words to design the motor for a high speed.

Application to the calculation of performance curves. It will be advisable, now, to sum up the foregoing theory and shew how it may be applied to the calculation of the performance curves of a motor of which full data are given.

Let it be supposed that the following data for a given motor are available:

Particulars of stator stampings, air ducts, slots, etc.
Particulars of rotor stampings, air ducts, slots, etc.
Windings of stator and rotor.
Normal working voltage and frequency.
Sufficient particulars of bearing, slip rings, etc.

Then the order of procedure is as follows:

1. Calculate the values of resistance per phase and leakage reactance per phase for stator and rotor, also the two values of h.

2. Calculate the saturation curve.

This curve is to shew the relation between the maximum induction in the gap and the ampere turns per pair of poles required to produce it. To obtain the inductions in the yoke and the core, the total flux per pole must be divided by the effective section of iron. The total flux per pole will be the area of one-half of a sine wave, the maximum of which is the induction assumed, and is, therefore,

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* The induction motors on the Burgdorf-Thun Railway are 8 pole motors for 40 frequency, apparently with closed slots on rotor and stator. Maximum power factor about ·8. See *Engineering*, July 27, 1900.

$$\frac{2}{\pi} \Delta \times \text{effective area of gap per pole.}$$

3. Calculate the data for constructing the characteristic semicircle. In doing so it is advisable to perform the calculation for more than one definite voltage, say, for instance, for the normal working voltage and for 5 per cent. above and below.

Now in the foregoing theory it has been assumed that the magnetising current is proportional to the flux per pole or to the maximum induction. This, of course, is not true unless the magnetomotive force for the iron is negligible. In fact, due to this magnetomotive force, there is not any definite semicircle for a given voltage.

The simplest way to make an allowance for this is to draw two semicircles, one, which corresponds to the state of saturation at open circuit, and another, which corresponds to the saturation at full load. The first is obtained in the ordinary way, and the second may be constructed by finding the magnetising current required to produce the main field which actually exists in the motor, and assuming that the open circuit magnetising current is proportional to this in the ratio of the open circuit field to the full load field.

The procedure is therefore as follows: Find first the full load current per phase; next find the internal voltage V_0 on open circuit = terminal voltage multiplied by $1 - \lambda_1$ (assuming a probable value for λ_1). Work out also the internal voltage on full load V_1 by deducting the primary drop from the terminal voltage. Assume a probable value of the full load power factor, $= \cos \phi$; then the primary drop can easily be shewn to be approximately

$$[\text{resistance drop} \times \cos \phi + \text{leakage drop} \times \sin \phi].$$

For each of these two voltages V_0 and V_1 find the corresponding value of the maximum induction in the gap; these values will follow from the formula

$$\mathsf{E} = \sqrt{\frac{3}{2}}\, 4rfh\mathsf{A}_1 \Delta 10^{-8} \times \text{effective gap area per pole (p. 21).}$$

For each of these inductions read off from the saturation curve the ampere turns per pair of poles, and hence find the magnetising currents using the formula

$$\left.\begin{array}{r}\text{ampere conductors per pole} \\ \text{or ampere turns per pair of poles}\end{array}\right\} = \mathsf{C}\sqrt{2}\,\frac{6h\mathsf{A}_1 l}{\pi^2} \text{ (p. 20).}$$

The magnetising current for open circuit condition needs no modification; that for the full load condition must be increased in the ratio $\frac{V_0}{V_1}$. The two magnetising currents so found may be used to construct two semicircles, between which the actual locus of the point A (figure 3) will lie.

To construct these semicircles the leakage factor must be found. This is done as already explained by dividing the leakage reactance volts by the main volts for the same current and adding the correction in table 1, p. 14. The two quantities λ_1 and λ_2 so found for the stator and rotor when added together give the value of σ, the leakage factor. If C_0 be the magnetising current C_0/σ will be the diameter of the semicircle, and $\frac{1}{2}C_0/\sigma + C_0$ will be the distance from the point O to the centre of the circle.

4. For the purpose of estimating the slip at any load the value of μ_2 should be found expressed in terms of the primary winding; *i.e.* if the ratio be $\frac{C_0 r_2 \sqrt{3}}{V_0}$ for the rotor winding this must be multiplied by the square of the ratio of the stator and rotor turns or by $\left(\frac{h_1 A_1}{h_2 B_1}\right)^2$. As shewn on page 20 the slip can be found graphically, being proportional to $\frac{\mu_2}{\sigma}$.

5. Work out the iron loss in watts.

6. Estimate the bearing friction and windage loss.

7. The performance curves can then be calculated by assuming different currents, and measuring off the power factor from the semicircle diagram. From the same diagram scale off the rotor currents corresponding to the assumed stator currents (the secondary current is equal to $\frac{XA}{1-\lambda_1}$ (figure 3) multiplied by the ratio of the primary to the secondary turns). Knowing the primary and secondary currents and the power factors the rest of the calculations are made without difficulty.

[In the example given below three terminal voltages are taken, 2850, 3000, and 3150 volts respectively, and it so happens that the internal voltage on open circuit for 2850 volts is practically the same as the internal voltage on full load with 3000 volts, and so on. Thus only four calculations as shewn are sufficient for three terminal voltages. The performance curves as a motor are calculated for a voltage 5 per cent. below normal and for a generator 5 per cent. above, as these are probable voltages in actual working.]

PART 2.

PRACTICAL DESIGN AND EXAMPLES OF ACTUAL MOTORS.

Specification No. 1.

Three-phase induction motor for railway work.

Particulars. Primary voltage 3000 volts.

Frequency 25.

One hour rating 250 H.P. at 730 R.P.M.

Temperature rise for one hour rating (shop test) 75° C.

	Stator	Rotor
External diameter	766 mm.	500 mm.
Internal diameter	504 „	232 „
Number of poles	4	4
Number of slots	60	72
Slot depth	50 mm.	45 mm.
Slot width	16·4 „	9·6 „
Slot opening	2 „	2 „
Gross length of laminations	400 „	400 „
Air ducts	3—10 mm.	3—10 mm.
Winding		
Conductors per slot	25	4
Size of conductor, bare	4·0 mm. diam.	3·3 mm. × 18 mm.
„ „ insulated	4·35 „ „	3·8 „ × 18·5 „
Distribution	uniform	4 slots full 4 half full
End connections	coils as in figure 14	diamond shaped*

Slip rings

Three slip rings each 250 mm. diameter and 35 mm. wide.
On each ring two carbon brushes (low resistance quality).
Cross-section of each brush 30 mm. × 30 mm.

The sketch in figure 14 shews the general arrangement of the stator windings, and the conductors in the stator and rotor slots.

Calculations. (For notes as to the calculations of the performance curves see pages 26 to 28).

Stator Windings, resistance and leakage.

Mean length of one end connection, turned up type 84 cm.

Mean length of one end connection, straight type 73 cm.

Average 78·5 cm.

Length of mean turn $2(40+78{\cdot}5)=237$ cms.

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* In this design it is assumed that diamond shaped end-connections can be used with slots practically closed. This is not the universal practice among manufacturers; but the authors believe that it is employed by Messrs Siemens Brothers Dynamo Works at Stafford and possibly by others.

Total length per phase = 250 × 2·37 metres = 593 metres.

Resistance per phase (hot) = $\frac{593}{55 \times 12{\cdot}57} \times 1{\cdot}2 =$ **1·03 ohms.**

Total weight of stator copper = 3 × 59300 × ·1257 × 9 × $\frac{1}{1000}$ = 201 kgms.

Leakage. Volts with 1 ampere per phase.

1. Across the slot internally.

   Maximum induction $\frac{4\pi \,.\, 25}{16{\cdot}4} = 19$.

   Total flux per slot $\frac{1}{2} \times 19 \times 4{\cdot}5 \times 37 = 1580$ C.G.S. lines.
   Effective flux per slot = $\frac{2}{3} \times 1580 = 1050$ C.G.S. lines.
   Leakage E.M.F. per phase = $500 \times 2\pi \times 25 \times 1050 \times 10^{-8} =$ **·825 volt.**

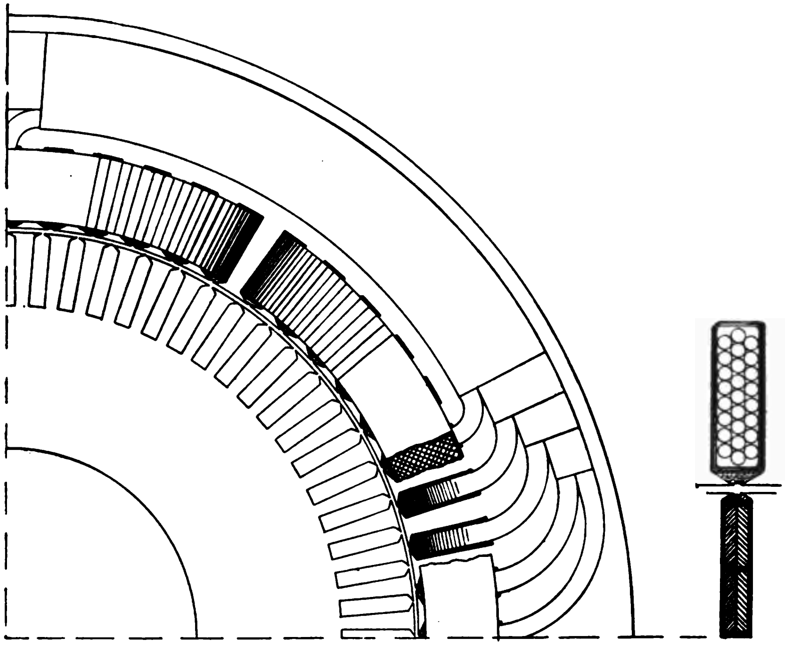

FIG. 14. Arrangement of stator windings and conductors in stator and rotor slots for 250 H.P. three-phase railway motor.

2. Across the top of slot.

   Suppose equivalent to 3 mm. broad and 5 mm. long.

   Flux per slot = $\frac{4\pi \,.\, 25}{5} \times {\cdot}3 \times 37 = 700$ C.G.S. lines.

   Leakage E.M.F. per phase = $500 \times 2\pi \times 25 \times 700 \times 10^{-8} =$ **·55 volt.**

3. Across the air gap through top of opposing tooth.

   For average case take path 5 mm. wide, 2 mm. long, in series with 14 mm. wide, 2 mm. long.

   Flux per slot = $\frac{4\pi \,.\, 25}{10} \left(\frac{1}{\frac{2}{5} + \frac{2}{14}} \right) 37 = 2140$ C.G.S. lines.
   Leakage E.M.F. per phase = $500 \times 2\pi \times 25 \times 2140 \times 10^{-8} =$ **1·68 volts.**

4. Round the end connections, *i.e.* round 125 conductors.

   a. Turned up type. Suppose path equivalent to 6 cm. wide and 30 cm. long.

   $$\text{Total flux per end connection} = \frac{4\pi \,.\, 125}{300} \times 6 \times 84 = 2640.$$

   b. Straight type. Suppose path equivalent to 1 cm. wide and 7·5 cm. long in series with 8 cm. wide and 22·5 cm. long.

   $$\text{Total flux} = \frac{4\pi \,.\, 125}{10} \left(\frac{1}{\frac{7 \cdot 5}{1} + \frac{22 \cdot 5}{8}} \right) 37 = 570.$$

   Average of the two types $= \frac{1}{2}(2640 + 570) = 1605$ C.G.S. lines.

   Leakage E.M.F. per phase $= 500 \times 2\pi \times 25 \times 1605 \times 10^{-8} = 1 \cdot 26$ **volts.**

   v_1 = total leakage volts per phase $= \cdot 825 + \cdot 55 + 1 \cdot 68 + 1 \cdot 26 = 4 \cdot 32$.

Rotor Windings, resistance and leakage.

Mean length of one end connection = 50 cm.

Length mean turn ≐ 2 (40 + 50) = 180 cm.

Total length per phase = 48 × 1·8 metres = 86·5 metres.

$$\text{Resistance per phase (hot)} = \frac{86 \cdot 5}{55 \times 59 \cdot 5} \times 1 \cdot 2 = \cdot 0317 \text{ ohm.}$$

Total weight of copper $= 3 \times 8650 \times \cdot 595 \times 9 \times \frac{1}{1000} = 139$ kgms.

Leakage. Volts with 1 ampere per phase.

1. Across the slot internally.

   $$\text{Maximum induction} = \frac{4\pi \,.\, 4}{9 \cdot 6} = 5 \cdot 24.$$

   Total flux per slot $= \frac{1}{2} \times 5 \cdot 24 \times 4 \times 37 = 388$ C.G.S. lines.

   Effective flux per slot $= \frac{2}{3} \times 388 = 258$ C.G.S. lines.

   Leakage E.M.F. per phase $= 96 \times 2\pi \times 25 \times 258 \times 10^{-8} = \cdot 039$ **volt.**

2. Across the top of slot. Same as for stator.

   $$\frac{4\pi \,.\, 4}{5} \times \cdot 3 \times 37 = 111 \text{ C.G.S. lines.}$$

   Leakage E.M.F. per phase $= 96 \times 2\pi \times 25 \times 111 \times 10^{-8} = \cdot 0168$ **volt.**

3. Across the air gap through top of opposing teeth.

   For average case take 7 mm. wide and 2 mm. long in series with 12 mm. wide and 2 mm. long.

   $$\text{Flux per slot} = \frac{4\pi \,.\, 4}{10} \left(\frac{1}{\frac{2}{7} + \frac{2}{12}} \right) 37 = 410 \text{ C.G.S. lines.}$$

   Leakage E.M.F. per phase $= 96 \times 2\pi \times 25 \times 410 \times 10^{-8} = \cdot 062$ **volt.**

4. Round the end connections, *i.e.* round 4 × 6 conductors.

Suppose average path equivalent to 5·4 cm. long and 4 cm. wide in series with 22 cm. long and 8 wide and insert a factor of ⅔ to allow for the presence of the other phases.

$$\text{Flux per end connection} = \frac{4\pi \,.\, 4 \,.\, 6}{10} \left(\frac{1}{\frac{5 \cdot 4}{4} + \frac{22}{8}} \right) 50 = 366 \text{ lines.}$$

Leakage E.M.F. per phase $= 96 \times 2\pi \times 25 \times \frac{2}{3} \times 366 \times 10^{-8} = \cdot 0368$ **volt.**

Total leakage E.M.F. per phase $= \cdot 039 + \cdot 0168 + \cdot 062 + \cdot 0368 = \cdot 155$ **volt.**

Slip Rings and Brushes.

Brush cross-section per ring = 18 sq. cm.

Take resistance per square centimetre ·075 ohm.

$$\text{Brush resistance per ring } \frac{\cdot 075}{18} = \cdot 00416 \text{ ohm.}$$

r_2 = secondary resistance per phase $= \cdot 0317 + \cdot 0042 = \cdot 036$ **ohm.**

Brush friction.

Assume pressure per sq. cm. = ·1 kgm.

Coefficient of friction = ·3.

$$\text{Peripheral speed at 730 r.p.m.} = \frac{\pi \times 38 \times 730}{60} = 1450 \text{ cm. per sec.}$$

$$= 14{\cdot}5 \text{ metres per sec.}$$

$$\text{Friction loss for 3 rings} = 3 \times 18 \times {\cdot}1 \times {\cdot}3 \times 14{\cdot}5 \times 9{\cdot}81$$

$$= 230 \text{ watts.}$$

Summing up:

Primary. r_1 = resistance per phase = 1·03 ohm.

v_1 = leakage volts per ampere = 4·32 volts.

$$h_1 = \frac{125}{\dfrac{\pi \times 50{\cdot}2}{12}} = 9{\cdot}53.$$

$A_1 = 1.$

$A_5 = {\cdot}208.$

$A_7 = {\cdot}156.$

Secondary. r_2 = resistance per phase (total) = ·036 ohm.

v_2 = leakage volts per ampere = ·155 volts.

$$h_2 = \frac{24}{\dfrac{\pi \times 50{\cdot}2}{12}} = 1{\cdot}83.$$

$B_1 = {\cdot}988.$

$B_5 = {\cdot}133.$

$B_7 = {\cdot}0523.$

Saturation curve. Data.

	Air gap	Stator teeth	Rotor teeth	Yoke	Core	
Effective section per pole	1455	600	560	540	600	sq. cms.
Length per pair of poles	·4	10	9	say 50	say 33	cms.
Total volume	—	12700	10900	53500	30400	c.cms.
Total weight	—	99	85	417	237	kgms.

Choosing different values of the maximum induction in the gap, the ampere turns for the various parts of the magnetic circuit can be calculated as follows:

Max. induction			Flux per pole $\times 10^6$	Induction		Ampere turns					
Gap	Stator teeth	Rotor teeth		Yoke	Core	Gap	Stator teeth	Rotor teeth	Yoke	Core	Total
3000	7250	7800	2·77	5120	4600	956	13	12	55	33	1070
5000	12100	13000	4·61	8520	7660	1590	40	48	85	50	1810
6000	14500	15500	5·54	10250	9220	1910	116	152	120	63	2350
7000	16900	18100	6·46	12000	10800	2220	360	592	200	97	3470
8000	19300	20700	7·39	13600	12300	2540	1500	2320	610	210	7180

	Primary current			Open circuit current
	45	45	45	
V = terminal volts	2850	3000	3150	3150
V_0 = terminal volts $(1-\lambda_1)$	2780	2920	3060	3060
V_1 = internal volts, *i.e.* terminal volts − primary drop	2640	2790	2940	3060
Δ_1 $(=V_1/\sqrt{\tfrac{3}{2}}\,4rfhA_1 10^{-8} \times$ gap area) ...	6140	6500	6850	7100
C_1 = magnetising current for Δ_1 ...	·7·5	8·5	10	11·75
C_0 = proportional open circuit current	7·9	8·9	10·4	11·75
$\dfrac{v_1\sqrt{3}C_1}{V_1}$	2·11 °/o			
Correction for distribution	·22 °/o			
λ_1	2·33 °/o	2·33 °/o	2·33 °/o	2·33 °/o
$\dfrac{v_2\sqrt{3}C_1\left(\dfrac{h_1A_1}{h_2B_1}\right)^2}{V_1}$	2·03 °/o			
Correction for distribution	·07 °/o			
λ_2	2·1 °/o	2·1 °/o	2·1 °/o	2·1 °/o
$\sigma = \lambda_1 + \lambda_2$	4·43 °/o	4·43 °/o	4·43 °/o	4·43 °/o
C_0/σ	178	200	235	264
$\tfrac{1}{2}C_0/\sigma + C_0$	97	109	128	144
$\mu_2 = \dfrac{C_1 r_2\sqrt{3}}{V_1} \times \left(\dfrac{h_1A_1}{h_2B_1}\right)^2$	·0049	·0052	·0058	·0066
μ_2/σ	·11	·12	·13	·15

From these figures four circles can be drawn, the essential parts of these being shewn in figure 17, numbered 1, 2, 3, and 4 respectively.

Iron loss with 45 amperes primary current per phase about 730 r.p.m.

Take an average terminal voltage of 3000 volts.

Stator yoke. Maximum induction = 11000 C.G.S. lines per sq. cm.

Watts per kgm. at 25 frequency = 1·6.

Total watts = 417 × 1·6 = **665 watts.**

Stator teeth. Maximum effective density = 15500.

Watts per kgm. at 25 frequency = 2·8.

Total watts = 2·8 × 99 = **275.**

Sum = 665 + 275 = **940 watts.**

Additional iron loss due to higher rotating fields (rough approximation).

$$\frac{\text{primary current}}{\text{magnetising current}} = \frac{45}{8{\cdot}46} = 5{\cdot}3.$$

$$\left\{5 \times \left(5{\cdot}3 \times \tfrac{1}{5}\frac{A_5}{A_1}\right)^{1{\cdot}5} + 7 \times \left(5{\cdot}3 \times \tfrac{1}{7}\frac{A_7}{A_1}\right)^{1{\cdot}5}\right\} \times 940$$

$$= \{{\cdot}5 + {\cdot}28\}\ 940 = \mathbf{730\ watts.}$$

$$\left\{6 \times \left(5{\cdot}3 \times \tfrac{1}{5}\frac{B_5}{B_1}\right)^{1{\cdot}5} + 6 \times \left(5{\cdot}3 \times \tfrac{1}{7}\frac{B_7}{B_1}\right)^{1{\cdot}5}\right\} 940$$

$$= \{{\cdot}32 + {\cdot}05\} \times 940 = \mathbf{350\ watts.}$$

Allow for eddy currents in end plates say 500 watts.

Total = 940 + 730 + 350 + 500 = **2520 watts.**

2520 watts are equivalent to a primary current per phase of

$$\frac{2520}{3000\sqrt{3}} = \mathbf{{\cdot}485\ amperes.}$$

Bearing friction and windage: allow say **2500 watts.**

Performance curves.

Motor at 2850 volts, 25 frequency.

Primary current ...	8·5	15	25	35	45	55	65
Secondary current ...	0	62·6	119	175	227	280	332
Primary C²R	220	700	1930	3780	6250	9350	13000
Secondary C²R	0	420	1530	3320	5570	8500	11900
Estimated iron loss ...	940	1200	1450	1850	2520	3400	4500
Friction loss	2730	2730	2730	2730	2730	2730	2730
Total losses	3900	5050	7650	11680	17070	23980	32130
Cos ϕ		·8	·9	·92	·917	·91	·89
Input, kw.	1·16	59·1	111	159	204	248	286
Output, kw.	– 2·73	54·05	103·3	147·3	186·9	224	253·9
H.P.	– 3·65	72	138	197	250	300	339
Efficiency, °/ₒ		91·8	93·2	92·8	91·6	90·4	88·7
Slip, °/ₒ	0	·74	1·4	2·07	2·75	3·5	4·2
Torque in lbs. at 1 ft. radius	– 26	510	990	1420	1810	2180	2500

These curves are shewn plotted in figure 15.

Generator at 3150 volts, 25 frequency.

Primary current	11·75	15	25	35	45	55	65
Secondary current ...	0	49	115	172	224	278	330
Primary C²R		700	1930	3780	6250	9350	13000
Secondary C²R		260	1430	3200	5400	8350	11800
Iron loss		1350	1500	2000	2700	3600	4700
Friction loss		2730	2730	2730	2730	2730	2730
Total losses		5040	7690	11710	17080	24030	32230
Cos ϕ		·6	·84	·885	·913	·916	·92
Output, kw.		49	115	169	224	275	326
Input, kw.	2·73	54	122·7	180·7	241·1	299	358·2
H.P.	3·65	72·3	164	242	323	400	480
Efficiency, °/ₒ		93·5	93·9	93·7	93	92	91
Slip, °/ₒ		·5	1·25	1·85	2·4	3·0	3·7
Torque in lbs. at 1 ft. radius	26	506	1140	1670	2220	2730	3250

These curves are shewn plotted in figure 16.

Notes on the design. In making preliminary calculations it is useful to know approximately the open circuit current. This can be estimated in the following way.

Suppose C = the primary current corresponding to the maximum power factor, and σ the estimated leakage factor; then, the open circuit current being C_0, the radius of the semicircle is $\frac{1}{2}\mathrm{C}_0/\sigma$.

$$\therefore\ \{\tfrac{1}{2}\mathrm{C}_0/\sigma + \mathrm{C}_0\}^2 - \mathrm{C}^2 = \{\tfrac{1}{2}\mathrm{C}_0/\sigma\}^2,$$

or

$$\mathrm{C}\frac{2\sigma}{\sqrt{(1+2\sigma)^2-1}} = \mathrm{C}_0.$$

In this case if $\mathrm{C} = 45$ and $\sigma = \cdot 045$,

$$\mathrm{C}_0 = \frac{\cdot 09 \times 45}{\cdot 423} = \mathbf{9\cdot 36\ amperes.}$$

The next stage in the preliminary calculations is to choose the diameter and nett length of the rotor, and from them calculate h and Δ_1, thus

$$\Delta_1 = \frac{3\cdot 4 h \mathrm{A}_1 r}{p\delta}\mathrm{C}_0,$$

and

$$\mathrm{E} = (1+\lambda_1)\sqrt{\frac{3}{2}} \times \frac{8\pi r^2 f b}{p} h \mathrm{A}_1 \Delta_1 10^{-8}.$$

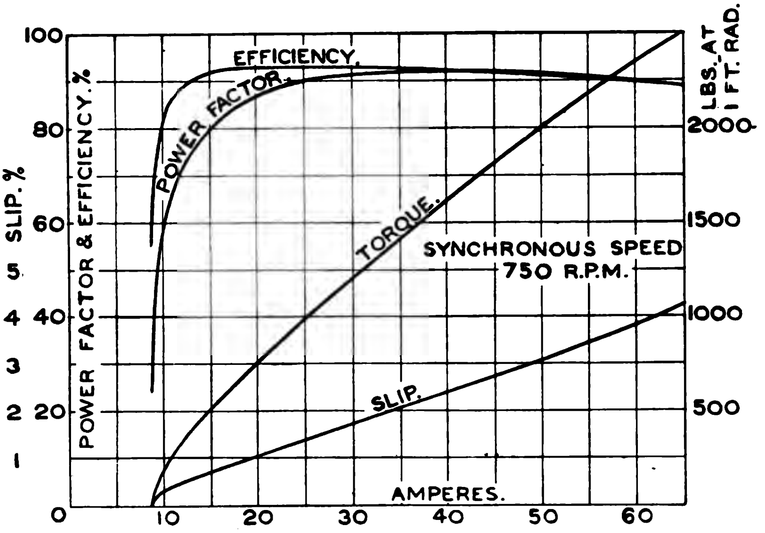

FIG. 15. Performance curves for 250 H.P. three-phase railway motor. 2850 volts, 25 frequency.

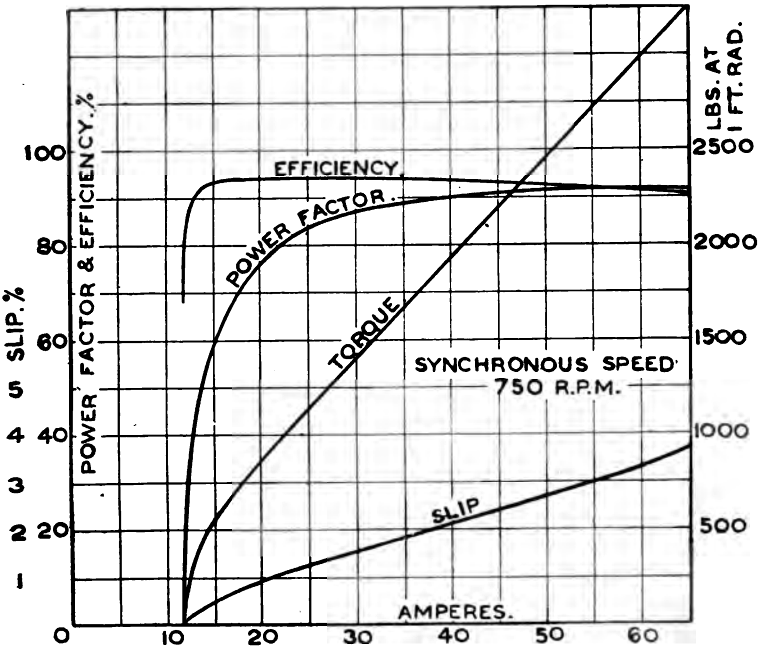

FIG. 16. Performance curves of 250 H.P. three-phase railway motor as generator at 3150 volts, 25 frequency.

In the first equation allow say 20 per cent. of the magnetising current for the iron and take $C_0 = {\cdot}8 \times 9{\cdot}6$ amps.

From these equations assuming $A_1 = 1$, $p = 4$, $\delta = {\cdot}2$, $r = 25$, $f = 25$, $\lambda_1 = {\cdot}02$ and $b = 37$,

$$\frac{\Delta_1}{h} = 816 \text{ and } \Delta_1 h = 66600,$$

whence $$h = 9{\cdot}04 \text{ and } \Delta_1 = 7400,$$

which are approximately the values taken.

Saturation of the teeth. The maximum of the fundamental sine wave is quite appreciably less than the maximum total induction. The higher rotating fields, however, which are superimposed on the main field do not travel with it, but at $\frac{1}{5}$ and $\frac{1}{7}$ of the synchronous speed. These fields are like ripples of fairly considerable amplitude passing across the surface of the fundamental wave and they do not *on the average* produce any appreciable effect on the reluctance of the teeth.

Induction in the yoke and the core. In the same way these higher rotating fields do not practically affect the main reluctance of the yoke and the core.

In both cases, however, the iron losses are increased by the presence of these fields, as already indicated.

The effect of saturation on the leakage factor. This effect is twofold. In the first place saturation increases the ratio of the magnetising current to the main E.M.F. produced thereby. This would tend to raise the leakage factor. On the other hand saturation of the teeth tends to suppress the leakage field, and consequently diminish the leakage factor.

The calculation of the former effect is included in the calculation of the saturation curve; the estimation of the latter effect is extremely difficult. In the above design it has been assumed that the two effects neutralise each other and that the leakage factor remains constant within the limits taken.

Capacity of the motor. This motor would work for an hour with about 45 amperes, 2850 volts, and continuously with an effective working current of 22 to 24 amperes (R.M.S. current) with an average iron loss due to the motor being at work for about 40 or 50 per cent. of its time. This is naturally a somewhat rough estimate but corresponds approximately with the capacity that would be obtained by the application of the rules given in Chapter 2, Vol. I.; the proportions and distribution of the C^2R losses and the iron losses are so different in this case that only an approximate estimate can be formed.

Calculation of an auxiliary motor to work in cascade with the 250 H.P. motor already designed. Before proceeding to the calculation of an auxiliary motor, it is necessary to obtain an expression for the terminal E.M.F. of this motor.

To find this expression the power factors of both motors must be assumed.

Let V be the terminal voltage of the main motor, C_1 the primary current, $\cos\phi$ the power factor of the main motor, and $\cos\phi_0$ the power factor of the auxiliary motor.

Then the E.M.F. induced in the secondary of the main motor by the rotating field is

$$e=\frac{N-n}{N}\times\frac{k_2B_1}{k_1A_1}\times\sqrt{(V-C_1r_1\cos\phi-C_1v_1\sin\phi)^2+(C_1v_1\cos\phi-C_1r_1\sin\phi)^2},$$

and if C_2 be the secondary current, the terminal voltage of the auxiliary motor is $\left(\text{putting } v_2'=\frac{N-n}{N}v_2\right)$

$$=\sqrt{e^2-(C_2v_2'\cos\phi_0-C_2r_2\sin\phi_0)^2}-C_2v_2'\sin\phi_0-C_2r_2\cos\phi_0.$$

This gives the terminal volts of the auxiliary motor, and if instead of v_2' and r_2, which apply only to the secondary winding of the main motor, $v_2'+v_0'$ and r_2+r_0 be substituted to include the corresponding values for the primary of the auxiliary motor, the above expression gives the primary back E.M.F. of this motor, *i.e.* the terminal voltage corrected for primary drop.

It is evident, therefore, that the whole calculation depends upon a correct estimate being formed of the two power factors. This, of course, can only be done by practice obtained by trial and error.

In applying these expressions to an actual calculation the value of e may be taken as

$$\frac{N-n}{N}\times\frac{k_2B_1}{k_1A_1}(V-C_1r_1\cos\phi-C_1v_1\sin\phi);$$

in other words it is sufficient to regard $C_1r_1\cos\phi+C_1v_1\sin\phi$ as the primary drop.

Similarly the terminal voltage of the auxiliary motor for speeds near the cascade synchronous speed may be taken as

$$e-C_2v_2'\sin\phi_0-C_2r_2\cos\phi_0,$$

in other words it is sufficient to regard $C_2v_2'\sin\phi_0+C_2r_2\cos\phi_0$ as the secondary drop.

As already remarked, each point in the curve for cascade working has to be obtained by a process of trial and error. This process will be very much facilitated if the two diagrams are plotted to suitable scales.

If the "main" motor diagram be drawn with a certain scale for its primary circuits, another scale will be required for measuring the secondary currents. The length AX represents the secondary current multiplied by $(1-\lambda_1)$. But by adjusting the scale this length can be made to represent the actual current. In that case the ratio between the scales for the primary and secondary currents will be

$$\frac{h_1 A_1}{h_2 B_1} \times (1-\lambda_1).$$

If now this same scale be used for the primary currents of the auxiliary motor, compass measurements can be transferred with great facility from one diagram to the other.

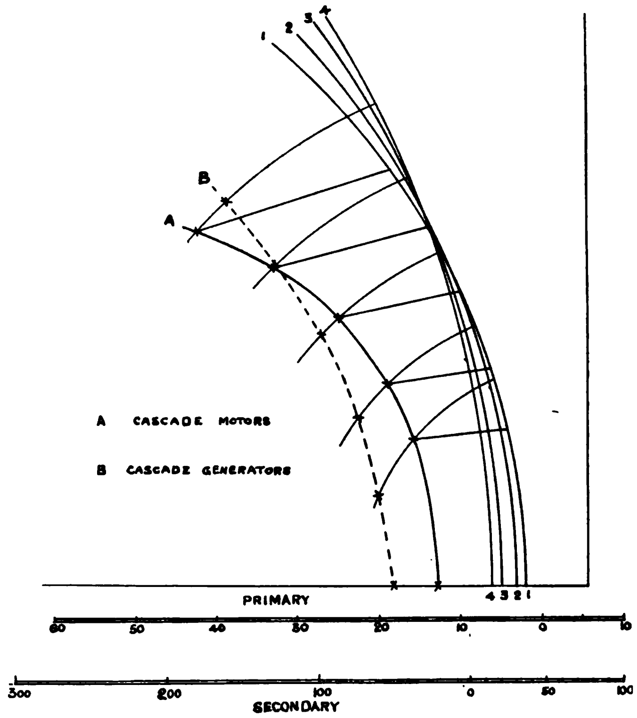

Fig. 17. Circle diagrams for Main Motor. Induction motors in cascade.

The method is therefore as follows (see figures 17 and 18):— Choose the primary current for the main motor; on the main diagram, figure 17, set off an arc of a circle with this current as radius; on this circle select some point as a first trial. From this point find the main secondary current, which is also the auxiliary primary current. With this current by means of the auxiliary diagram, figure 18, find the wattless current of the auxiliary motor, and set off this current on the main diagram along the line from the far end of the diameter through the point selected. If this length, so set off, reaches to the characteristic semicircle of the main motor, the point selected is correct; otherwise a fresh trial must be made. The auxiliary wattless current will of course depend on the auxiliary terminal voltage, and this must be worked out for each trial as shewn above.

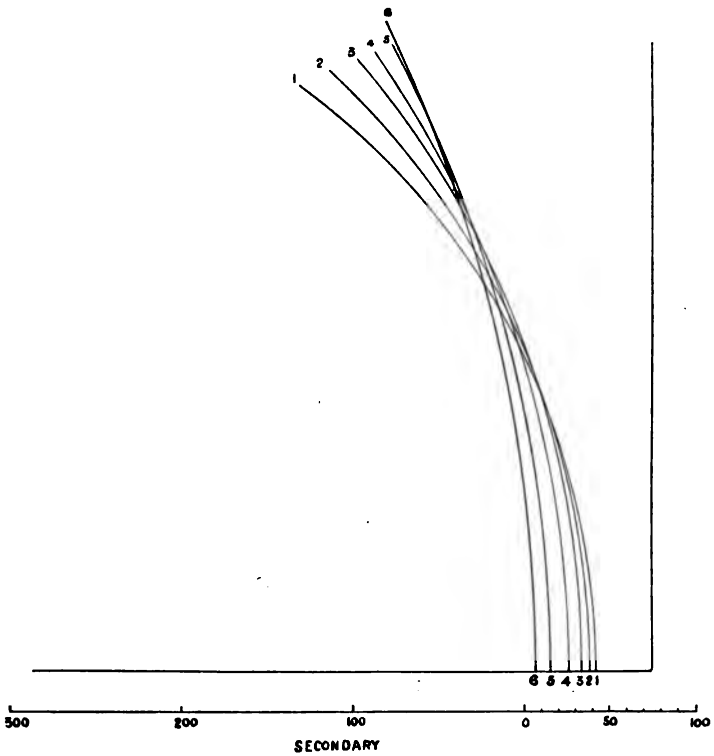

FIG. 18. Circle diagrams for Auxiliary Motor. Induction motors in cascade.

In designing the auxiliary motor two points are to be noticed. In the first place, it must be regarded in its proper relation to the main motor so far as its capacity is concerned. In all the traction systems existing at present in which the cascade connection is employed, the auxiliary motor is never connected directly to the line; its only function is to receive the energy transmitted through the main motor. Its capacity is, therefore, not identical with that of the main motor, and will in general be appreciably less, in consideration of the fact that the main motor is working for part of the time during which the auxiliary motor is out of action. In other words the auxiliary motor works more intermittently than the main motor, and advantage can be taken of this by reducing somewhat the section of the copper.

In the second place the range of voltage applied to the auxiliary motor is greater than that applied to the main motor. Taking into account the probable drop in the line as was done in finding the performance curves of the main motor, the variation is considerably increased by the reversal of the internal drop in the main motor when the two machines are working as generators in cascade. This variation of voltage will be seen on examining the calculations for the performance curves given below; but it is mentioned here so that the designer may take account of it in preliminary calculations. It is wise, therefore, to proportion the magnetic circuit with a view to the maximum terminal voltage which will occur when the machines are working as generators in cascade.

Specification No. 2.

Three-phase auxiliary motor for use in cascade connection with a 250 H.P. *motor.*

	Stator	Rotor
External diameter	736 mm.	500 mm.
Internal diameter	504 „	270 „
Number of poles	4	4
Number of slots	60	72
Slot depth	36 mm.	45 mm.
Slot width	14·3 „	8·1 „
Slot opening	2 „	2 „
Gross length of laminations	400 „	400 „
Air ducts	2—10 „	2—10 „
Winding.		
Conductors per slot	4	4
Size of conductors, bare	5·6 mm. × 12 mm.	2·5 mm. × 18 mm.
Size of conductors, insulated	6·1 mm. × 12·5 mm.	3·0 mm. × 18·5 mm.
Distribution	3 slots full 4 slots half full	4 slots full 4 slots half full
End connections	diamond-shaped	diamond-shaped

Slip rings and brushes.

Three slip rings each 250 mm. diameter and 35 mm. wide, on each ring two carbon brushes (soft quality), cross section of each brush 30 mm. × 30 mm.

Stator particulars.

Resistance per phase $= r_1 = \cdot 025$ ohm.

Leakage volts per ampere $= v_1 = \cdot 05$ (at 12·5 frequency).

$h_1 = \frac{20}{13 \cdot 1} = 1 \cdot 527.$

$A_1 = \cdot 984.$

$A_5 = \cdot 105.$

$A_7 = \cdot 016.$

$h_1 A_1 = 1 \cdot 5.$

Rotor particulars.

Resistance per phase $= r_2 = \cdot 044$ ohm.

Leakage volts per ampere $= v_2 = \cdot 083$ (at 12·5 frequency).

$h_2 = \frac{24}{13 \cdot 1} = 1 \cdot 83.$

$B_1 = \cdot 988.$

$B_5 = \cdot 133.$

$B_7 = \cdot 052.$

$h_2 B_1 = 1 \cdot 8.$

Magnetic circuit.

	Air gap	Stator teeth	Rotor teeth	Yoke	Core
Cross-section per pole	1500	684	684	550	480
Effective length per pair of poles	·4	7	9	say 25	say 14
Total volume, c.c.		13300	13000	56000	25600
Total weight, kgms.		104	101	440	200

Saturation curve.

Gap density	Tooth density	Flux per pole $\times 10^6$	Yoke density	Core density	Ampere turns per pair of poles				
					Gap	Teeth	Yoke	Core	Total
4000	8760	3·83	7000	8000	1270	27	32	22	1350
6000	13100	5·75	10500	12000	1910	100	62	56	2130
7000	15300	6·7	12200	14000	2230	272	108	130	2740
8000	17500	7·66	13900	16000	2540	800	225	320	3880
9000	19700	8·62	15800	18000	2870	3000	540	980	7400

Characteristic diagrams.

Primary current					260	330
Cos ϕ...					$-\cdot 92$	$-\cdot 92$
V = terminal voltage	190	210	230	250	270	280
$V_0 = V(1 - \lambda_1)$	186	206	225	245	265	274
V_1 = terminal voltage − primary drop					272	282·5
Δ_1					7750	8050
Magnetising current for $\Delta_1 \left(= \frac{AT}{50} \right) = C_1$					68	77
Proportional open circuit current $= C_0$	36	41	47	54	66·3	74·6

$\frac{C_1 v_1 \sqrt{3}}{V_1}$	1·68 %					
Correction for distribution	·04 %					
λ_1	1·72 %					
$\frac{C_1 v_2 \sqrt{3}}{V_1} \times \left(\frac{h_1 A_1}{h_2 B_1}\right)^2$	1·93 %					
Correction for distribution	·07 %					
λ_2	2·00 %					
$\sigma = \lambda_1 + \lambda_2$, %	3·72	3·72	3·72	3·72	3·72	3·72
$\frac{C_0}{\sigma}$	970	1100	1260	1450	1780	2000
$\frac{1}{2}\frac{C_0}{\sigma} + C_0$	521	591	677	779	956	1075
$\frac{C_1 r_2 \sqrt{3}}{V_1} \times \left(\frac{h_1 A_1}{h_2 B_1}\right)^2 = \mu_2$	·01	·0105	·011	·0116	·013	·0143
$\frac{\mu_2}{\sigma}$	·27	·28	·3	·31	·35	·38
$\frac{C_0}{\sigma} \times \frac{h_1 A_1}{h_2 B_1}$	810	873	1050	1200	1480	1660

NOTE. For the first four columns it is unnecessary to calculate the values of V_1 as the saturation curve for that range does not depart appreciably from a straight line.

Performance curves of the two motors in cascade at 2850 volts

Main primary volts	2850					
,, ,, current $= C_1$	18·5	28	35	45	55	65
$C_1 \cos \phi$	0	18·1	25	33	39	44
$C_1 \sin \phi$	18·5	21·4	24·5	30·6	38·5	47·5
Drop $= C_1 \cos \phi \times 1·8 + C_1 \sin \phi \times 7·5$	140	200	230	290	360	435
Main primary internal volts $= V_1$	2710	2650	2620	2560	2490	2415
,, sec. int. volts $= \frac{V_1}{10·54}$	257	252	248	243	236	229
,, sec. current Aux. primary current $\} = C_2$	54	118	159	213	267	320
,, ,, watt current $= C_2 \cos \phi_0$	0	98	146	198	248	290
,, ,, wattless current $= C_2 \sin \phi_0$	54	58	63	79	102	133
Main secondary drop $= C_2 \cos \phi_0 \times ·063 + C_2 \sin \phi_0 \times ·134$	7·3	14	18·5	24	31	38
Aux. primary terminal voltage	250	238	230	219	205	191
Aux. sec. current	0	90	130	178	223	270
S = slip below speed of auxiliary field, %	0	2·2	3·4	5·4	7·4	9·6
$\frac{S}{2}$ = slip below cascade synchronism, %	0	1·1	1·7	2·7	3·7	4·8
Speed	375	371	369	364	361	357
Main primary C^2R	1060	2410	3780	6250	9350	13000
,, secondary C^2R	310	1500	2730	4900	7700	11000
,, iron loss	1200	1500	1900	2600	3400	4500
,, friction	1000	1000	1000	1000	1000	1000
Aux. primary C^2R	220	1040	1900	3400	4200	7700
,, secondary C^2R	0	1060	2220	4200	6600	9600
,, iron loss	400	500	600	700	850	1140
,, friction	1000	1000	1000	1000	1000	1000
Total losses	5200	10000	15130	24050	34100	48900
Input, kw.		89·5	123	164	194	219
Output, kw.	– 5·2	79·5	108	140	160	170
Combined efficiency, %	—	89	87·6	85·4	82·5	77·6
Cos ϕ	0	·645	·715	·735	·713	·68
Combined B.H.P.	– 7	107	144	187	214	228
Combined torque, lbs. at 1 ft. radius	– 99	1520	2060	2710	3120	3380

These curves are shewn plotted in figure 19.

Performance curves of the two machines as generators in cascade at 3150 volts.

Main primary volts	3150					
,, ,, current $= C_1$	24	28	35	45	55	65
$C_1 \cos\phi$	0	−11·1	−20·6	−31	−39·5	−47·6
$C_1 \sin\phi$	24	25·4	28	32·2	38	44·7
Drop $= C_1 \cos\phi \times 1{\cdot}8 + C_1 \sin\phi \times 7{\cdot}5$	180	170	177	190	215	250
Main primary internal volts $= V_1$	2970	2980	2970	2960	2935	2900
,, secondary internal volts $= \frac{V_1}{10{\cdot}54}$	282	283	282	281	278	275
,, secondary current $= C_2$ ...	68	100	146	203	257	312
Aux. primary watt current $= C_2 \cos\phi_0$	0	−64	−120	−181	−238	−290
,, ,, wattless current $= C_2 \sin\phi_0$	68	76	82	92	104	120
Main secondary drop $= C_2 \cos\phi_0 \times {\cdot}063 + C_2 \sin\phi_0 \times {\cdot}134$	9	6	3·5	1	−1	−2
Aux. primary terminal voltage ...	273	277	278·5	280	279	277
,, secondary current	0	58	108	162	210	260
$S =$ slip above speed of aux. field, %		1·3	2·5	3·7	4·8	6
$\frac{S}{2} =$ slip above cascade synchronism, %		·65	1·25	1·85	2·4	3
Speed	375	377	380	382	384	386
Main primary C^2R	1770	2410	3780	6250	9350	13000
,, secondary C^2R	500	1080	2300	4450	7100	10500
,, iron loss	1300	1600	2000	2700	3400	4500
,, friction	1000	1000	1000	1000	1000	1000
Aux. primary C^2R	350	750	1600	3100	5000	7300
,, secondary C^2R	0	440	1540	3500	5800	8800
,, iron loss	500	600	700	800	900	1100
,, friction	1000	1000	1000	1000	1000	1000
Total losses	6420	8880	13920	22800	33500	47200
Output, kw.	—	60·6	112·5	169	216	260
Input, kw.	+6·4	69·5	126·4	191·8	249	307
Combined efficiency, %	—	87	89	88	86·6	85
Cos ϕ	0	·4	·59	·69	·72	·73
Input, combined B.H.P.	8·6	93	169	256	333	410
Combined torque, lbs. at 1 ft. radius	120	1300	2350	3540	4580	5600

These curves are shewn plotted in figure 20.

Two speed induction motors for traction purposes. Quite recently an alternative arrangement to the cascade connection has been brought out by Messrs Brown, Boveri & Co., and has been put into use on the Simplon tunnel railway. This arrangement consists in winding the motor in such a way that there are two alternative numbers of poles, that is to say, there are two possible synchronous speeds.

The only method of producing this effect that has been so far applied to electric traction is to divide the windings on rotor and stator in such a way that the number of poles can be changed by altering the connection between the various parts of the windings.

The simplest arrangement is to use two alternative speeds, one of which is twice the other, as for this condition the connections for the windings are not very complicated.

Consider a motor which is wound as an 8-pole motor thus:

Pole number	1	2	3	4	5	6	7	8
Polarity	N	S	N	S	N	S	N	S

If now it be desired to change the connections so that only four poles are produced the polarities will be as follows:

Pole number	1	2	3	4	5	6	7	8
Polarity for 8 poles	N	S	N	S	N	S	N	S
Polarity for 4 poles	N	N	S	S	N	N	S	S

If then there is one winding for each of the eight poles, it is necessary for the production of only four poles to reverse the direction of current in coils 2, 3, 6, 7. Wind the eight coils, therefore, in two groups 1, 4, 5, 8, and 2, 3, 6, 7; a reversing switch between the two groups will effect the necessary change of polarity.

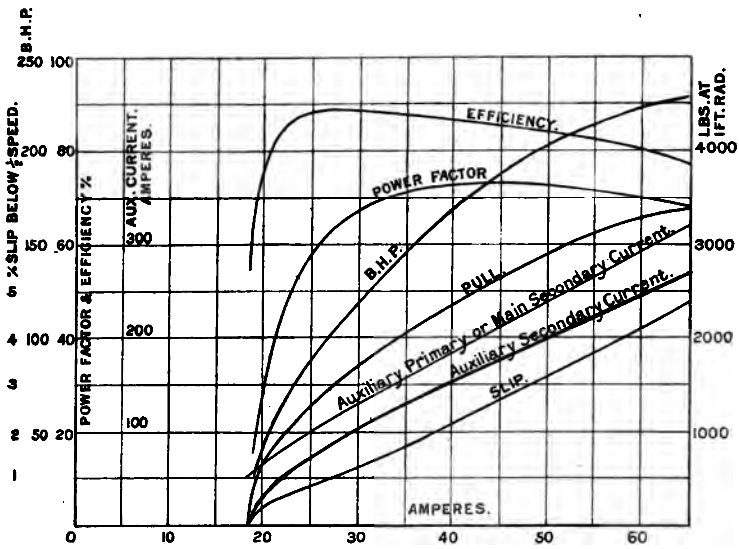

Fig. 19. Performance curves of two three-phase railway motors in cascade at 2850 volts.

In practice it is simpler, as far as the end connections are concerned, to arrange the winding so that there is only one complete coil for each alternate pole; thus, for example, an 8-pole motor would have 12 complete coils or 4 per phase, each of the four coils embracing say a North pole. Between each pair of coils would be a South pole just as if there were special windings for these poles. To change over to a 4-pole field all that is necessary is to reverse alternate coils; thus assuming eight poles as before the polarities would be as follows:

Pole number	1	2	3	4	5	6	7	8
Polarity for 8 poles	N	S	N	S	N	S	N	S
Polarity for 4 poles	N		S		N		S	

so that only coils 3 and 7 need be reversed.

An example of this type is given on pp. 55—57, and it will be seen that it is only necessary to tap the windings at six points both on the stator and on the rotor.

Three-phase direct current motor. Cases may arise in which it is convenient or necessary to design a motor suitable for working from a three-phase alternating current supply over parts of the line, and from a direct current supply over other parts.

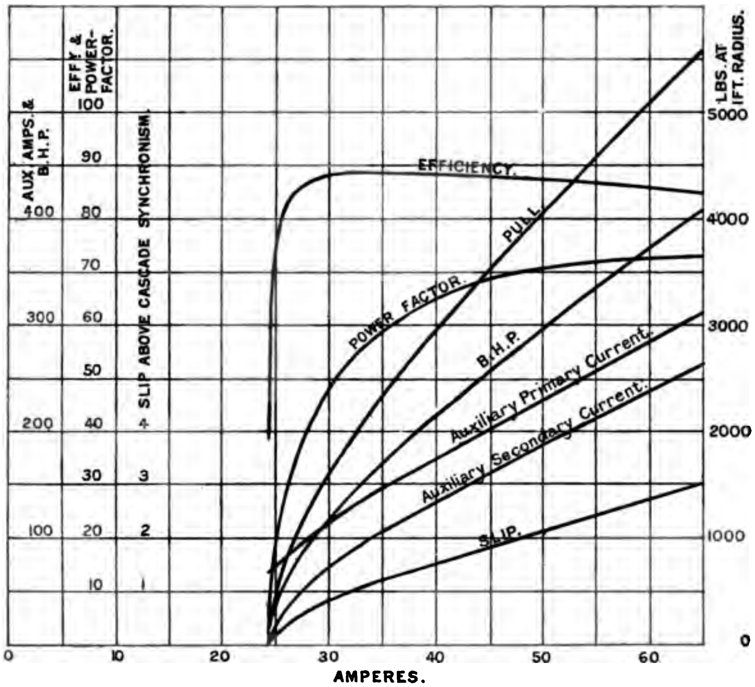

FIG. 20. Performance curves of two three-phase railway motors in cascade as generators at 3150 volts.

In such a motor, the primary winding will be on the stator, and one of the three windings will be utilised for excitation with direct current while the two other windings will be connected in series therewith and will be utilised as compensation for the ampere turns on the rotor or armature.

The rotor must be wound in such a way as to be suitable for operation with direct current or with alternating. This is done by taking an ordinary direct current armature and tapping its windings at four equally spaced points per pair of poles, the taps being connected to slip rings suitably mounted on the shaft.

When supplied with direct current the three stator windings are put in series, connected as already described, and in series connection with the brushes on the commutator. This will produce a series wound direct current motor differing only from the standard type in that it has compensating windings on the field. This difference acts advantageously as it tends to help commutation.

When supplied with three-phase currents the stator windings are connected up in star connection, and rotor currents are taken off through the four slip rings.

It is advisable to have four slip rings and not three; the latter arrangement produces a genuine three-phase winding on the rotor which must be clearly distinguished from what is usually called a three-phase winding such as the stator is wound with, but which is in reality a 6-phase winding.

In a true 3-phase winding the currents are not so favourably distributed as in a 4-phase winding, and the addition of a single slip ring is justified.

As an example of the possibility of such a motor, take the case of the auxiliary motor the design for which has just been given.

Let the stator windings be rearranged so that the two sets of coils belonging to each phase are connected in parallel instead of in series.

Wind the rotor with 320 conductors connected up as a multiple circuit winding to a 160 part commutator.

Then with 440 amperes direct current,

the armature ampere turns will be $\frac{440}{4} \times 160 = 17{,}600$,

and the compensation ampere turns will be $\frac{440}{2} \times 80 = 17{,}600$.

The excitation ampere turns per pair of poles will be

$$20 \times \frac{440}{2} = 4400.$$

The flux per pole will be

$$8200 \times \tfrac{2}{3} \times 1500 + \tfrac{1}{2} \times 8200 \times \tfrac{1}{3} \times 1500 = 10{\cdot}25 \times 10^6.$$

The back E.M.F. at 450 r.p.m. will be

$$10{\cdot}25 \times 10^6 \times 80 \times \frac{450 \times 4}{60} \times 10^{-8} = 246 \text{ volts};$$

the applied E.M.F. would be about 265 volts.

This motor, therefore, on a three-phase supply would be rated at 500 volts, 25 frequency, 220 amperes, 730 r.p.m. and on a direct current supply 265 volts, 220 amperes, 450 r.p.m.

A pair of such motors could be used on a car, and controlled by connecting in cascade or singly, or by the usual series parallel method with direct currents. When connected in cascade the two rotors would have to be put in series, as it would be impossible to supply three-phase current from the main rotor to the auxiliary stator.

When running on alternating currents the direct current brushes must be lifted off the commutator, otherwise violent sparking would occur.

Change of primary voltage for continuous running. From the calculations already made, it will be obvious that induction motors can be designed with high power factors even when starting with considerable overloads. In most cases, however, the torque required to keep a train in motion is considerably less than that required for acceleration; and it will be manifest that if the motors be designed to give maximum power factor and efficiency under the starting conditions, the behaviour of the motors under running conditions will be much less favourable. If, then, the running conditions are important, that is, if the frequency of starting is not high, some measures must be taken for adapting the motors to both sets of conditions.

The simplest method is to change the primary voltage. The maximum power factor of a motor being an intrinsic quantity independent of the applied voltage, it is manifest that, within reasonable limits imposed by the internal losses, it is possible to obtain any required torque with the maximum power factor of the motor. The two sets of conditions can, therefore, be met by applying the normal voltage for accelerating, and reducing the voltage for running.

The most obvious method of effecting this is to wind the primaries for delta connection for accelerating, and reconnect them in star for running. This reduces the effective voltage per phase in the ratio of $\sqrt{3}$ to 1.

If the motors be supplied by means of a transformer on the car, the change of voltage can be effected by altering the transformer connection on the secondary side from star to delta, or on the primary side from delta to star, or by connecting the motor to different points of the secondary winding.

Finally, it is possible to avoid all changes of connection in certain cases in which a train is composed of several motor cars, viz. by reducing the number of motors propelling the train. Thus, for instance, if there were 2 motor cars each equipped with 2 motors, all 4 motors might be used to accelerate the train, and only one to keep it running.

Construction of three-phase traction motors. In dealing with the construction of these motors there is very little to be said in a general way that has not already been said in connection with direct current motors for tramways and railways.

The chief differences are to be found in the substitution of slip rings for a commutator and in the relatively very short air gap.

The first of these differences is, without doubt, a clear gain; but there is practically nothing in this particular in which the three-phase traction motor differs from the three-phase stationary motor except the fact that brushes must be used which will collect the current equally well with both directions of rotation.

The short air gap was at first looked upon as a great disadvantage inherent in these motors. It was anticipated that the wear of the bearings would be sufficient to cause actual contact between stator and rotor after a short period of working. Experience, however, shews that with properly designed bearings this danger is not so serious as was supposed; the motors made by Messrs Ganz and Co. for the Valtellina Railway have run 100,000 miles without requiring renewal of bearings.

Examples of three-phase motors. The development of three-phase alternating current traction has taken place almost entirely on the Continent. Three or four prominent Continental firms have been associated with this development, viz. Messrs Siemens and Halske, Messrs Brown, Boveri and Co., Messrs Ganz and Co., and others. The first two of these firms were chiefly responsible for the early work, whereas Messrs Ganz and Co. have made the greatest advances recently.

In 1892 Messrs Siemens and Halske equipped an experimental track in Charlottenburg, and tested thereon a street car mounted with a three-phase motor suitable for 500 to 600 volts, 50 frequency, about 1400 revolutions per minute. This installation was exhibited at Chicago in 1893.

On the Lugano tramways equipped by Messrs Brown, Boveri and Co. in 1895, each car had one 20 H.P. induction motor for 400 volts, 40 frequency.

A few years later, in 1898, the installation of three-phase railways was begun in Switzerland. The conditions that obtain on a mountain railway are eminently suitable for this system, as the load is fairly uniform, the speed required is constant, and there is an excellent opportunity for economising energy by using the motors as generators during the descent. On the **Gornergrat Railway**, equipped by Messrs Brown, Boveri and Co., each locomotive contains two 90 H.P. motors at 800 r.p.m., 540 volts, 40 frequency. On the **Jungfrau Railway**, the locomotives built by Messrs Brown, Boveri and Co. and by the Oerlikon Co., have each two 125 H.P. motors at 760 r.p.m., 500 volts, 38 frequency. On the **Stanstaad-Engelberg Railway**, equipped by Messrs Brown, Boveri and Co., each locomotive contains two 75 H.P. motors at 650 r.p.m., 750 volts, 32·5 frequency, and each motor car with two 35 H.P. motors, at 400 r.p.m.

The Burgdorf Thun Railway works under the same conditions as the standard Swiss Railways; the passenger trains are composed of motor cars and trailers and the freight trains are drawn by locomotives.

*Each motor car is equipped with four 64 H.P. three-phase motors for 600 r.p.m. at 750 volts, 40 frequency. All four motors are connected with their stators in parallel, that is to say, the cascade method of control is not employed. These motors are wound for eight poles and have a mechanical clearance between stator and rotor of 1·5 mm. They are geared to driving wheels of 1020 mm. diameter, the gear ratio being 2·96 : 1. The weight of each motor with its pinion is 1350 kgms. or 3000 lbs. The general appearance is very similar to that of a standard direct current traction motor, the method of suspension being precisely the same.

Each locomotive is equipped with two 150 H.P. motors mounted on the same shaft. These motors are wound for 16 poles, the synchronous speed being 300 r.p.m.; they are geared to driving wheels of 1230 mm. diameter, the gear ratio being 1·188 : 1 or 1·372 : 1 alternatively. The clearance between rotor and stator in this case is 3 mm. These motors are not geared direct to the driving axles in the ordinary way, but drive through connecting rods. This Railway was equipped by Messrs Brown, Boveri and Co.

Berlin-Zossen Railway. In the famous high speed experiments on the Berlin-Zossen line three-phase motors were employed both by Messrs Siemens and Halske and by the A. E. G.

The first car built by the former company was equipped with four direct drive motors. Each motor was rated at 250 H.P. normal and 750 H.P. maximum, and was mounted directly upon the driving axle, the diameter of the driving wheel being 1250 mm. or 49·3″. The rotor was wound as the primary for a voltage of 1150 volts, 50 frequency, which voltage was obtained from a transformer carried on the car, which transformed down from 10,000 volts obtained from the overhead trolley wires. For starting purposes, when a large torque is required, the secondaries of the transformer are connected in star, and the primary voltage on the motors is under these circumstances about 1850 volts.

In this case the stator formed the secondary and was wound for a voltage of about 650 volts at standstill.

At 225 kms. per hour (for which speed the equipment was designed) the speed of the motors would be 960 r.p.m.

The weight of each motor, excluding the axle and the wheels but including the supports, was about 4100 kgms. or 9000 lbs.

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* See the paper by Professor Carus-Wilson before the Institution of Mechanical Engineers, or *Engineering*, July 20 and 27, 1900.

Figure 21 shews the cross-section of this motor through the centre line of the axis and figure 22 gives a view of the rotor and stator core plates and conductors.

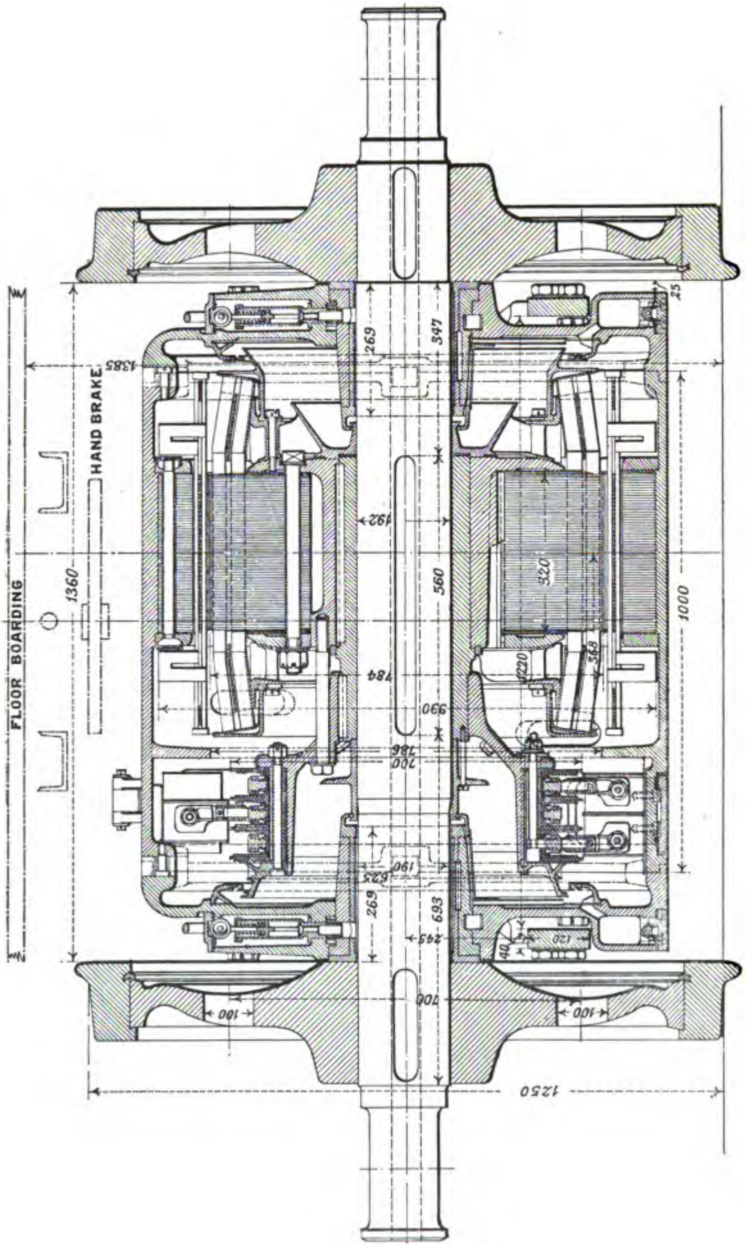

FIG. 21. Section of Siemens and Halske 250 H.P. three-phase gearless railway motor. (Berlin-Zossen.)

The motor car equipped by the A.E.G. contained also 4 motors of the same capacity, viz. 250 H.P. normal and 750 H.P. maximum each. The primaries of these motors were wound for 435 volts, 50 frequency, synchronous speed being 1000 r.p.m. In this case the rotors are not built directly on the driving axles, but on hollow sleeves which are coupled to the driving wheels by means of flexible couplings.

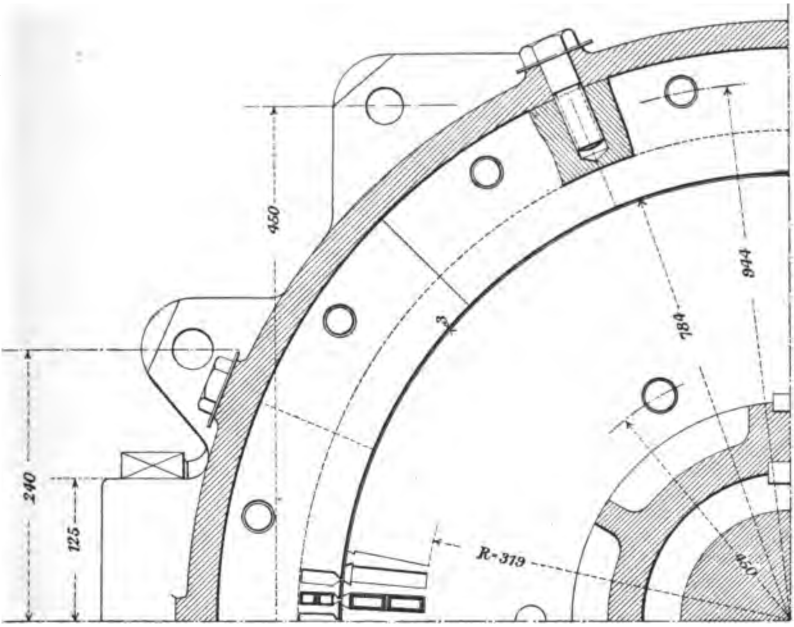

FIG. 22. Rotor and stator core plates of Siemens and Halske 250 H.P. three-phase gearless railway motor. (Berlin-Zossen.)

Messrs Siemens and Halske subsequently constructed a locomotive for experiments on the same line, and equipped it with geared motors, the particulars of which are approximately as follows:

	Stator		Rotor
Voltage	... 10,000		700
Frequency	... 50		
External diameter	990 mm.		680 mm.
Internal diameter ...	684 mm.		390 mm.
Air gap	...	2 mm.	
Length of laminations	300 mm.		300 mm.
Number of slots ...	... 72		90
Type of slots ...	... open		half-closed
Conductors per slot	... 72		4
Connection	... star		star
Number of poles ...	... 6		6

In March, 1903, Messrs Ganz and Co. received an order for three electric locomotives for the extension of the passenger traffic on the Valtellina Railway.

The motors with which these locomotives are equipped are duplex motors, that is to say, two distinct motors on a common shaft and in the same case, one being the main and the other the auxiliary motor.

The main primary is proportioned for 3,000 volts, 15 frequency, and all the windings for eight poles. Figure 23 shews the general arrangement, in which it will be seen that there is only one set of collector rings outside the bearings and that the motors drive through cranks. (The mounting and mechanical connection are shewn in figure 24.)

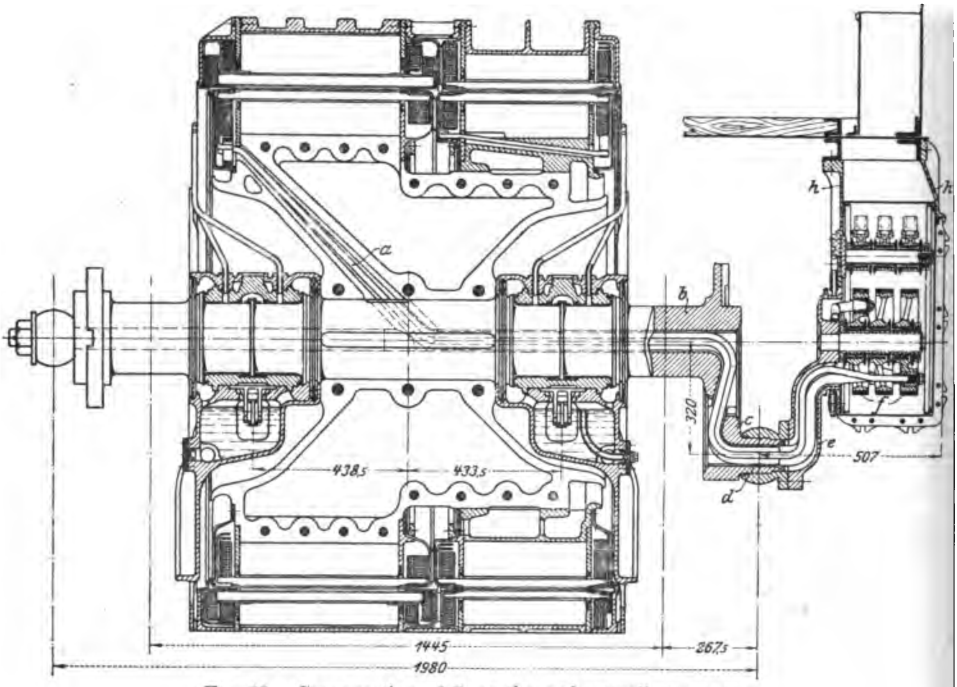

FIG. 23. Cross-section of Ganz three-phase railway motor for the 1904 Valtellina locomotives.

The two rotor windings are permanently connected together by internal connections, and both sets are connected to the slip rings. These connections are clearly shewn in the figure, being marked *a*, running along inside the motor from the left-hand rotor winding to the hollow shaft and so through the right-hand bearing and through the crank to the external slip rings.

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The distance between the outside bearings of the motor is 1,445 mm., which is greater than the standard gauge of 4 ft. 8½ in. (1,435 mm.); thus it is evident that this type of construction and suspension permits the full space between the wheels to be utilised for the motor.

The normal speeds of the locomotive are 64 km. per hour and 32 km. per hour, the diameter of the driving wheels being 1,500 mm. The motors are together capable of producing a maximum pull of about 12 tons, the normal pull with the cascade connection being about 6 tons. This overload is obtained with a line voltage reduced to 2,200 or 2,300 volts, which shews that such motors can be designed to operate satisfactorily even with 25 per cent. drop of voltage.

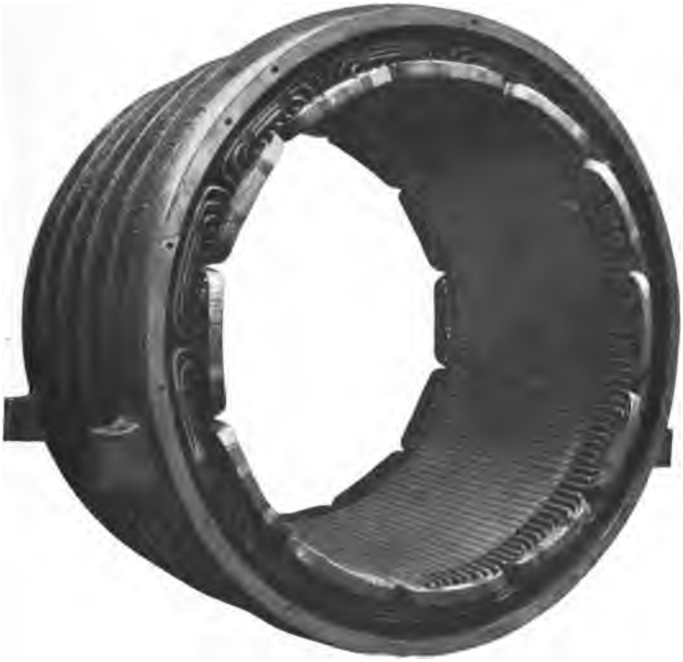

FIG. 25. Stator of three-phase railway motor for Simplon Tunnel locomotive. (Brown, Boveri & Co.)

The efficiency in single connection is about 95 per cent. and in cascade about 88 per cent.

The power factor in single connection is 95·6 per cent. and in cascade is 79 per cent.

Three-phase motors for the Simplon Tunnel. For this railway three-phase locomotives have been made by Messrs Brown, Boveri & Co., which are equipped with two-speed induction motors. Each motor is rated at 450 horse-power with a voltage of 2,700 to 3,000 volts, 16 frequency. The diameter of the driving wheel is 64½ inches, and the motors drive through coupling rods in the same way as on the Valtellina Railway locomotives. The two speeds are 42 and 21 miles per hour approximately, which correspond to 220 and 110 r.p.m. respectively. The maximum output of each motor is 1150 horse-power and the weight of each complete is 10¾ tons. The stator of this motor is shewn in figure 25 and the rotor in figure 26.

The arrangement for changing the number of poles in the stator windings is shewn diagrammatically in figure 27. The winding consists of six separate parts connected to form a closed circuit as shewn. This circuit is tapped at the six points of junction and leads from these points go to a 9-pole throw-over switch. Calling these points 1, 2, 3, 4, 5 and 6 and the windings between these points 12, 23, 34, 45 and 56 respectively, it will be seen that to produce an 8-pole field the points 1, 3 and 5 are connected to the three points X, E and Y of the supply, and for a 16-pole field, the points 2, 4 and 6 are connected to X, E and Y, while 1, 3 and 5 are connected together. The result of these connections is shewn diagrammatically in figure 28, from which it appears that windings 16, 65, are in series between X and Y and so on for the 8-pole field, whereas for the 16-pole field one of each pair of windings is reversed, and 61 and 65 are in parallel between Y and the neutral

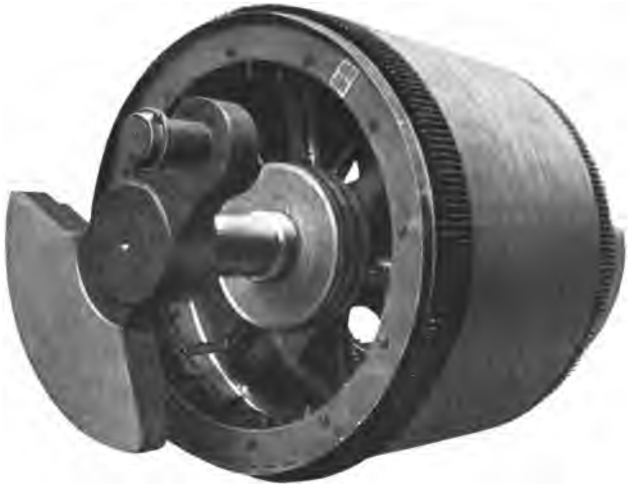

FIG. 26. Rotor of three-phase railway motor for Simplon Tunnel locomotive. (Brown, Boveri & Co.)

point. Thus not only are the necessary connections made, but the voltage applied to each of the six windings is altered in the ratio of $\frac{1}{2}$ to $\frac{1}{\sqrt{3}}$.

The rotor is wound with two separate three-phase windings, the terminals being connected to two sets of slip rings on the shaft. Thus, when the number of poles is changed on the stator, no modifications are required on the rotor, one set of brushes being merely raised or lowered.

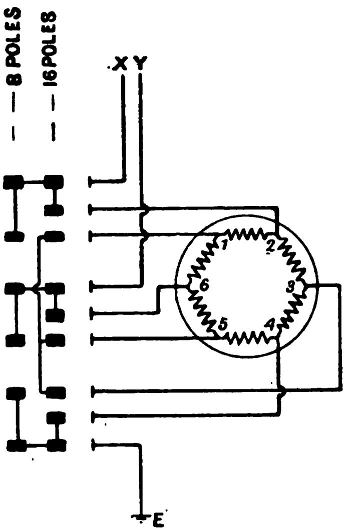

FIG. 27. Diagram of connections for three-phase railway motors for Simplon Tunnel locomotives. (Brown, Boveri & Co.)

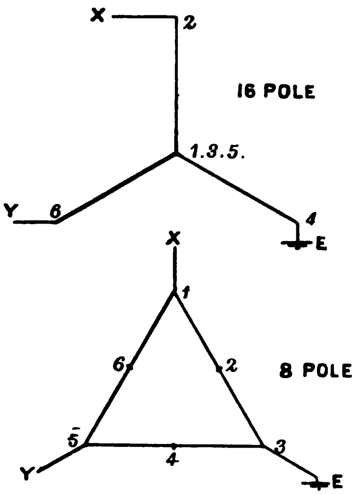

FIG. 28. Vector diagram for connections of three-phase railway motors for Simplon Tunnel locomotive.

Three-phase D.C. motor. As already mentioned, three-phase induction motors can be made for operation on alternating as well as direct current circuits. Such a motor has been designed for use on the London and Port Stanley, Ontario Railway in Canada, which is being equipped by Messrs Bruce, Peebles and Co. Each car is driven by two motors of 65 H.P. each. The stators are wound for 1000 volts alternating or 500 volts direct current; the rotors have commutators and four slip rings. Arrangements are made for cascade single control, and for series parallel control. The general arrangement of the motor is shewn in figure 29.

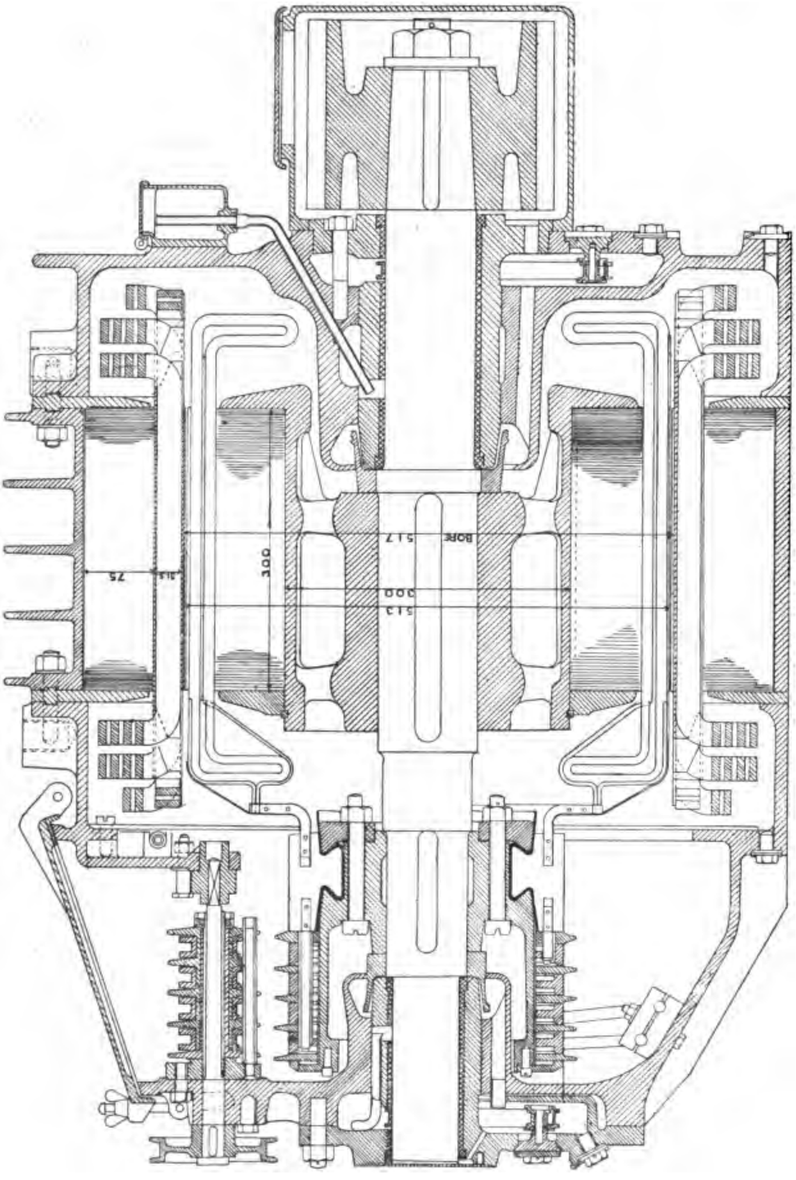

FIG. 29. General arrangement of three-phase direct current motor. (Bruce, Peebles & Co.)

CHAPTER 2.

THE CONTROL AND EQUIPMENT OF ELECTRIC ROLLING STOCK ON THE THREE-PHASE SYSTEM.

General. In considering the equipment of a tramway or railway car with polyphase apparatus, it must be stated at the outset that, although the system is by no means new, the extent to which it has been applied has not been sufficient to develop any apparatus which may be called standard. As already shewn in Chapter 1 of this Volume, most of the actual installations on this system are on a few mountain railways in Switzerland, in addition to the Burgdorf Thun line, and the important electrifications of the Valtellina line in the north of Italy and the Simplon tunnel. And in these few cases there exists a considerable diversity of apparatus, in so much that there is scarcely a single installation which does not differ in some respect from all the others.

In its simplest form the equipment of a car will consist of one or more motors, a controller, single or in duplicate, resistances, motor cut-out switches, automatic cut-out and the necessary current collectors. In addition to these items, cars for use on railways must be equipped with motor compressors for operating the brakes, and the necessary switches for governing them.

Dealing first with the controllers, these may be divided into two sections, viz. those which govern motors connected permanently in parallel, and those which are employed when the cascade system of control is used.

Rheostatic controllers. The functions of the rheostatic controller are normally three in number: (1) to connect the primaries of the motors to the line, (2) to provide for forward or backward running, (3) to regulate the speed by varying the external resistances in the secondary circuits. In special cases there may be other functions, such as that of altering the voltage at the terminals of the primary windings as already mentioned (page 47).

In figure 30 such a controller is shewn diagrammatically and consists of three barrels, viz. the reversing barrel, the motor cut-out barrel and the main barrel. The reversing barrel is the simplest, and effects its purpose by exchanging two of the three leads from the line. It will be obvious that the result of such an exchange will be the reversal of the direction of the revolving field in both motors. The motor cut-out barrel is merely an illustration, and could be modified in many ways. Its function, however, is to enable the driver to cut out either motor when disabled; and the connections must be such that it can be disconnected effectively at either end of the car. The purpose of the main barrel is to regulate the speed of the car, and the method of

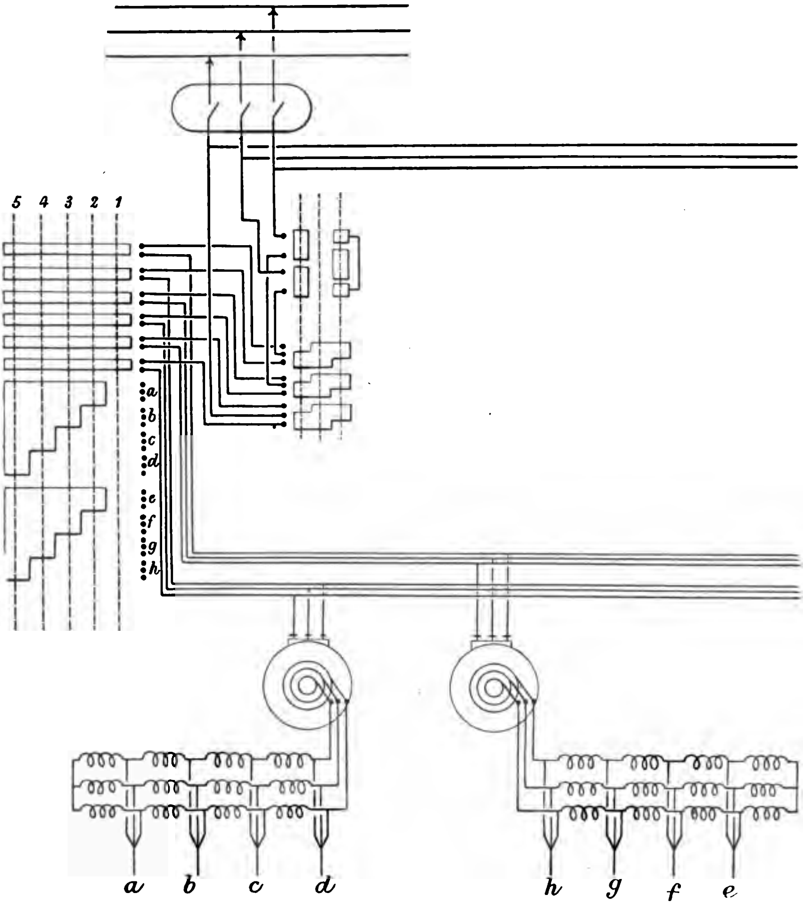

FIG. 30. Diagram of rheostatic controller for two three-phase railway motors.

carrying this out is sufficiently obvious from the figure; the fingers being arranged in eight sets of three, *a*, *b*, *c*, *d*, etc., each set being supposed joined by a three-core cable to the proper points of the two rheostats.

As already mentioned, the arrangement shewn is capable of many modifications, just as in tramcars for direct current working there are several different forms of controllers, all of which are capable of carrying out the same operations. In particular, the order of contacts on the main barrel might require to be modified if a magnetic blow-out were employed. As there are three different phases, all the contacts carrying current of one phase would probably be grouped together. On the other hand each separate contact might be fitted with its own blow-out. In any case arrangements would have to be made for short circuiting all the blow-out coils in the final position, so as to avoid as far as possible the generation of eddy currents in the solid masses of metal.

Other modifications also are fairly obvious, such as the connections for varying the grouping of the primary windings from delta or star, for the use of a transformer to reduce the line voltage, or for the grouping of the secondary windings of the transformer in order to vary the voltage of the supply to the motors.

It is of interest to note the comparison between the above and the parallel case of a direct current tramcar. In the latter the number of cables that run from the controller at one end to that at the other end varies from 13 to 18, and the number of contact fingers in each controller varies from 27 to 44. In the case of the three-phase equipment in figure 30 there are 11 three-core cables, or 33 single cables, and the controller contains 49 contact fingers.

Rheostatic controller for railways. A controller, such as is outlined in figure 30, would be suitable for small mountain railways; but for cases where the requirements are more severe, such as in motor-coaches for fast and heavy service and in locomotives equipped with large motors, certain modifications are practically necessary.

These modifications are in the direction of subdivision of the whole into a number of separate parts for the different functions, and also in the mechanism by which such parts are operated.

It should be noted that, in the controller in figure 30, no circuits are opened during the forward movement of the main barrel. If, therefore, the primary switch were used not only as a protective device but also as a switch for connecting and disconnecting the line and the motors, no provision would be required in the controller for the breaking of arcs, for the circuits could be opened by the primary switch before the barrel is moved backward.

It follows, therefore, that if all switching off be done by a separate main switch the rheostat barrel can be regarded simply as a means for changing the connections of the various circuits without the necessity of opening any.

Such a function is almost invariably fulfilled by a barrel as being the simplest means that can be devised. For heavy service, however, the currents are large and the barrel is necessarily bulky. It becomes apparent that such an apparatus will differ from a tramcar controller to such a degree that the driver will require some assistance in operating it. Moreover, in many cases, it will be more economical to depart from the standard tramway practice of putting a complete controller at each end of the car, and to provide only one such barrel which may be operated either by suitable gearing or by auxiliary means from either end.

The control of the Berlin-Zossen car. The motor car which was equipped by Messrs Siemens and Halske for experiments on the Berlin-Zossen line provides an illustration of the above remarks. On this car there were four motors, and consequently four distinct sets of regulating resistances. If then the controllers had been arranged so that these resistances were regulated directly as in the case of an electric tramcar, each controller would have had to be capable of accommodating a very great number of contacts, so much so that auxiliary power would be required to operate it, and further the power wiring of the car would be extremely costly. Instead of this, the resistances, of which there were twelve sets, were arranged along the sides of the car, and twelve distinct barrels were fitted immediately below them so that the electrical connections might be as short as possible. These twelve barrels, six along each side, were mechanically connected through gearing to the hand controller at each end, and in order to facilitate the working compressed air cylinders were supplied and so arranged that all the friction of the gearing etc. was eliminated, and the controller handle could be moved with a comparatively slight pressure. The resistances for this car are shewn in figure 32.

The control of motors in cascade. Although there are several possibilities of controlling a pair of three-phase motors, only one method besides the simple rheostatic control has been employed so far. This method is what is known as the cascade single control, and consists in connecting the two motors in cascade for accelerating below cascade synchronism and for braking above that speed, and connecting the main motor only to the line for accelerating above cascade synchronism. Moreover, in the only examples at present in existence, both motors have been wound for the same number of poles, so that the two economical speeds are full speed and half speed.

The functions of the controller required to operate a pair of motors on this method will therefore be as follows:

(1) To connect the primary of the main motor to the line for forward or backward running; to connect the secondary of the main motor to the primary of the auxiliary motor; to connect the secondary of the auxiliary motor to the rheostat.

(2) To vary the resistances in the rheostat as the car runs up to the speed of cascade synchronism, finally short circuiting the auxiliary secondary.

(3) To break the connections of the main secondary to the auxiliary primary, either before or after the auxiliary secondary has been opened, and connect the main secondary to the rheostat.

(4) To reduce the resistance in the rheostat, and finally short circuit it.

Provided the controller is capable of carrying out these functions in the order required, all the operations of accelerating and braking can be performed.

Cascade control in general. In regard to this system of cascade working the objection is often put forward that it is wasteful of material in that for a considerable portion of the total time half of the motor equipment is not working and is nothing but a dead weight.

Taken simply by itself the objection is not of much value. It would assume a greater importance if the assertion were ever made that this cascade method was applicable to all conditions of electric traction. No responsible engineer, however, would propose to apply this method to cases in which the stops are few and far between, and a uniform speed is sufficient.

In fact the objection must be limited to the pointing out of what *may be* a handicap when all the advantages and disadvantages of one system are compared with those of another system. (See also Chapter 4 of this Volume.)

In any case the remark cannot be directed against cascade working in general. It is theoretically perfectly possible to utilise all motors to an equal extent; that is to say, motors can be controlled in cascade and in parallel. Further, it is also theoretically possible to use motors with different numbers of poles, and to connect them in cascade, parallel and single.

The value of these possibilities will depend on three considerations, viz. the practicability of the design, the practicability of the control, and assuming these to be favourable, the advantages to be gained by employing the different systems.

The first consideration has already been dealt with, and will be referred to, together with the third consideration, in Chapter 4 of this Volume.

Cascade parallel control. With regard to the practicability of the control, consider first the cascade parallel method. This will naturally require two motors of like numbers of poles.

There are several alternatives as follows:

(1) The main motor may be wound for a high voltage. In this case the auxiliary motor must be wound for the same voltage, and a step-up transformer must be inserted between the main secondary and the auxiliary primary.

(2) Both motors may be wound for a low voltage, and the line voltage may be high. In this case the whole current must be taken from a transformer on the car.

(3) The line voltage may be low; in which case the transformer can be dispensed with.

Of these alternatives the second and third may be taken together, the difference being but slight.

The scheme of control for the first of these alternatives is shewn in figure 31, and the same scheme with slight modifications would apply to the second and third alternative.

The control is supposed to be based on the use of contactors, governed by a master controller, and would be equally applicable to a single car or to a multiple unit system. It should be observed that all the rheostat switches would be relatively simple in that they are only required to connect together three wires; whereas each of the other switches connect together three pairs of wires.

The method of operation would be as follows:

(*a*) Close switches 1, 2 and 3, and gradually cut out the resistances in the lower rheostat until all are short circuited. The motors will then be running in cascade.

(*b*) Close 4, open 2, and insert in the secondary circuits of both motors the proper amount of resistance, and then close 5.

(*c*) Gradually cut out resistances in both rheostats until the motors are running at full speed in parallel.

To such a system most of the developments that have been worked out for direct current controllers could be easily applied, such as the automatic operation of the contactors in their proper order, and the automatic regulation of the speed of switching on.

The problem of constructing and working contactors for alternating currents will be dealt with in Chapter 6 of this Volume on the control of single phase motors; but it may be said here that there is no insuperable difficulty.

It appears, then, that the cascade parallel system is not impracticable on the score of the control, although the actual switching apparatus may be a little bit more cumbersome.

Cascade-parallel-single control. This method of control differs only from the last in that one of the motors reaches its synchronous speed before the other. For instance, suppose the frequency to be 25,

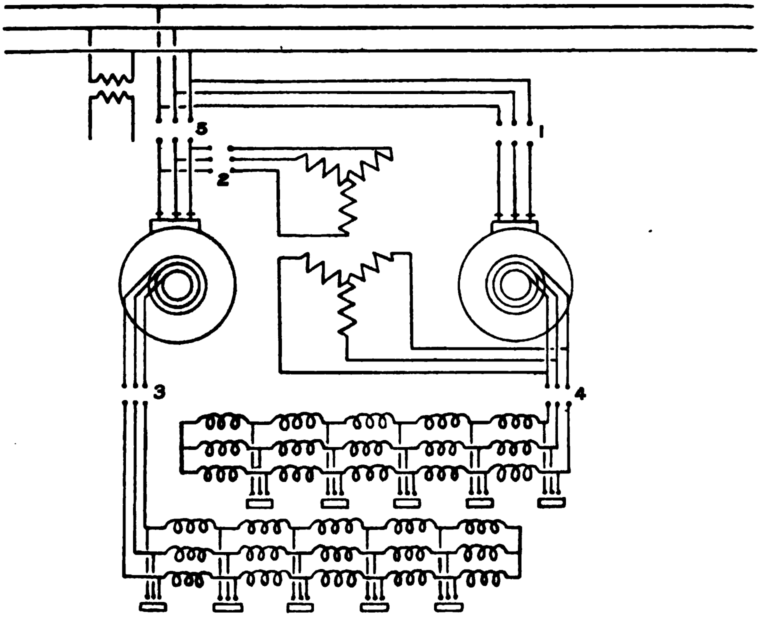

FIG. 31. Diagram of cascade parallel control for two three-phase railway motors.

and one motor to be wound for four poles while the other is wound for six. Then the three running speeds will be $\frac{25 \times 120}{4+6} = 300$ r.p.m.; $\frac{25 \times 120}{6} = 500$ r.p.m.; and $\frac{25 \times 120}{4} = 750$ r.p.m.

Or again suppose both motors to be 4-pole motors, and one of them geared to a higher speed than the other; the same effect will be produced.

With such a combination there are several ways of connecting and operating the motors. As an example let A be the 4-pole motor and B the 6-pole motor. Then one method would be as follows:

(1) Connect the motors in cascade, using A as the main motor.

(2) Cut out resistance in the secondary circuit of B until cascade synchronism is reached.

(3) Connect the secondary of A to its own resistances; open the cascade switch; reinsert resistance in the secondary of B, and connect the primary of B to the supply.

(4) Cut out resistances in both secondaries until the synchronous speed of B is reached. By this time the secondary of B will be short circuited, a certain amount of resistance being left in the secondary of A.

(5) Cut out motor B altogether.

(6) Cut out resistances in the secondary of A up to the maximum speed.

The operations for regenerative braking will depend on circumstances. If the train coasts down to the synchronous speed of B from this speed to cascade synchronism, the braking will be effected by both motors in cascade.

Apparatus for the control of three-phase locomotives and motor coaches. In studying the practical details of the equipment of three-phase cars it may be best to consider first the regulating resistances, for upon them depend other parts of the apparatus.

Resistances. Two types are employed, viz. the metal rheostat and the liquid rheostat. Both these types are used extensively in practice with stationary three-phase motors, and each seems to have its own advantages. Many engineers consider that the metal rheostat is more reliable and less subject to temporary disablement than a liquid rheostat. It is regarded as being mechanically superior as being rigid, whereas the liquid in the other type is liable to evaporation and leakage, and needs circulation for cooling purposes. On the other hand the liquid rheostat requires no such electrical connections as does the metal rheostat, and moreover lends itself more readily to automatic regulation.

Another point may be mentioned which would appear to favour the liquid rheostat more particularly when used for cascade control. It has been shewn in a former chapter that the overload capacity of a three-phase motor is very much lowered if anything in the secondary circuit causes the current in that circuit to lag behind the volts. This is shewn plainly in the diagram for the cascade motors; and it will be apparent that any self-induction in the external secondary circuit of the auxiliary motor will have a prejudicial effect on the overload capacity of the cascade combination.

Of the two types of resistance the metal rheostat is bound to have some self-induction, whereas the liquid rheostat if it has not a pure resistance may have a capacity, which will cause the current to lead.

As an example, the grid resistances described in Chapter 4, Vol. I. may be taken. The total resistance of the batch was ·0996 ohm, and the self-induction of the batch so connected was found to be 60×10^{-6} henry when carrying a current of 50 amperes. Assuming the coefficient to be constant, at a frequency of 25, the reactance

$$= 2\pi \,.\, 25 \,.\, 60 \,.\, 10^{-6} = \cdot 0094 \text{ ohm}.$$

This shews that iron grids have by no means a negligible self-induction as compared with their ohmic resistance, and that the current would lag about 5½ degrees behind the volts at its terminals; or to put it in another way the leakage coefficient of the motor at starting would be increased by an amount proportional to this reactance.

An example of metal rheostats is afforded by the high-speed car equipped by Messrs Siemens and Halske for experiments on the Berlin-Zossen line. The resistances are made of high resistance material called "Kruppin" and are in the form of thin strips 45 mm. wide by 2 mm. thick. These strips are supported, as shewn in figure 32, on longitudinal rods from which they are insulated by porcelain. The resistances are enclosed in sheet iron cases and are attached to the top and bottom girders of the car body. The sheet iron covers are made in the form of shutters so that the resistances may be adequately ventilated by the motion of the car. Immediately under these frames are situated the barrels, and connections are made to the fixed contacts which press thereon by means of short lengths of cable.

In the motor car equipped for the same experiments by the A. E. G. a water rheostat was employed, consisting of a set of electrodes which could be immersed in a 5 per cent. solution of carbonate of soda. The electrodes are specially arranged so that the resistance is suitably graduated by forcing the liquid into the rheostat chamber. This is effected by means of a centrifugal pump which drives the liquid from a tank through a large number of cooling tubes into the chamber by one opening and out again by a second opening back to the tank. So long as the second opening is free the level of the liquid in the chamber remains below the level of the electrodes, but as soon as the opening is restricted the level begins to rise. In this way, the resistance is adjusted and at the same time the fluid is kept circulating through the cooling tubes. The electrodes are so shaped that at the bottom they are separated by a considerable distance, whereas at the top they are very close together. When the liquid has been forced up to its maximum height, the resistance has been reduced to an amount equal to the internal resistance of the secondary windings of the motor, and the final step consists in short circuiting the rheostat with a switch.

Another example of a liquid rheostat will be referred to in the following pages in the description of the equipment of the locomotives on the Valtellina Railway.

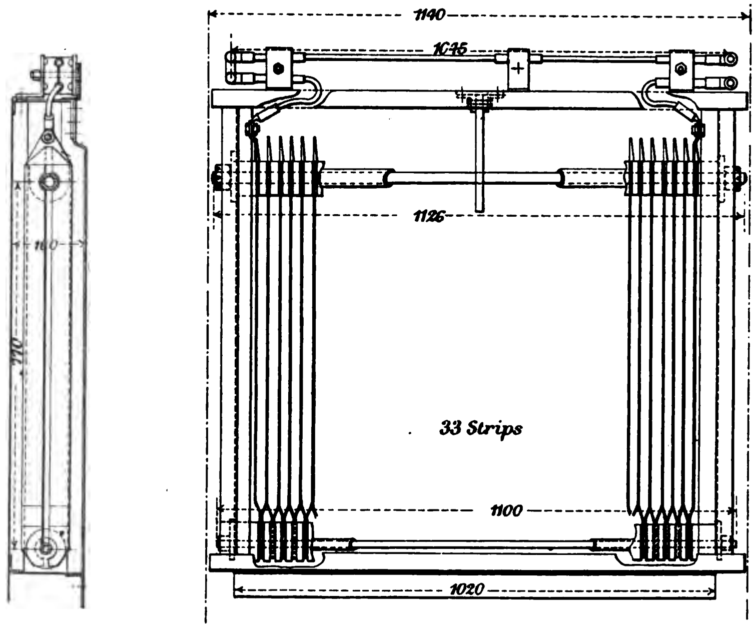

Fig. 32. Details of the control resistances for the Berlin-Zossen high-speed car. (Siemens and Halske.)

The 1904 locomotives on the Valtellina Railway. The electrical equipment of these locomotives, manufactured by Messrs Ganz and Co. of Buda-Pesth, may be taken as the most recent example of the type of apparatus employed by this firm.

The two distinguishing features of their system are (1) the cascade single control with main and auxiliary motors, (2) the use of fluid rheostats automatically operated by the agency of the motor current.

The cascade single system of control has already been dealt with in a general way, and the special motors manufactured for the purpose are shewn in figure 23, Chapter 1, of this Volume. Each motor is in reality duplex, or may be described as consisting of a pair of motors mounted on the same shaft and in the same case. The two rotors are permanently connected electrically, and also to a single set of slip rings. The operations of control are, therefore, as follows:

(1) The three-phase 3000 volt supply is connected to the main stator, and the auxiliary stator is connected to the rheostat; this provides for cascade running as the two rotors are permanently connected.

(2) The rheostat is gradually cut out and short circuited.

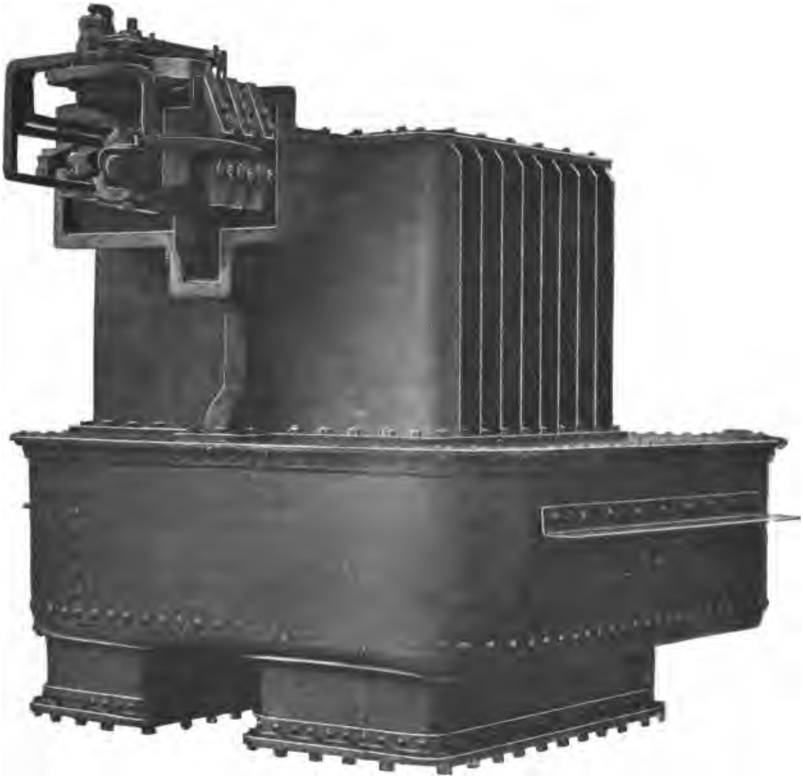

FIG. 33. Head of liquid rheostat on the 1904 locomotives for the Valtellina Railway. (Ganz & Co.)

(3) The resistances are reinserted, and their connection to the auxiliary stator opened, and a connection to the slip rings substituted. The motors are then working singly.

(4) The rheostat is gradually cut out as before.

The rheostat, figure 33, consists of a tank into which iron plates dip, the plates being connected to the three phases. Fluid is forced into this tank by compressed air, the influx being regulated by a valve governed by a magnet excited by the motor current. As the level of the fluid rises, the resistance between the phases falls, and the rate of decrease of the resistance is thus automatically controlled. When the liquid has risen as far as possible, the three phases are automatically short circuited by means of a metallic switch. In order to prevent the overheating of the liquid, cooling tubes are provided through which the liquid flows. The short circuiting switch is clearly shewn in figure 33. The valve for controlling the rheostat is shewn in figure 34, which gives one section of it. This piece of apparatus is very complicated and a full description of it would fill several pages. It is only possible here to refer to it briefly and to indicate the various functions which it has to perform. In the figure *f* represents an electromagnet excited by the secondary currents from the motor. This magnet attracts an armature attached to a lever arm *e* which carries a projection *g*. This projection bears upon the upper surface of a valve head. This valve head is peculiar and consists of circular discs as shewn supported by two flexible membranes of different diameters. Compressed air at a reduced pressure supplied from the master controller (see below) is admitted to the space between these membranes, and as the upper one has the greater diameter, it produces an upward force and unless otherwise prevented lifts the valve

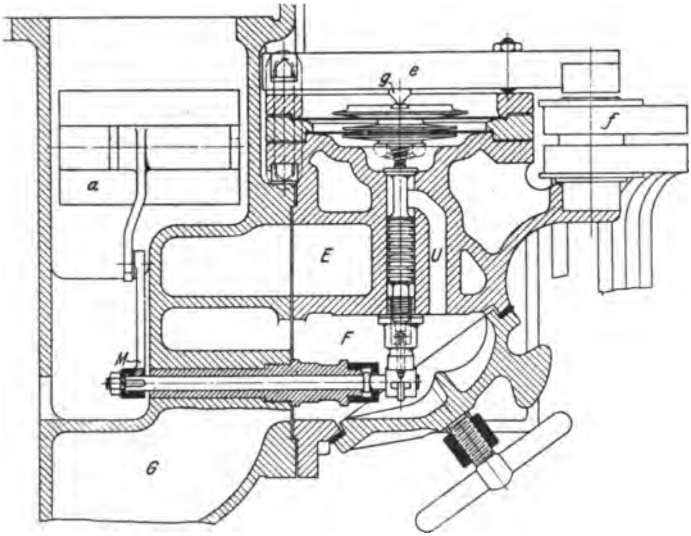

FIG. 34. Controlling valve for the liquid rheostat on the 1904 locomotives for the Valtellina Railway. (Ganz & Co.)

attached. This valve is not seen in the figure, being behind the long piston valve shewn. As a consequence, compressed air under full pressure enters the space below the membranes. To the left of the figure is shewn a float *a* which is connected with the spindle M, at the right-hand end of which is a cross bar turning with it. When the float is below its proper position this cross bar is tilted and lifts the piston valve. Compressed air is thus admitted to the passage U which communicates with the chamber G, the water reservoir of the rheostat. This pressure forces the water out of the reservoir into the chamber in which the electrodes are placed and, as long as the current does not exceed its proper value, continues to do so. If the current rises beyond this value the supply of air is stopped by the depression of the lever *e* owing to the attraction of the magnet.

The level of the liquid rises in the rheostat chamber until the float *a* which is governed by this level reaches a certain position, at which point the cross bar attached to the spindle M permits the piston valve to close; at the same time, by means not shewn in the figure, the short circuiting switch seen in figure 33 is automatically closed by means of an air cylinder.

If through leakage past the piston valve the level of the liquid in the rheostat chamber rises too high, the float rises with it and tilts the cross bar at the end of M so that another valve is lifted which opens the chamber G to atmosphere. For reinserting resistance, the process is reversed and compressed air is admitted to the rheostat chamber and the liquid is forced back into the reservoir*.

The complete electrical equipment of each locomotive consists of the following separate parts:

(1) Two high tension bow collectors, described below; each collector takes current from two overhead wires and therefore two high tension connections lead away therefrom.

(2) Four high tension fuses, to which each phase of the two collectors are connected. These four fuses are connected by pairs in parallel and form the terminals of the supply to the motors etc.

(3) Two lightning arresters with choking coils, one for each phase.

(4) Two high tension three-phase main switches operated by compressed air supplied from the master controller.

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* For a full description see *Zeitschrift des Vereines deutscher Ingenieure*, Jahrgang, 1905.

(5) One change over switch or speed switch for changing the connections between the motors and the rheostats.

(6) Two double motors.

(7) Two rheostats.

(8) Two overload relays for the main high tension switches.

(9) Two compressors and motors, with three-phase transformers and fuses, and a hand pump for putting up the collector at starting.

(10) The necessary voltmeters and ammeters, two of each.

(11) Two master controllers, one at each end.

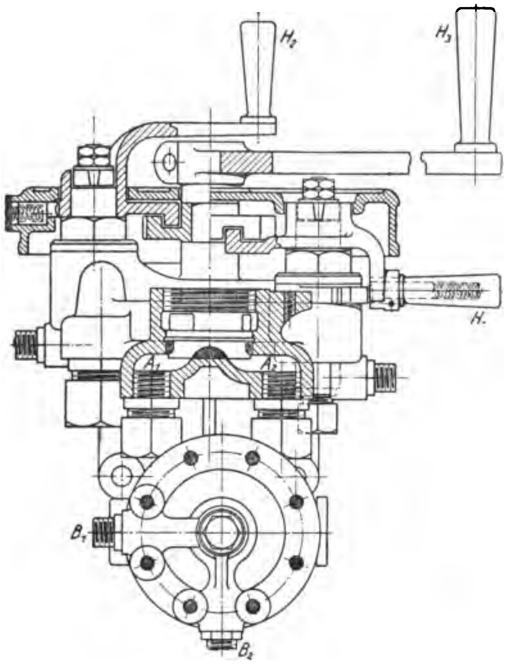


FIG. 35. Pneumatic master controller for the 1904 locomotives for the Valtellina Railway. (Ganz & Co.)

Of these parts, it is perhaps only necessary to speak of the master controllers. These controllers govern the whole equipment by means of compressed air, and they consist therefore of the necessary air valves with handles for operating them. Figure 35 gives a section of this part of the equipment and shews that it contains three handles H_1, H_2, H_3. The handle H_1 opens and closes a valve through which compressed air is supplied to the high tension switches; H_2 controls a valve which operates the change over switch; and H_3 regulates the starting and acceleration, by supplying air under reduced pressure to the controlling valves of the rheostats.

A general view of the locomotive is shewn in figure 24, Chapter 1, of this Volume.

Three-phase locomotives for the Simplon tunnel. These locomotives are in many ways of great interest as being representative of the most recent practice in three-phase work. They are manufactured by Messrs Brown, Boveri and Co., and, although they are in many respects similar to the 1904 locomotives on the Valtellina Railway built by Messrs Ganz and Co., yet they embody many differences in detail and are well worth careful examination.

It may be worth while to state one or two of the requirements for which the locomotive was designed.

(*a*) Each locomotive to be capable of bringing a train of total weight of 400 tons from rest up to a speed of 20 miles per hour in 55 seconds on a straight course with a gradient of less than 1 in 1000.

(*b*) To be capable of bringing a train of total weight of 250 tons from rest up to a speed of 40 miles per hour in 110 seconds under the above mentioned conditions, the time taken to change over from the lower to the higher speed being included in the 110 seconds.

The last paragraph in the second condition is important, as it lays stress on the necessity for a rapid change of connections between the two conditions of running. With a single fluid rheostat which is employed for both cascade and single connection (as in the Valtellina locomotive) it is necessary to empty the tank before the single connection can be put into operation. This is bound to take time, whereas if two distinct sets of resistances are employed as is the case in the Simplon tunnel locomotive, the time between the two conditions of working can be reduced to that necessary for operating the change over switch.

A general view of this locomotive is shewn in figure 36, the main driving motors having been already referred to and illustrated in Chapter 1 of this Volume.

The diagram of connections of the locomotive is shewn in figure 37, from which the general arrangement can be seen. The equipment contains two bow collectors P raised and lowered by compressed air cylinders *g*, one safety double pole line switch O with overload relays, one sheet iron high tension chamber L interlocked with the collectors so that the chamber can only be opened when the collectors are down,

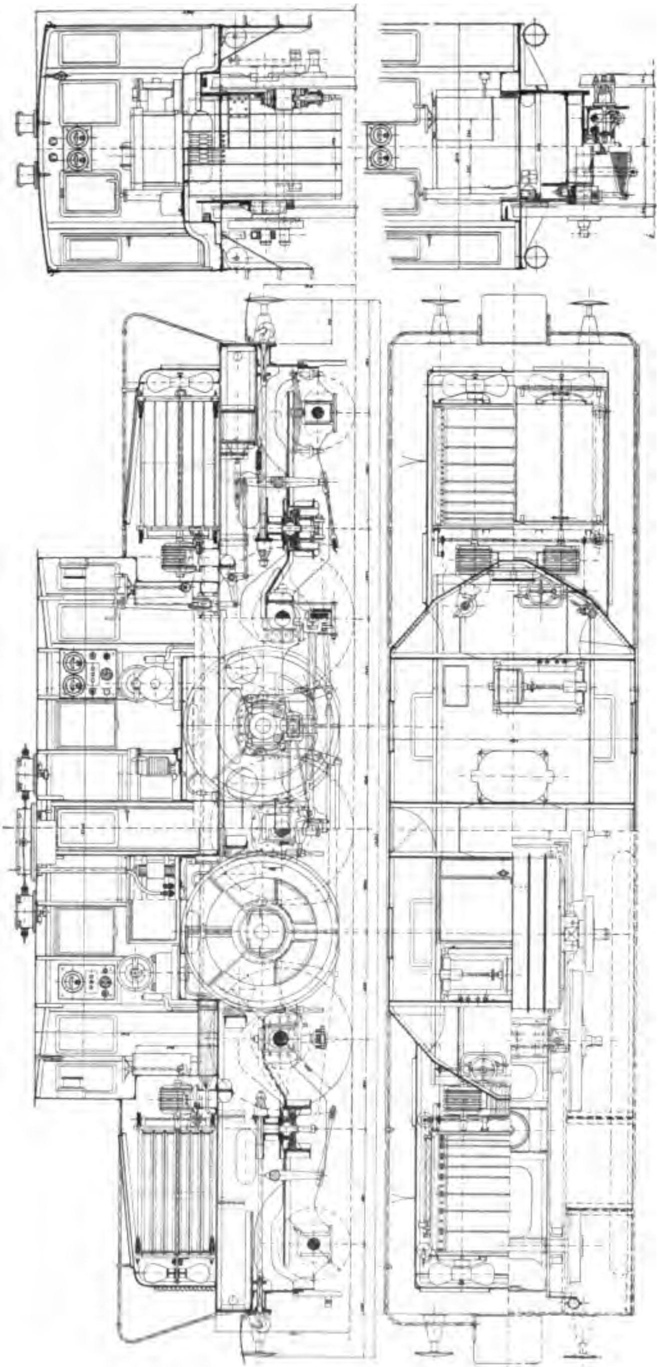


FIG. 36. General view in elevation and plan of the three-phase locomotive for the Simplon tunnel. (Brown, Boveri & Co.)

one reversing switch L_2, one pole changer L_1, and one main switch L_3 for each motor, two main motors D, two rheostats A, each with a pair of ventilating fans B, two main controllers C, two air compressors F with motors supplied from the transformers E, ampere meters, voltmeters and fuses.

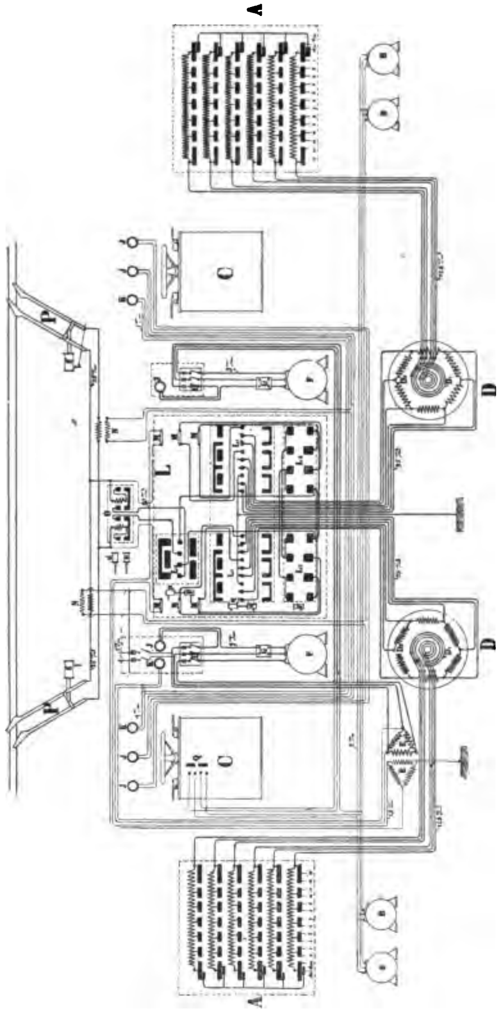


FIG. 37. Diagram of the electrical connections on the three-phase locomotives for the Simplon tunnel. (Brown, Boveri & Co.)

The master controller is shewn in figure 38 and contains two handles and a hand wheel, and various parts operated by these handles. The two first are connected to pneumatic valves which control the supply of air to the reverser cylinder and the pole changing switch cylinder;

and the hand wheel is connected to the shaft which runs the full length of the locomotive, and by means of bevel wheels and chains at each end works the moving contacts of the rheostats.

The reverser and the pole changer both consist of wooden cylinders on which are mounted copper contact blocks; both switches are enclosed in oil and are operated by compressed air relays or cylinders. The connections of the reverser do not require any remarks, and those of the pole changer have been given already in Chapter 1, p. 56 of this volume.

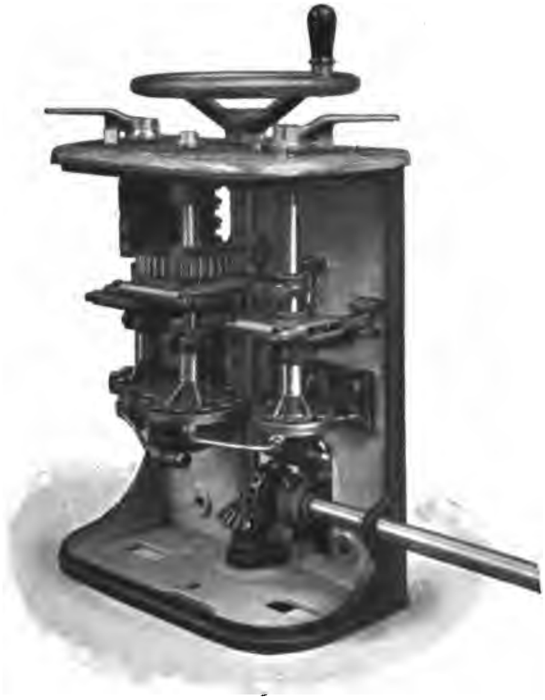


FIG. 38. Master controller for the three-phase locomotives.

The rheostats are placed on the two ends of the locomotive outside the cab and enclosed in sheet iron casing. They are built of rheostane wire, a very high resistance material, stretched on suitable framework and insulated by means of porcelain cups. These frames are made to slide in slots, and means are provided whereby they can be taken out for inspection or repair. A very ingenious arrangement has been devised for ventilating these resistances; this consists of small ventilating motors and fans which are driven by current from the rotor circuits, and the connections are such that as soon as the rheostats are short circuited and are no longer dissipating heat, the supply to these motors is simultaneously short circuited.

The bow collectors are referred to below.

Compressed air is supplied by two Christensen compressors electrically driven. These compressors supply the Westinghouse air brake, the whistle, the sanding gear and all the pneumatic relays or cylinders. Each compressor supplies 14·5 cub. ft. of air per minute at 110 lbs. per sq. in. to four cylindrical reservoirs mounted at the ends of the locomotive. The compressor motors are controlled by automatic switches which cut off the supply when the pressure in the reservoirs rises to 110 lbs.

Three-phase current collectors. The simplest form of current collector on a three-phase circuit consists of two independent trolley wheels, one for each of the overhead wires, the third phase being supplied from the rails. This form has obvious limitations, and for speeds above that commonly employed on electric tramways, other types are necessary. It may be said, that the complexity of the collector depends to a certain extent on the speed for which the car and its equipment have been designed. Three designs may be referred to in order of complexity, viz. that on the Valtellina locomotive, that on the Simplon tunnel locomotive, and that on the experimental car built by Messrs Siemens and Halske for the Berlin to Zossen line.

The current collector on the Valtellina locomotive is shewn in figure 39. The collector itself consists of two steel tubes covered with electrolytic copper separated from each other horizontally by insulating material about 175 mm. in length, the diameter of the rollers being about 75 mm. These two rollers collect current from the two overhead trolley wires and current is taken from the rollers by carbon brushes. The roller is supported at both ends by a light frame made of steel tubes, the two frames being pivoted at their bases. These bases are mounted on the roof of the locomotive supported on porcelain insulators with cast iron caps. Each end frame is connected by a connecting rod c to a glycerine dash-pot, and the collector is elevated by means of compressed air supplied through the pipes n_1, n_2. The pressure is transmitted to the collector by means of the pistons d_1, d_2 which are forced outwards to the left. These pistons are connected to a cross bar which is attached to the left-hand end of the spring shewn in the drawing. The other end of the spring, which is not seen in the figure, is fixed to the bottom of a lever arm attached to the frame of the collector, and hence any force applied to the left-hand end of the spring is transmitted by the spring tension to the collector and raises it into contact with the trolley wires. Part of the frame is shewn cut away in order to explain the action of the pistons. Provision is made for two pressures between the collector and the overhead wires corresponding to high and low speed. For the smaller pressure compressed air is supplied through pipe n_1; through this pipe the air flows into the space at the back of the right-hand piston d_1 and pushes both pistons to the left until piston d_1 comes against the fixed stop *u*. If now a greater pressure is required air is admitted through the pipe n_2 and through the opening in *u* to the back of piston d_2 and in the space *t* between the pistons. This latter piston, having a greater effective area, exerts a greater force and provides for an increased pressure on the trolley wires.

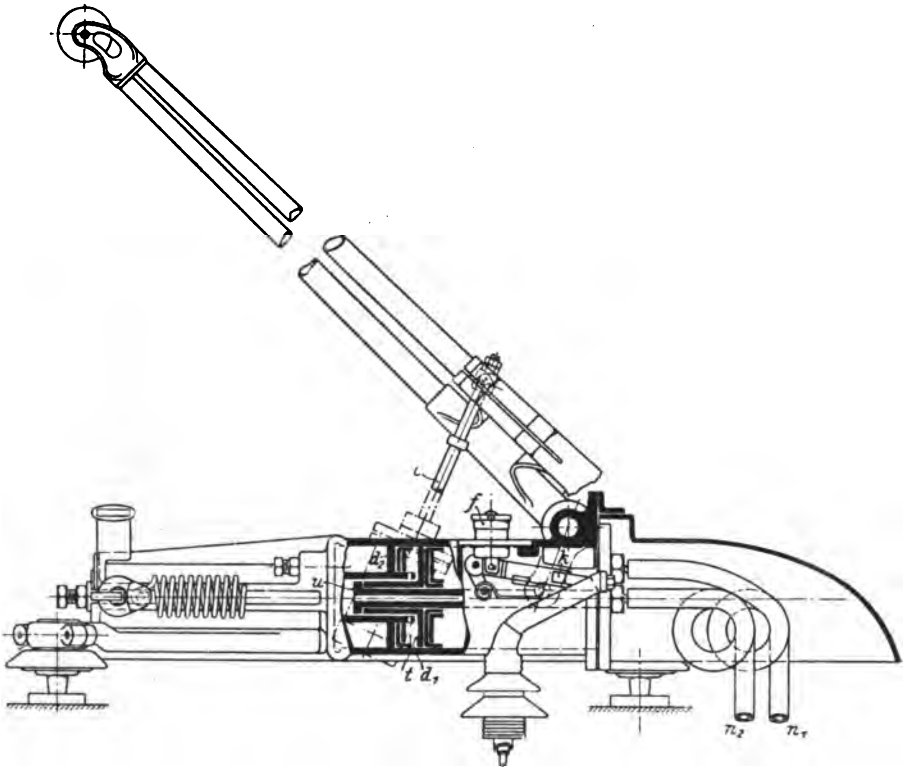


FIG. 39. Current collector for the 1904 locomotives for the Valtellina Railway. (Ganz & Co.)

At the lower speed the pressure on each wire is 14·5 lbs. and at the higher speed 18·7 lbs.

When the air pressure is released the collector falls by its own weight, and in so doing comes into contact with the stud *f* and depresses it, and with it the switch arm *k*, and thus disconnects the whole of the collector and the base from the high tension wiring of the locomotive which may be alive through the other collector.

When both collectors are down, the locomotive is of course entirely disconnected, and in order to start working when there is no pressure in the air reservoirs, a small hand pump is provided whereby sufficient pressure can be applied to the pistons of the collectors to elevate one or the other of them and so obtain current for working the compressor.

The current collector on the Simplon tunnel locomotive. On these locomotives the current collector consists of two parts capable of independent movement. Figure 40 gives a view of the locomotive from which a general idea of the collectors may be obtained. One part consists of the frame which is pivoted to the supports on the roof of the locomotive body and can be raised and lowered by the driver. The other part consists of the two small frames pivoted to the extremity of the main frame, each of the two small frames forming a collector from one of the overhead wires. In this way, small irregularities of level in the overhead wires are taken up by slight movements of the collectors themselves, and large differences of level by the movements of the whole collector gear including small and large frames.

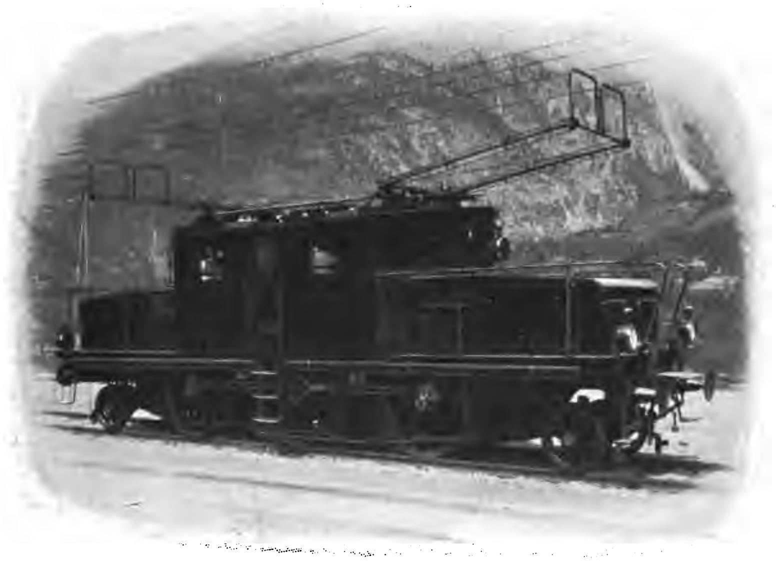


FIG. 40. General view of three-phase locomotive for Simplon tunnel shewing current collectors. (Brown, Boveri & Co.)

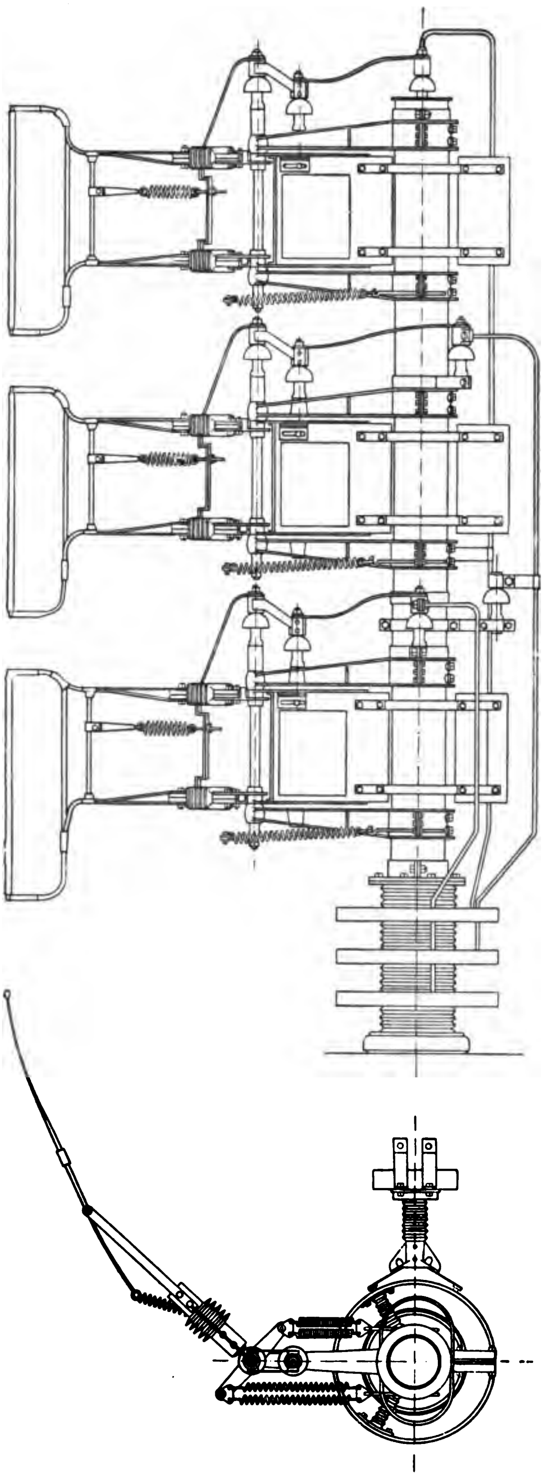


Fig. 41. Current collector for the three-phase high-speed car on the Berlin-Zossen line. (Siemens and Halske.)

The current collector on the Berlin-Zossen high speed car. The current collector on the Siemens car may be said to possess a triple spring system. The general arrangement is shewn in figure 41, in which the upper part represents the three collectors in elevation, and the lower part the same in plan. From the latter it will be seen that at the first stage the springs are four in number working on to a cross bar pivoted at its centre and carrying the frame to which the outer portions of the collector are attached. The second stage of the spring system consists of a single helical spring which acts on the outermost frame carrying the contact bar. This frame is pivoted as shewn and is free to move subject to the action of the spring relatively to the intermediate frame. The third stage of the spring system is a pair of flat steel springs, to the ends of which the contact bar is attached. This contact bar was first made of brass with an aluminium insertion, but subsequently it was made entirely of brass as the high speed gave rise to rapid wear on the aluminium.

The unusual speed also necessitated compensating wind-vanes so that the pressure on the wires could be made independent of the wind pressure on the collectors. These vanes are shewn in the figure on both sides of the mast.

By this system of triple spring the motion due to collectors passing points of support of the wires is divided up among the various moveable parts, so that for the small but very rapid movements only a very light frame including the collector bar responds, whereas for more extensive but slower movements all the frames will participate.

CHAPTER 3.

OVERHEAD CONSTRUCTION FOR THREE-PHASE RAILWAYS.

General. The function of the overhead construction for three-phase electric railways is the same as that for direct current tramways, and necessarily has many points of similarity. In direct current tramways, there is generally only one overhead wire from which current is collected by the trolley wheel or the bow; but occasionally where it is thought advisable to use an insulated return two overhead wires are employed. In precisely the same way it is most usual to utilise the track rails as one conductor, and in that case for three-phase railways two overhead wires are necessary; if the track rails are not used, it is of course necessary to put up three wires, all insulated from each other and from the ground. It is apparent, therefore, that in all cases, there must be at least two insulated conductors at different potentials, and it is this fact which constitutes the essential feature of the distributing system for three-phase railways.

The two types of construction. The two alternatives mentioned in the previous paragraph, viz. those cases in which either two or three insulated wires are used, mark the two different types of construction. In the former case it is usual to arrange the two conductors in a horizontal plane roughly over the centre of the track; in the latter case, of which there is only the example of the Berlin-Zossen line, the three conductors are supported at the side of the track in a vertical plane. A little consideration will shew that at points and crossings the complexity of the construction with two overhead wires at different potentials is much greater than is the case with a single trolley wire, in fact this consideration is one of the most important points in the comparison between three-phase and single phase systems.

General characteristics. It might be suggested that the three-phase distribution system should be identical with that adopted on many direct current railways, viz. the system of third and fourth rails on the track. Such a suggestion would leave out of account one of the advantages of alternating currents, viz. the possibility of supplying high voltage current to the train. If third and fourth rails were installed, it would probably be scarcely worth while to depart from the standard direct current system. There are many cases in which the voltage on the overhead conductors on three-phase lines is not greater than is the practice on direct current railways; but this is because a simple and more or less standard construction has been possible without requiring any higher voltage. In cases where the power to be distributed is considerable, as on the Valtellina Railway and the Zossen line, voltages of 3000 and 10,000 have been used.

It will be evident that the standard trolley wheel so largely used on tramways is unsuitable for most railways. The speed in the latter case is generally much higher, and there are not the same facilities for replacing the trolley wheel on the wire when it has come off, as would almost certainly happen at points and crossings. In practically all cases, the overhead construction is designed for a collector of the bow type with which contact may be made over a reasonable width, and with which considerable lateral movement of the trolley wire is possible without there being any danger of the collector leaving the wire. Examples of such collectors have already been described and illustrated in chapter 2 of this Volume.

It is, of course, advisable that the wires should not always make contact at the same point of the collector, as the resulting wear would soon give rise to a groove, and would soon put the collector out of action. It is the usual practice, therefore, to stagger the wires backwards and forwards over the width occupied by the working surface of the bow so as to distribute the wear.

When the voltage is about 500 or so, the overhead construction is generally of a simple type, being practically the same as would be put up for a direct current tramway with two overhead wires. When the voltage is greater and up to about 3000, with two overhead wires, the construction is still similar, but, as is natural, the insulation is designed on a totally different scale. Various illustrations are given below of that on the Valtellina Railway and the Simplon Tunnel Railway. For higher voltages with three overhead wires the construction on the Zossen line will be briefly described.

The above remarks deal only with the equipment of single or double tracks; the equipment of crossings and junctions requires careful examination and is considered separately.

Overhead construction on the Valtellina Railway. The point of chief interest in connection with any high voltage overhead construction is undoubtedly the system of insulation. For transmission lines there is very little difficulty in providing sufficient insulation;

all that is necessary being to design a suitable porcelain insulator to which the wire is attached at the points of support. When wires are used, however, for conveying current to an electric train, a different system of support is necessary. The wire has to be gripped so as to present a working contact surface to the collector without any obstruction; the support must be flexible; and the insulator must be capable of withstanding or avoiding any shocks due to the rapid passage of the collector.

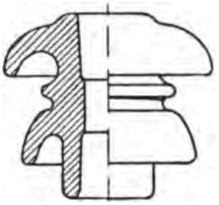


Fig. 42. Strain insulator for overhead construction on the Valtellina Railway.

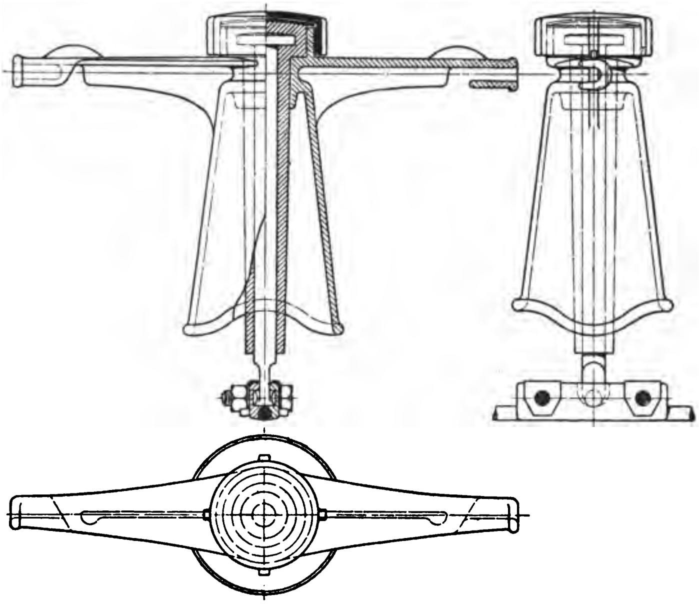


Fig. 43. Insulating hanger for overhead construction.

On the Valtellina Railway, equipped by Messrs Ganz and Co., the type of insulation is not very different from usual tramway practice, the chief innovation being the provision of porcelain strain insulators in the span wires. In addition to this insulating hangers are used of the ordinary type, but with insulating bolts longer than usual surrounded with a sheet iron petticoat. The bolt is about 6½ inches long, and is attached at its free end to a mechanical clip which grips the trolley wire. The strain insulator is in the form of a porcelain shackle with a bolt through the centre and a band round the middle. Figure 42 shews the strain insulator, and figure 43 the insulating hanger. Figure 44 shews a typical structure consisting of two wood poles and a cross bar. One of the poles also carries the three high-voltage transmission wires. On each pole is

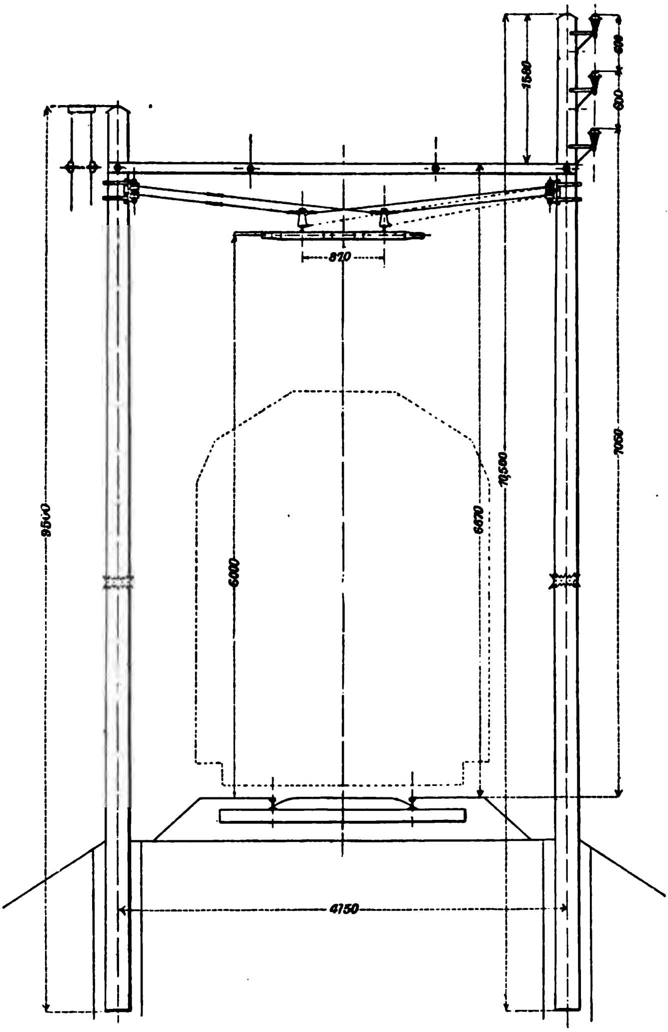


FIG. 44. General arrangement of overhead construction on the Valtellina Railway.

fastened a clip which supports two porcelain shackles; from these go two span wires which take the insulating hangers. Figure 45 shews a construction with a wood pole and a side bracket. It will be seen, therefore, that a suitable trolley wire clip is provided, also double insulation, one of which is of porcelain; and that the method of support is flexible, and is such that no direct shocks are received by the porcelain insulators.

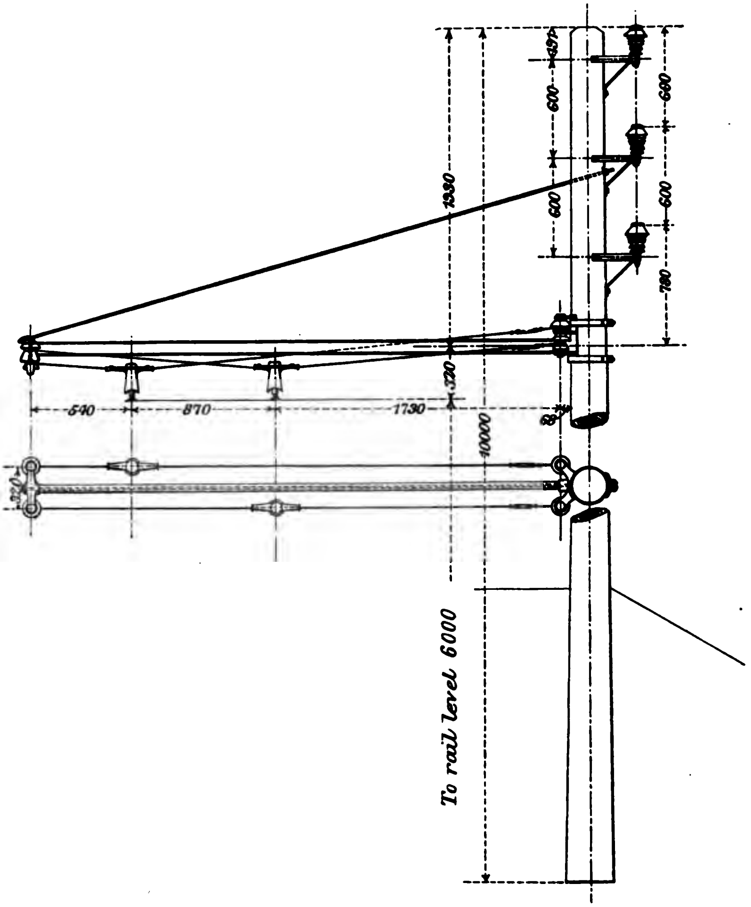


FIG. 45. Side bracket overhead construction on the Valtellina Railway.

Overhead construction on the Simplon Tunnel Railway. The conditions in the Simplon Tunnel are very severe as regards the difficulty of effective insulation. The atmosphere in the tunnel is hot and damp, and consequently the insulating qualities of the system of support have to be on a higher scale than on the Valtellina Railway. The combination of porcelain and rubber composition is adhered to, but the arrangement adopted is quite different. A single span wire is used, insulated at both ends by means of a porcelain shackle, and on this span wire are fastened the trolley wire supports. In the Valtellina construction, the arrangement with two span wires, one per phase, secured double insulation, porcelain and rubber composition, between the phases. In the Simplon Tunnel, as there is only a single span wire, it is necessary to provide trolley wire supports which are themselves doubly insulating.

The arrangement actually adopted has been designed by Messrs Brown, Boveri and Co., and is clearly shewn in figure 46. It consists of a hard rubber bolt fixed into a bronze cross piece, which at both ends is screwed into a porcelain petticoat insulator. The two porcelain insulators are similarly screwed into bronze caps which are bolted on to the span wire. To take up the shocks due to the passage of the collectors asbestos fibre is inserted between the insulated bolt and the cross piece, and both inside and outside the porcelain insulator. Thus double insulation is inserted between the trolley wire and the span wire, both the porcelain and the rubber being designed to stand a working voltage of 18,000 volts. The attachment of the span wire and porcelain shackle to the tunnel wall is shewn in figure 47.

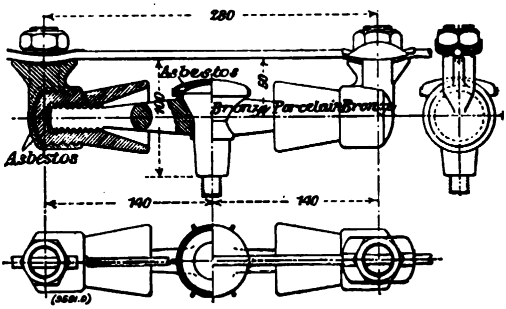


FIG. 46. Trolley wire insulators in the Simplon Tunnel.

Distribution of current on the Berlin-Zossen Railway. The method and apparatus for supporting the contact wires designed by Messrs Siemens and Halske for the Berlin-Zossen Railway are quite different from the foregoing examples. This difference is chiefly due to two circumstances, viz. the use of 10,000 volts, and the unprecedented speed (120 miles per hour):

The high voltage and the frequency of 45 induced the engineers to use three insulated wires rather than to bond the track for one conductor. This system eliminates any disturbance of telegraphs and telephones due to earth currents (provided the three phases are equally loaded, the centre point of the star being earthed). The high speed required side collection so that the collectors should not leave the wires as they passed each point of support.

The actual arrangement is quite simple and is shewn in figure 48. A channel iron is bolted to a wooden pole and bent round at its ends, between which is stretched a vertical span wire. This span wire is insulated at the top and bottom by four link insulators in series; and the three insulating supports are attached to the span wire as shewn in the figure. These insulating supports consist of a metal body which clips the span wire and on which a porcelain petticoat insulator is mounted. On the head of the porcelain insulator is fixed a cap which grips the trolley wire. On this cap there is also a horizontal loop of wire through which passes a vertical wire stretched between the ends of the channel iron. These loops are designed as safety devices; if any one of the trolley wire breaks, the tension in the nearest unbroken span pulls over the insulator from its normal position and brings the loop into contact with the vertical wire and thus earths the broken wire.

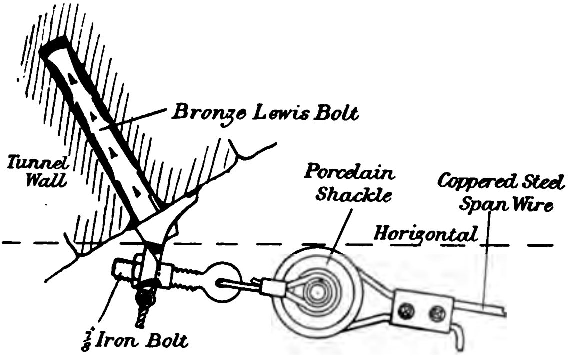


FIG. 47. Attachment of the span wire to walls in the Simplon Tunnel.

Three-phase overhead line construction at points and crossings. It has frequently been urged against the three-phase system for traction that the overhead construction at points and crossings is so complicated as to make the general adoption of this system practically impossible. There can be no doubt that such construction for three-phase systems is not so simple as for single phase or direct current systems; but such a statement is not sufficiently precise to be of much value. In order to come to any rational conclusion, an engineer must have before him the necessary materials for a decision; examples are therefore given in figures 49 and 50, shewing possible constructions for a cross-over road between two parallel tracks and for a double track junction, on the three-phase system. These are merely two particular cases and as in practice there will be many different arrangements of tracks it is well to keep in mind the principles on which such constructions must be laid out.

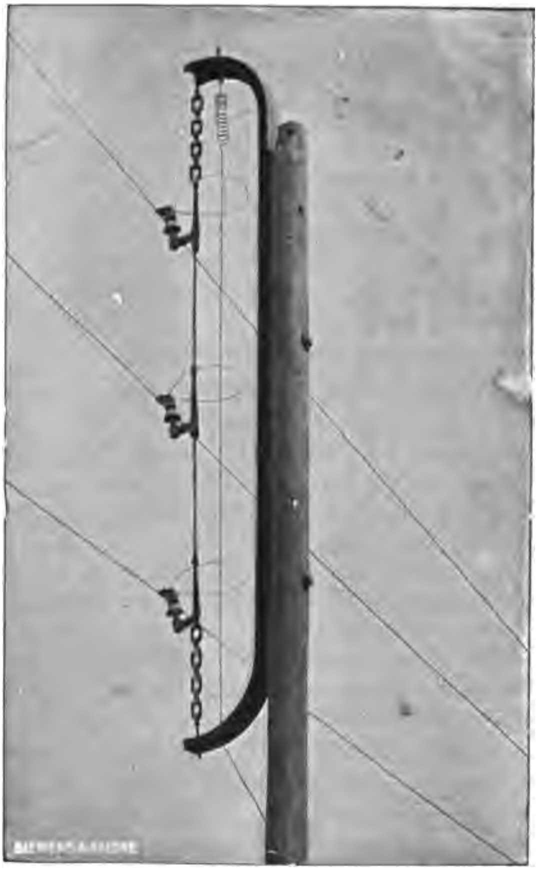


FIG. 48. High tension trolley wire suspension on the Berlin-Zossen Railway.

It may be assumed, in general, that in a train containing several motor cars, each motor car is a separate unit and is not connected electrically with other motor cars, except by auxiliary circuits (control wires, etc.). It is only necessary, therefore, to consider the passage of a single car over any special bit of track; but at the same time it should be assumed to be fitted with two sets of collectors, one at each end of the car. The requirements, therefore, for any particular overhead construction may be laid down as follows:

(1) No live wire of one phase must trespass on the space traversed by a collector of the other phase.

(2) A continuous contact (whether on live or dead wires) must be provided for each collector (assuming separate collectors for each phase, *i.e.* four in all on a car) at all points of its travel.

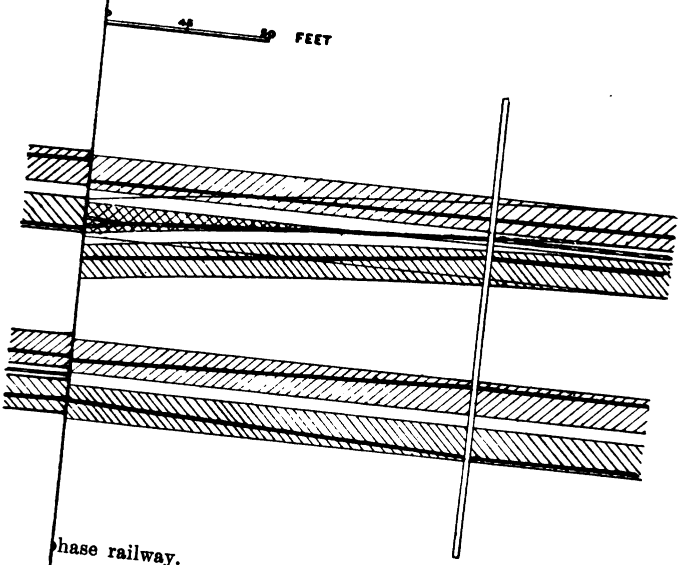
(3) There must be no possibility of a short circuit between the two phases due to a dead wire bridging between two live collectors of different phases.

The first of these requirements needs no comment, and the second is due to mechanical considerations, it being not advisable to allow a collector to travel freely in the air at high speeds. The third requirement is obvious enough and is inserted as a warning; it is quite possible with some constructions for the front collector of one phase and the back collector of the other phase to be touching the same dead wire, both collectors being alive at the time. Such an arrangement must inevitably produce a dead short circuit.

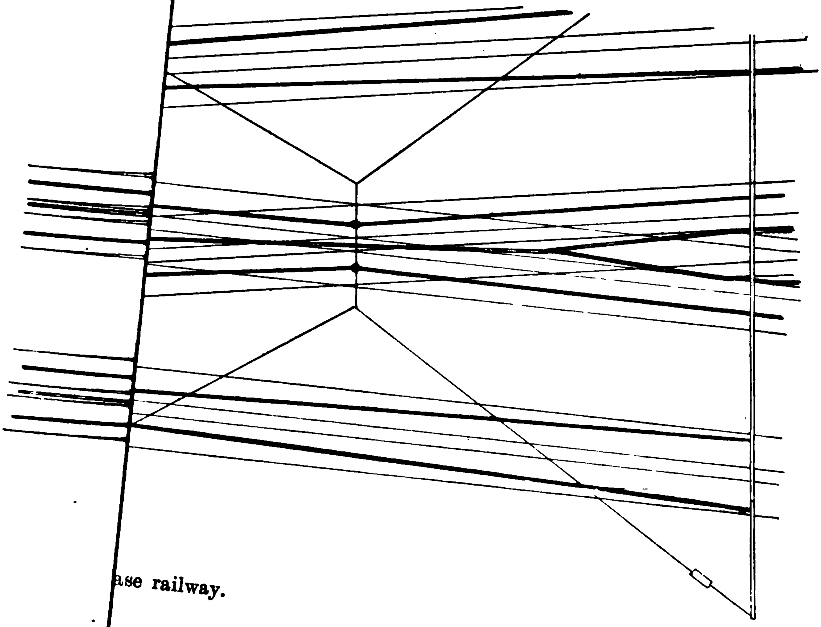
As a preparation for laying out the wires, draw out two parallel strips representing the spaces traversed by the collectors of the two phases separated from each other by a narrow strip representing the clearance between the live portions of the two collectors. If one phase be indicated by hatching downwards from left to right and the other from right to left the spaces which may contain live wires will be clearly marked. Down the centre of each track will be a narrow unshaded strip indicating clearance, and where two tracks cross there will be a space cross-hatched indicating the crossing of different phases. Further, where tracks cross there must also be a clearance space outside each collector, which may be left unshaded. These different portions are clearly shewn in figure 49.

The continuous support for each collector is provided by live wires in the single-hatched spaces and must be provided in all other spaces by dead wires. In the simplest construction the live and dead wires are mechanically continuous, being separated electrically by section insulators. These insulators must be placed as near as practicable to the boundaries of the single-hatched spaces. The problem is, therefore, how to dispose the dead wires so that the above requirements may be fulfilled.

Consider now the left-hand half of figure 49 for the cross-over, and more particularly that part which includes the converging tracks. Starting from the centre of the figure there are in this part four live wires numbered 1, 2, 3 and 4. Of these numbers 1 and 4 continue without interruption on to and past the points. The centre wires 2 and 3 if continued would have to leave the single-hatched strips in which



hase railway.



ase railway.

they start, and they must therefore terminate in section insulators as shewn in the figure. From these insulators two dead wires start and both must follow as closely as practicable the curves of their respective lines. For this purpose the two dead wires are drawn together by a clip in the cross-hatched space, from which point they continue together to the apex of this space, and thence to the centre of the clearance strip. At this point a span is put over the double track and the dead wires are supported by suitable insulated supports, and from there they continue along the clearance strip until the next span where they are anchored off, somewhat above the level of the live wires. Figures 51, 52 and 53* illustrate this arrangement, being taken from photographs of actual constructions on the Simplon Tunnel Railway.

In order that a continuous support may be provided it is necessary that each bow before it reaches the apex of the cross-hatched space should be travelling also on the live wire of its proper phase. This is ensured by the proper disposition of the live wire.

The third requirement is met by arranging the dead wires so that they never enter any single-hatched space.

Figure 49 relates to a cross-over road of standard 4 ft. 8½ ins. gauge between two tracks 6 ft. apart, the slope of the crossing being 1 in 6. An inspection of the figure will shew that provided the distance between the front and rear collectors on a motor car is not less than 25 ft. the gap on any of the lines will be bridged. This distance is not excessive, and there is generally no difficulty in arranging the collectors 30 ft. apart or more. (On the Valtellina motor cars the distance is 38 ft., on the first type of locomotive the distance is 23 ft.)

Figure 50 shews a similar arrangement for a double track junction with the same slope of crossing.

It is important to observe that the point at which a collector meets a fresh wire should be sufficiently near to a support that there may be no danger of the wire not riding up easily on to the collector. It may easily happen, if there is too much sag at this point, that the wire may foul the side of the collector and not rise up at all, in which case there must inevitably be a breakdown.

Complicated junctions. No definite statements can be made as to the possibility of equipping railway junctions such as occur outside large terminal stations. On any system, whether single phase or three-phase, there will be a good deal of difficulty, although it is highly probable there will be more difficulty with three-phase. Each case,

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* The authors are indebted to Messrs Brown, Boveri and Co. for these photographs.

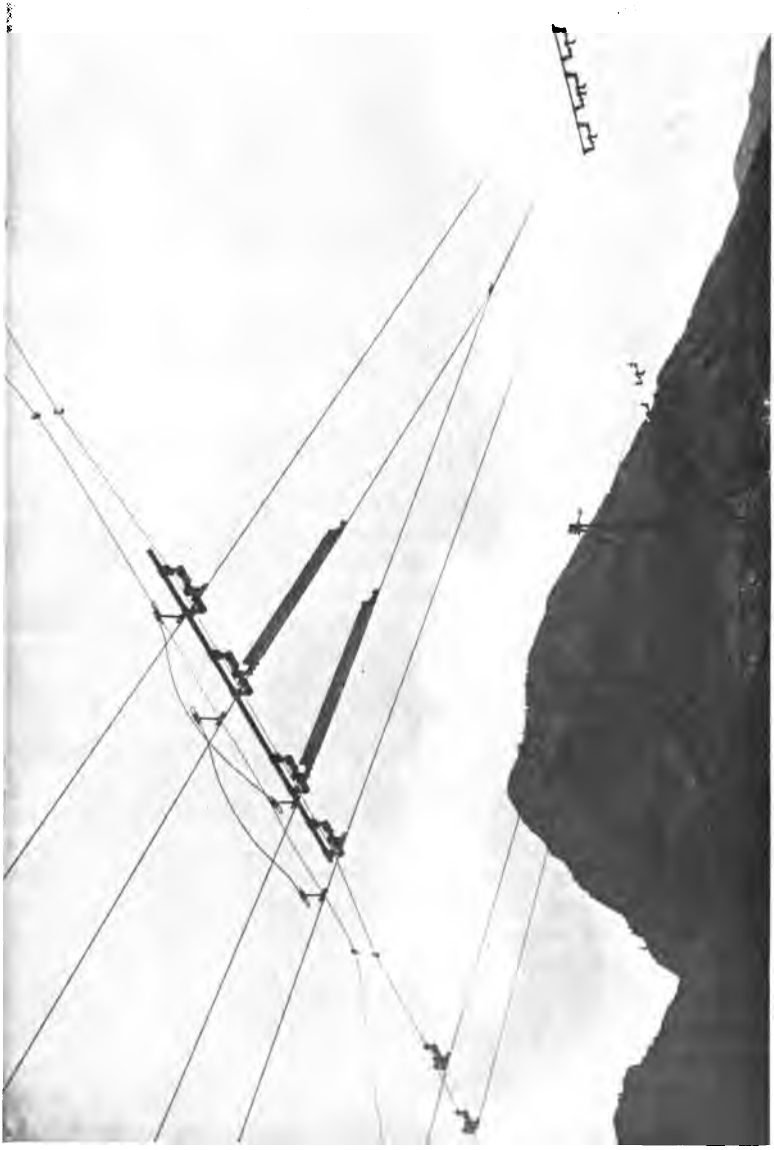

FIG. 51. Overhead construction at a junction on the Simplon Tunnel Railway.

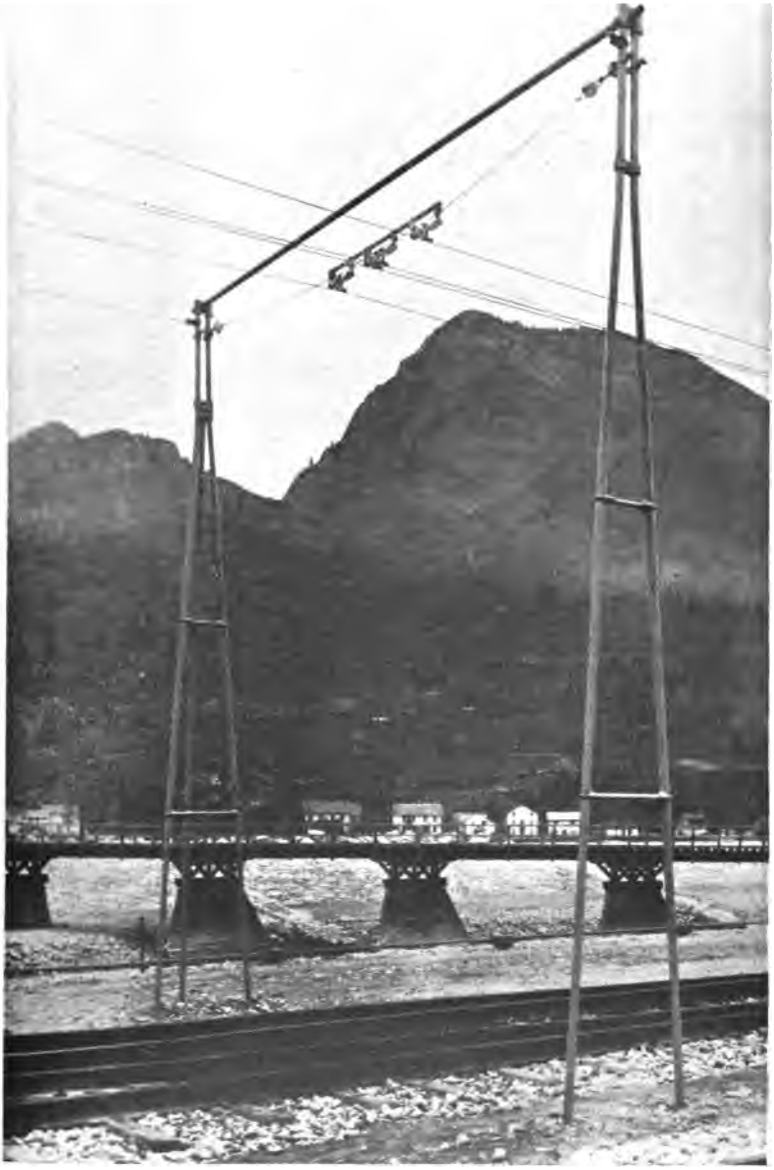

FIG. 52. Overhead construction at a junction on the Simplon Tunnel Railway.

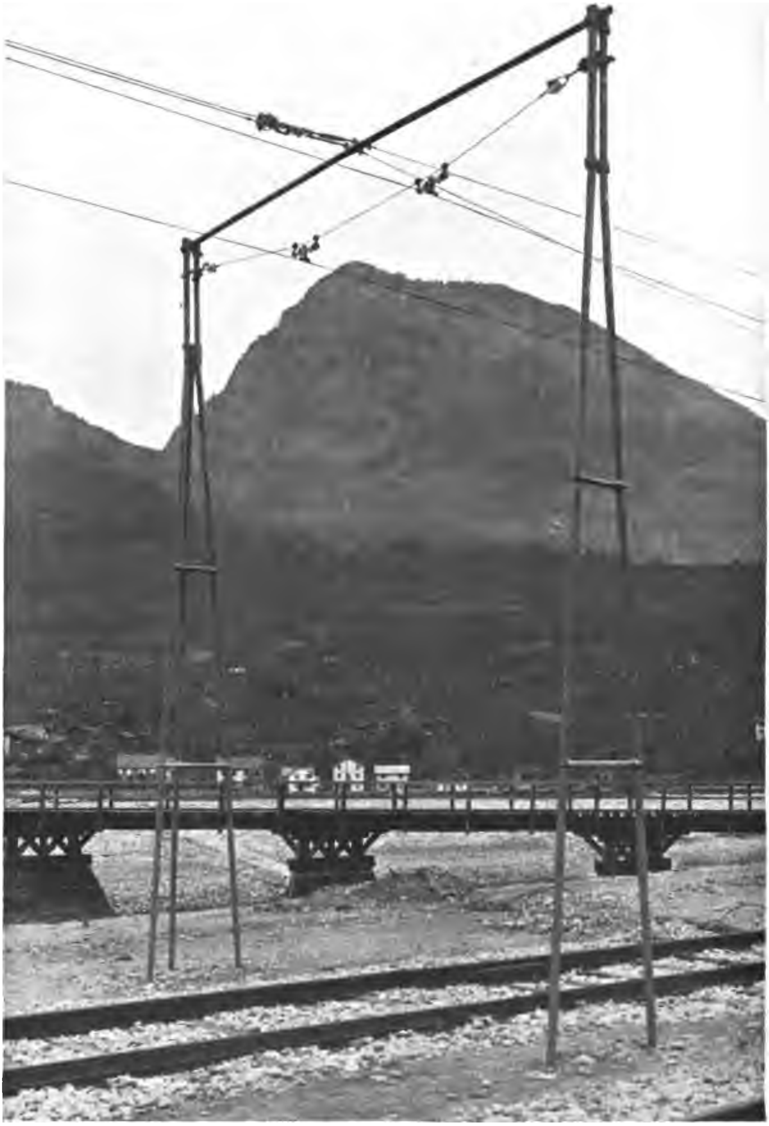

Fig. 53. Overhead construction at a junction on the Simplon Tunnel Railway.

however, must be dealt with individually, as in all probability no two places will have the same requirements.

It must be borne in mind that it is not sufficient merely to provide a continuous support, alive or dead, for the collectors, and such gaps as can be bridged by a motor car with two sets of collector gear. At such places it is absolutely essential to arrange the construction so that a sufficient subdivision is provided. Separate tracks must be capable of isolation in case of breakdown, and the isolation should as far as possible include a train section (or a motor car section) which can be made dead and cannot be bridged. These precautions must be combined with the sectionalising of cross-over roads in order that the divisions may be effective. It is evident, therefore, that a complete scheme for an overhead equipment outside a large terminus would require very careful examination from many points of view.

At such places there are frequently a large number of tracks, and it may be of interest in connection with the planning of the supports for the spans to quote the Board of Trade requirement as to clearances:

"No standing work (other than a passenger platform) to be nearer to the side of the widest carriage in use on the line than 2 ft. 4 ins., at any point between the level of 2 ft. 6 ins. above the rails and the level of the upper parts of the highest carriage doors." (*Railway Signalling* by H. Rayner Wilson.)

Thus the distance between tracks which will permit the erection of a pole will vary from one railway to another; but allowing the maximum width of carriage as 9 ft., and a space 1 ft. wide for the pole, the minimum distance between adjacent gauge lines will be 9 ft. 11½ ins., or say 10 ft.

Several interesting examples of large railway stations, shewing the arrangements of the various tracks, are given in *Railway Signalling* by H. Rayner Wilson.

Three-phase overhead construction at junctions and crossings for extra high pressure. In the two examples worked out above and shewn in figures 49 and 50 it has been assumed that a trolley line can be made mechanically continuous, but electrically discontinuous, by means of a section insulator. This is satisfactorily accomplished on the Valtellina Railway and on the Simplon Tunnel Railway; but in both these cases the voltage is not above 3300 volts. On the Berlin-Zossen experimental line the voltage was about 10,000 volts; but the type of construction on this line is hardly suitable for a general system which contains crossings and junctions. As will be seen later (Chapter 7 of this Volume) section insulators are in use on single phase lines where the voltage is 6000; but for higher voltages such as 10,000 or more it is almost necessary to adopt some method other than interposing an insulator mechanically continuous with both wires. Such a method consists in anchoring off both wires by means of porcelain strain insulators, the points of anchoring being such that the two wires run parallel to each other for a short length. Thus a continuous support is provided for the collector without there being any electrical connection between the two wires (see page 261, fig. 159). This type of section insulation will necessarily introduce fresh complications due to the space required and the number of supports, and the fact that the dead section is made alive by the bridging of the two parallel wires by the collector whenever a motor car passes the insulator. At present, however, this point need not be elaborated, as there has been no proposal to adopt a higher voltage than 3300 for three-phase overhead construction.

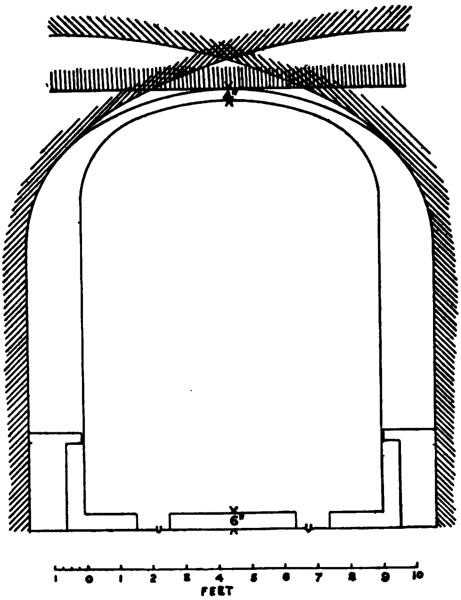

FIG. 54. Diagram shewing clearance between maximum rolling stock and minimum structure.

Overhead construction in tunnels and under bridges. There is a prevailing impression that on English main line railways the clearances between trains and bridges or tunnels is so small that it is quite impossible to erect an overhead construction which shall be equally satisfactory in tunnels, under bridges and in the open. This opinion is apparently confirmed by a glance at a diagram shewing typical outlines for the maximum rolling stock and the minimum structure. Such a diagram is shewn in figure 54, and from this figure it is at once apparent that the clearance between the minimum structure and the maximum rolling stock is so small as to preclude the possibility of supporting any live wires within it. The following consideration*, however, will shew that this difficulty has been much exaggerated.

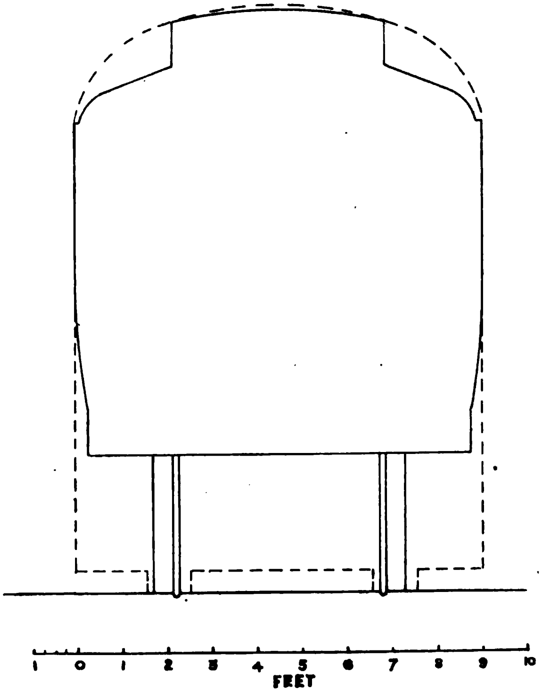

FIG. 55. Maximum rolling stock diagram.

The profile of the maximum rolling stock represents the space within which no permanent structure can be placed without fouling some part of one of the various types of rolling stock. It is in fact a compound drawing, not necessarily representing any actual coach section, but including all the different types such as passenger coaches and loaded goods-wagons (as illustrated in figure 55†). It is, therefore, absolutely necessary that any overhead construction must be designed so as to leave a reasonable clearance between it and the profile of the maximum rolling stock. This clearance will, of course, depend on circumstances, but for live high tension wires a minimum distance of 4 ins. will probably be safe.

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* This method of treating the problem is due to Mr C. F. Jenkin of Messrs Siemens Brothers Dynamo Works, who has made a special study of the design of overhead construction under arches and in tunnels.

† The passenger coach illustrated here is described in *Engineering* for April 21, 1905, p. 501. The profile of the maximum rolling stock is hypothetical, but is typical of those on several English railways which differ only slightly from one another.

The profile of the minimum structure is similarly a compound drawing as indicated in figure 54. The relation between the live wires and this minimum structure is quite different from that between the live wires and the maximum rolling stock, in that it is only necessary to treat each individual structure separately, and the wires may be supported in different positions for different structures independently of the minimum structure. This consideration, together with the fact that the great majority of structures allow more clearance than the minimum, very greatly facilitates the design of overhead construction.

As an example, take the case of the double tunnel shewn in figure 56. To prepare a design for such a case draw out first the tunnel outline and the profile of the maximum rolling stock in their proper relative positions. Next, draw lines parallel to these two profiles at 4 ins. distance from them, inside the tunnel profile and outside the maximum rolling stock. These lines will mark out the clearances that must be left, and the space between them represents the room available for the live parts of the overhead construction. In the figure, for the sake of clearness, the clearance spaces are shewn shaded, the space between them being left unshaded. In arranging the positions of the live wires and their supports it is necessary to bear in mind (1) that the wires sag between supports, and that the lowest points must not come within the clearance space, (2) that the collector rises and falls with the position of the wire and that in its highest position it must not be within the clearance from the tunnel, (3) that it is desirable, though not absolutely necessary, to zigzag the wire to prevent undue wear on the collector; for a short tunnel this precaution may be omitted. In the case shewn in figure 56 it has been assumed that the tunnel is cut in the rock and that, therefore, a pocket can be cut wherever necessary to receive the attachment of the span wire on which the trolley wires are supported. If the construction of the tunnel were such as not to permit the cutting of a fairly deep pocket it would be necessary to put the trolley wire as far as possible in the position shewn in thin lines with a dotted span wire. It has also been assumed that supports are spaced about 50 ft. apart so that the sag may not exceed 3 ins.; one pair of supports is shewn in full lines, and two other positions are indicated thus +, the centre of the cross marking the position of the trolley wire at its point of support. It will be observed that in the position shewn in full the trolley wire is 3 ins. above the rolling stock clearance and the collector at its nearest point is about 3 ins. below the tunnel clearance, to allow of travel to the highest point of the trolley wire. The type of insulation shewn is that installed in the Simplon Tunnel.

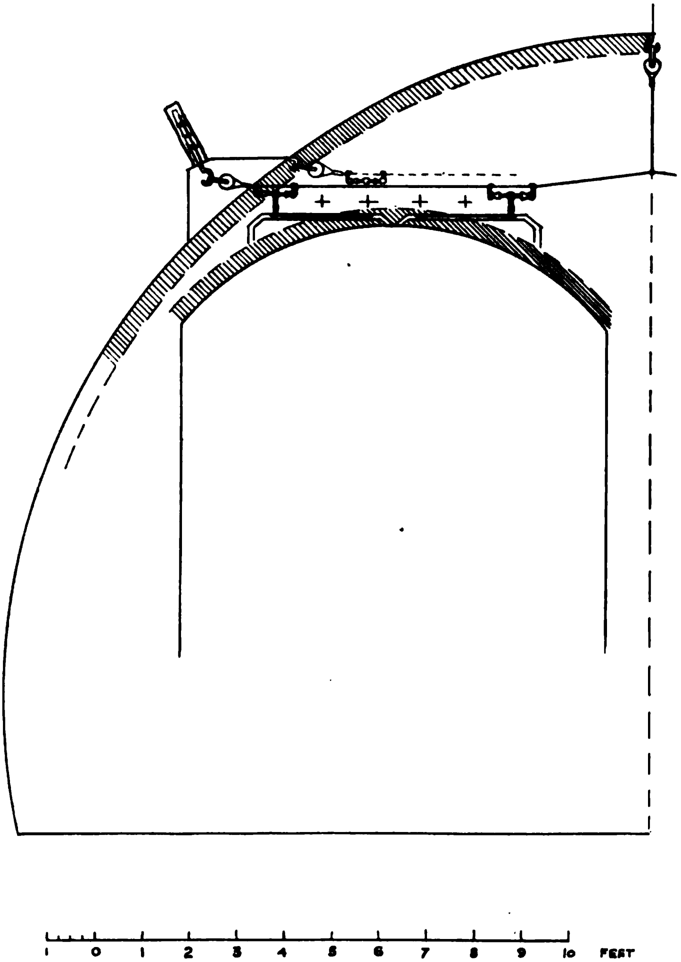

FIG. 56. Overhead construction in a double tunnel for three-phase railway.

If steam locomotives run over the same line as the electric trains it is advisable not to put the trolley wire supports and insulators too near the centre line of the track, otherwise the smoke and steam, and possibly the flame, from the funnel may destroy the insulating material.

For arches and very short tunnels it is generally not necessary to make provision for insulating supports.

It will be seen from the example given above that in many cases there is no impossibility in erecting a suitable overhead construction in a tunnel. When the tunnel profile is such as to render the construction impossible, there are only two alternatives, (1) to shift the track either vertically or horizontally so as to increase the clearance, or (2) to shift the structure, arch or bridge. Whether either or both of these alternatives is impossible rests with the engineer of the line.

CHAPTER 4.

CALCULATIONS OF ENERGY CONSUMPTION ON THREE-PHASE RAILWAYS.

In making calculations on the consumption of energy on the polyphase system of electric traction, the same preliminaries are necessary as in the direct current system, namely the estimation of the tractive resistance and of the weight of the train. The tractive resistance has been discussed already in Chapter 18, Vol. I., and also the estimation of the weight of the train.

Weight of the equipments. For estimating purposes the weight of a three-phase equipment may be taken as the same as that of a direct current equipment of equal capacity, with one exception. If the three-phase motors are wound for low voltage, and the distribution is at a high voltage, a transformer is necessary on each motor car.

It is not advisable to discuss here the design of transformers, as this is a special branch of electrical engineering. Only one point need be referred to, namely the modifications that may be introduced into the design if the transformers are required for intermittent service. As far as the windings are concerned, these must of course be proportioned for the R.M.S. current; but the iron of the magnetic circuit must be designed in relation to the way in which it is used. In some cases the high voltage is applied constantly to the primary winding although current is taken from the secondary intermittently; in other cases the secondary load is switched off by disconnecting the primary. In the former case, the iron loss continues without intermission, whereas in the latter case the iron loss continues only so long as the load is on the secondary. In some cases, the iron may be worked at a greater induction if the primary circuit is opened every time, and this may give rise to a diminution in the weight of the transformer.

Rheostatic control. Consider first the case of a short run with a simple rheostatic start and no electrical braking.

In such a case the supply of energy from the line is kept constant throughout the starting period until the motors attain their full speed; from this point the supply diminishes rapidly until it reaches another constant value which is just sufficient to maintain a constant speed.

(If there are any grades the supply will accommodate itself so as to keep the speed within a small percentage of synchronous speed above or below as the case may be.) At the proper point the supply is cut off and the train coasts until the application of the brakes.

As a concrete example, take the case already worked out in Chapter 18, Vol. I.; that is to say, assume a run of 3000 feet to be performed in 99 seconds. It is not necessary to consider again the heating of the motors; the method of calculation is in no way different from that already given. For the purpose of finding the consumption of energy assume a total load of 1 ton dead weight, and further assume a motor efficiency at the end of the period of constant acceleration of 88 per cent. and during steady running of 80 per cent.

Then, as in the case of the direct current system, the consumption of energy will depend partly upon the losses in the rheostats and partly on the losses in the brakes.

Suppose the train is accelerated at 1·5 feet per second per second up to 40 ft. per sec. Then the calculation is as follows:

Input to motors which is kept constant during acceleration

$$\frac{(1\cdot 5\times 70\times 1\cdot 06+10)\times 40}{550}\times \cdot 746\times\frac{1}{\cdot 88}=7\cdot 45\,\text{kw.}$$

(allowing for rotational inertia 6 per cent.).

Kw. input	Speed, ft. per sec.	Pull due to motors, lbs.	Tractive resistance, lbs.	Nett pull, lbs.	Acceleration, ft. per sec. per sec.	Time, secs.	Distance, ft.	Total Time, secs.	Total Distance, ft.
	0							0	0
7·45		121	10	111	1·5	26·6	534		
	40							26·6	534
·95		14	14	0	0	40	1600		
	40							66·6	2134
0		0	14	−14	−·2	13·5	524		
	37·3							80·1	2668
brakes									
	0							99	3000

The speed time curves for this run are shewn in figure 57. Proceeding in the same way as indicated in Chapter 18, Vol. I., the consumption of energy is found to be

236,000 watt seconds = 65·5 watt hours,

that is to say, the watt hours per ton-mile $= 65\cdot 5\times\frac{5280}{3000}$

$= 115.$

This result may be compared with the corresponding figure for the direct current system thus:

	Direct current	Three-phase
Max. power per ton	5·44 kw.	7·45 kw.
Watt hours per ton-mile	75·6	115

As with direct currents, the energy consumption could be improved at the cost of the maximum power taken by the train; but enough has been said to shew that for a service with frequent stops the rheostatic control is a serious handicap, and in a general way may be said to be prohibitive.

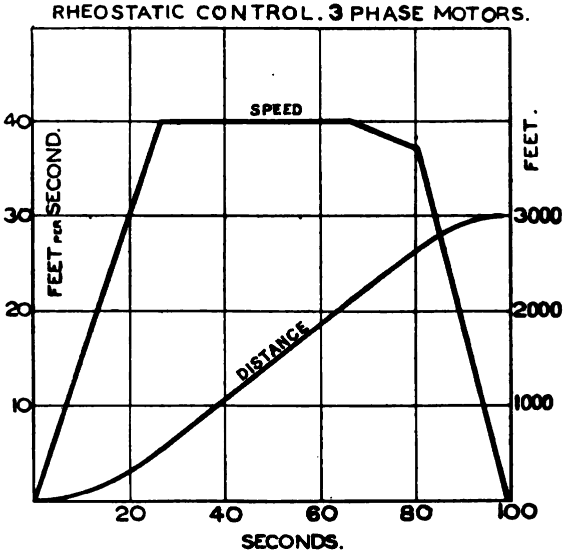

FIG. 57. Speed-time curves for typical run on three-phase railway with rheostatic control.

For other classes of service, however, the influence of the inefficient start is not so important, and it is not wise to make any sweeping statement as to the suitability or the reverse of this system.

Cascade-single control. Consider next the case of cascade-single control applied to the same problem.

As before, take a distance of 3000 feet, but in this instance reduce the running time to 95 seconds in order to make the result comparable with the previous calculations in which the retardation with brakes was assumed to be about 2 ft. per sec. per second, whereas in this case it will be more convenient to assume a figure of 3.

Take two motor cars each equipped with two main and two auxiliary motors the particulars of which have been worked out in Chapter 1 of this Volume; and let the train be made up of these two cars each weighing 36 tons, and sufficient trailers to make up a total of 136 tons.

Gear all the motors to 36 in. driving wheels by gearing with a ratio of 70 : 24; with these particulars a set of performance curves can be constructed, and the calculations will be as follows, allowing as before 6 per cent. for rotational inertia:

True kw. per pair of motors	Speed, ft. per sec.	Pull in lbs.			Acceleration, ft. per sec. per sec.	Time, secs.	Distance, ft.	Total	
		From motors	Track resistance	Nett				Time, secs.	Distance, ft.
	0							0	0
164		20000	1360	18640	1·84	10·7	105		
	19·7							10·7	105
191		12840	1360	11480	1·13	17·5	516		
	39·4							28·2	621
27		1770	1770	0	0	31·8	1279		
	40							60	1900
0		0	1770	- 1770	-0·2	23	870		
	35·5							83	2770
-169		-29000	1360	-30360	-3	5	140		
	20							88	2910
0		brakes	brakes	brakes	-3	7	70		
	0							95	3000

These curves are plotted in figure 58, the curve of power being for the true kw. per train. Integrating this curve, the watt hours per ton-mile are found to be 73·5; similarly the R.M.S. current of each main motor is found to be 24·3 amperes, and it is fairly evident that the auxiliary motors will not overheat.

The comparison now stands as follows:

	Watt hours per ton-mile	Maximum kw. per ton
Direct current series parallel control	75·6	5·44
Three-phase rheostatic control	115	7·45
Three-phase cascade, single control	73·5	5·62

shewing that the direct current system produces almost identical results with the three-phase cascade single system.

The following remarks, however, must be made:

(1) In the alternating current system the motors are approximately 50 per cent. larger than in the direct current system, and consequently the weight of the electrical equipment

will be about 8 tons higher. This must be allowed for in comparing the consumption of energy per train.

(2) In both cases the train is supposed to be perfectly controlled. This remark applies more especially, of course, to the alternating current system. In particular, no allowance has been made for an interval between the cascade and the single working (see p. 73). Moreover the driver is supposed to be able to operate the cascade regeneration and the brakes so as to maintain a uniform retardation.

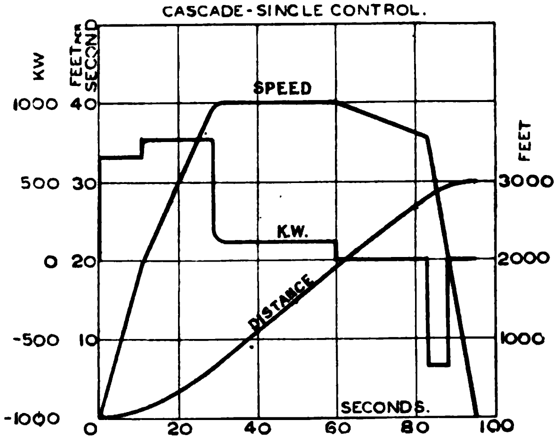

FIG. 58. Speed-time curves for typical run on three-phase railway with cascade single control.

(3) The question of line losses is not considered, since the two methods are not comparable in this particular. In the one case, viz. with direct currents, a low tension distribution is necessitated by the requirements and limitations of the motors; in the other case the voltage of distribution is not so limited, and as a matter of fact has been assumed to be 3000 volts. In this comparison, therefore, it is not necessary to emphasise the low power factor in the alternating current system.

(4) In the example worked out here the total input per train is 23,820 kw. secs., and the output during regenerative braking is 3,380 kw. secs., making the nett input 20,440. Thus the energy returned to the system is 16·5 per cent. of the nett input.

It is interesting to compare these results with the figures put forward by Herr Blathy before the Metropolitan District Railway Arbitration.

Distance between stops	2500 ft.
Running time	92 seconds.
Cascade acceleration	2·6 ft. per sec. per sec.
Single acceleration	1·44 " "
Maximum speed	25 miles per hour.
Speed at which cascade braking is applied	17½ miles per hour.
Watt-hours per ton-mile	71.
Recuperation of energy	9 °/o of nett input.
Maximum energy per ton (for 150 ton train)	6·7 kw.

Cascade-parallel control. In the same way, the case of cascade-parallel control can be examined when applied to a similar run. Take for instance two motors each of them the same as the auxiliary motor, the design for which is worked out in Chapter 1 of this Volume. Mount these on 36″ drivers with a gear ratio of 69 : 25, and supply them either from a low tension distribution or from a high tension line by means of a transformer carried on the car.

Take a train weighing in all 185 tons, including two motor cars each equipped with four similar motors. Start the train with the motors in cascade, and at half-speed switch over to parallel running. Then the calculation will be as follows:

True kw. per train	Speed, ft. per sec.	Pull in lbs.			Acceleration, ft. per sec. per sec.	Time, secs.	Distance, ft.	Total	
		Motors	Track resistance	Nett				Time, secs.	Distance, ft.
	0							0	0
656		18900	1850	17050	1·245	16·7	174		
	20·8							16·7	174
1275		20000	1850	18150	1·325	15·7	490		
	41·6							32·4	664
159		2400	2400	0	0	27·6	1166		
	42·3							60	1830
0					-0·2	21	840		
	38							81	2670
-680		-27400	1850	-29250	-2·14	7·5	225		
	22							88·5	2895
		brakes	brakes	brakes					
	0							99	3000

These curves are plotted in figure 59. Proceeding in the same way as with the other systems the watt-hours per ton-mile are 80, and the maximum power required per ton 6·9 kw.

From the point of view of consumption of energy and maximum power this system possesses no advantage over the cascade-single system. It has, also, precisely the same limitations as regards the number of speeds at which the train can run economically.

On the other hand it should be noted that the same motor capacity is sufficient for 185 tons with the cascade-parallel system as against 136 tons with the cascade-single system. Against this may be put the increase of cost due to the necessity of transformers and to the greater complexity of switch gear.

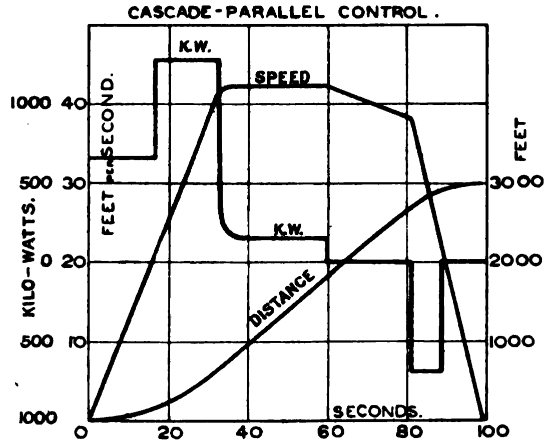

Fig. 59. Speed-time curves for typical run on three-phase railway with cascade-parallel control.

Balancing the advantages and disadvantages, it seems clear that, for a service such as has been supposed, the cascade-parallel system presents no advantage.

Such a service may be said to be at one end of the scale the variable of which is average speed combined with frequency of starting and stopping. At the other end of the scale is the class of service consisting of long non-stop runs. In such a service the starting requirements are of comparatively little importance, and it would, naturally, be out of the question to equip the train with a set of auxiliary motors to be used only for starting and stopping. It is quite clear that for this class of service all the motors must be in operation during that period which is of the greatest importance, viz. the period during which the train is running at full speed.

The choice then lies between the rheostatic control and the cascade-parallel control (always supposing it is decided to adopt some form of three-phase system). These two methods involve practically the same apparatus. It is highly probable that a transformer would be used in both cases, and in the rest of the apparatus there are only very slight differences such as the addition of a few switches to control the cascade connection. The number of rheostats is identical, since in both systems there must be as many as there are motors in parallel.

The advantage of one system over the other lies in the greater flexibility of the cascade-parallel system. The rheostatic system is capable of running economically only within 4 per cent. or 5 per cent. of the synchronous speed, above or below. If the requirements of the service are sufficiently met by this condition there is no reason for adopting any other system. An example of such requirements has already been quoted in the high-speed experiments on the Berlin-Zossen line.

If, on the other hand, the conditions of service are not so rigid, there are obvious advantages in providing for two economical running speeds. Such a provision is made by the use of the cascade-parallel system. The economy of the cascade connection, however, must not be exaggerated. Reference to the performance curves in Chapter 1 of this Volume will shew that the range of working over which the efficiency and power factor are high is excellent for the motors in parallel but comparatively limited for motors in cascade. (Theoretically this could be corrected by adjusting the voltage of the supply to the motors, but this would introduce several undesirable complications.) To make this point clearer, take an illustration. Suppose a train to be equipped with motors controlled on the cascade-parallel system, the performance curves of each motor individually being the same as those in figure 15, and for the motors in cascade as in figure 19, Chapter 1 of this Volume. Suppose, further, that these motors are mounted direct on 42″ driving wheels and supplied with current at 20 frequency. (This is simply to keep the speeds down to reasonable limits—the windings are assumed to be modified in accordance with their frequency so as to keep the performance curves unaltered.) Let the weight of the train be equivalent to 50 tons per pair of motors.

With such an equipment, the full speed of the train would be 73 miles per hour, and the half-speed 36·5 miles per hour. The maximum pull per pair of motors connected in cascade is about $\frac{3400}{1 \cdot 75}$ lbs. = 1940, which would be sufficient to start the train with an acceleration of about ·4 foot per sec. per sec. The motors would also supply sufficient pull for driving the train at half-speed up a 1 in 100 grade, this pull corresponding roughly to the maximum power factor.

Under these circumstances the torque required to maintain half-speed on the level would be about 1000 lbs. ft. which corresponds to a power factor of about ·54. The efficiency however remains high.

Thus a comparison can be made for running at half-speed on the level between cascade-parallel working and simple rheostatic working.

	Cascade-parallel	Rheostatic
Line current per pair of motors	23 amps.	30 amps.
Power factor	·54	·8
Efficiency	88 °/。	46·5 °/。

Thus the advantage of cascade working is considerable from the point of view of efficiency on the train, but not so marked as regards the efficiency of the distribution system.

Cascade-parallel single control. Consider next the possibility of using two motors with different numbers of poles. As already explained such a device makes possible three distinct economical speeds.

There are two ways of effecting this arrangement, viz. by using two motors with different numbers of poles geared with the same ratio of gearing to the same sized drivers, or by using two similar motors unequally geared.

In considering this system particular attention must be paid to the limit imposed by adhesion upon the pull exerted by a pair of wheels. In the case of the two motors designed in Chapter 1 of this Volume it can be shewn that the auxiliary motor exerts about 5 per cent. less pull than the main motor while both are working as motors, and about 3 per cent. more than the main motor while working as generators. If, however, a 4-pole and a 6-pole motor be connected in cascade the auxiliary motor will exert approximately 40 per cent. more pull than the main motor, and when working as generators about 50 per cent. more.

Another consideration which affects the practicability of the system is the question of the design of the motors. It has been pointed out already in Chapter 1 of this Volume that the success of the cascade working depends upon the use of motors with very high power factors, and further that the possibility of obtaining such high power factors depends almost entirely on the length of the pole pitch (keeping the air gap length constant). If then with a 4-pole motor of a given diameter a leakage factor of ·04 is possible, with a 6-pole motor of the same diameter the leakage factor would be between ·055 and ·06. This difference in leakage factor would lower the maximum power factor from 92·6 per cent. to 89·8 per cent. Such a difference would seriously affect the performance of the motors in cascade.

On the other hand, if this difficulty is avoided by the use of two similar motors unequally geared, there is a probability that when full speed is reached the gear velocity of the motor which is running light may be excessive, as also the relatively high speed of the motor itself.

From these considerations it would appear that this system would require special circumstances to make it of much value.

The problem of the suburban system. The conditions that have to be met on local service and on main-line service have already been discussed. It remains to study the requirements of a large suburban system. Such a service may be called a mixed service, and its requirements have been set out on pp. 384 to 387, Vol. I.

In order to study the problem more completely assume that the train has to maintain an average speed of 30 miles per hour with an average run of say 1½ miles, and ½ minute stops at intermediate stations. Suppose further that it has to cover a distance of 20 miles without a stop at an average speed of 50 miles per hour.

In considering the choice of system, it is safe to say that the rheostatic method of control may be disregarded; for all conditions of working except those in the long runs at high speed the efficiency would be so low as to be prohibitive. The choice must therefore fall on one of the cascade systems (or the alternative system used on the Simplon tunnel locomotives).

The maximum speed may be fixed at 60 miles per hour. Before going any further the same difficulty arises as was experienced in the case of the direct current system, viz. the tendency to high gear velocities if geared motors are employed. With alternating current motors this difficulty is accentuated, because for motors of equal capacity the distance between the centres of the motor and the car axles is greater with A.C. motors than with D.C. For this reason the gear wheels are larger and gear velocities greater, *cœteris paribus.*

For instance, suppose the main motor designed in Chapter 1 of this Volume, p. 29, were geared to a 42″ driving wheel to maintain a speed of 60 miles per hour the gear ratio would be 1·52 : 1 and the maximum gear velocity would be 2840 feet per minute. Even if this velocity were permitted the advantage of employing gearing with such a low ratio is scarcely sufficient to warrant the adoption of gearing at all. A large motor with a speed of 480 r.p.m., the speed required for a gearless motor, would probably have a higher efficiency than a geared motor of the same capacity running at 750 r.p.m.

Now a motor which has a full speed of about 480 r.p.m. must either be a 6-pole 25 frequency motor, or a 4-pole 15 frequency motor, and the choice lies between these two since the diameter of the driving wheel cannot be assumed less than 42″ unless very small motors are used.

If 25 frequency be adopted the nearest approach that can be obtained to the maximum speed desired will be 61 miles per hour with a 42″ wheel. With this size of wheel, after making the necessary allowances for clearance, the diameter of the rotor could be not more than 580 mm. The leakage factor of such a motor if wound for 6 poles would not be less than ·05 and a pair of motors with such a characteristic would not have a wide range of working in cascade connection.

If 15 frequency were adopted a 4-pole motor could be used with a 48″ driving wheel, a maximum speed of 63 miles per hour would be maintained, and the diameter of the rotor would be about 660 mm. which would allow of a leakage factor of about ·03. Moreover, with this frequency and the same size of driving wheel a 6-pole motor could be used and would have a leakage factor of about ·04 and two such motors would have a reasonably satisfactory range of operation in cascade connection. With larger driving wheels, and motors running at proportionately lower speeds, slightly better results could be obtained.

An alternative method in which a frequency of 25 could be employed is that in which the motors are connected to the driving wheels by coupling rods as illustrated in figure 24, which has the advantage that the diameter of the driving wheel does not limit the diameter of the rotor.

The three possibilities are therefore:

(1) Geared motors—which involves a high gear velocity.

(2) Direct coupled motors—which involves a supply at 15 frequency.

(3) Indirect coupled motors—with which a 25 frequency supply may be used.

To make a final choice between these alternatives, all the conditions of the problem must be studied, such as the weight of the trains and the grades to be surmounted and so on. As an example, however, suppose the conditions of service to be those already given, and further that the maximum gradient is 1 in 80, and that the train exclusive of any equipment weighs 240 tons.

For such conditions it would seem desirable to provide for an acceleration of say ·8 ft. per second per second and if possible for three economical speeds.

In this case the best solution would seem to be to provide a locomotive with two 6-wheel trucks, each truck mounted with two compound motors spring-supported on the bogie and connected to the driving wheels by means of cranks and connecting rods. Each motor would be composed of two separate stators and two separate rotors with one set wound for 4 poles and the other for 6 poles. If such motors were supplied with current at a frequency of 15, and were connected to 48″ driving wheels, the three economical speeds would be about 25, 42, and 63 miles per hour respectively. The locomotive would weigh approximately 60 tons, and would be able to exert a pull of about 10 tons. The total weight of the train would be 300 tons, and this pull would produce a starting acceleration of about ·85 ft. per second per second on the level, and would be more than sufficient for hauling the train at 63 miles per hour up a 1 in 80 grade.

The output of each compound motor connected for cascade running would be 2·5 tons pull at 25 miles per hour or 375 H.P. maximum. There would be no necessity for using the two halves of each motor in parallel, and consequently a single set of slip rings would be sufficient; the design being similar to that of the motors in the latest locomotives on the Valtellina Railway*.

The question may be raised as to why a multiple unit train is not proposed. In the first place there is the difficulty of putting sufficiently large motors on the bogies of the motor cars without raising the carriages considerably. Unless the motors are large enough, it is impossible to ensure a sufficient range of working in cascade, as already explained. Beyond this, however, there is another reason. Induction motors have speed-torque characteristics very similar to those of shunt motors; that is to say, a small variation in the speed produces a very large change in the torque. Consider, then, the result of making up a train with motor cars some of which contain new driving wheels, and others worn wheels. The wear of railway wheels may amount to two inches on the diameter. If the reduction of diameter is say 3 per cent. of the original, the speed of the corresponding motor will be such that, if all the other wheels are new, that motor will be doing no work. On the other hand, if there be one new wheel, and all the others be worn 3 per cent., the motor coupled to the new wheel will have perhaps double its proper load thrown on to it. The result at the end of a day's working will probably be the charring of the insulation, even if the motor does not burn out.

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* This chapter was written in February, 1906, the authors having no knowledge of any proposals for a similar case that had been put forward or were being contemplated. They are, therefore, interested to note the account in the *Elektrische Kraftbetrieb und Bahnen* for February 24, 1907, of the locomotives built by Messrs Ganz and Co. for the Italian State Railways. Each of these locomotives has one 8-pole motor and one 12-pole motor, three economical speeds being thus obtained by using each motor singly and the two motors in cascade.

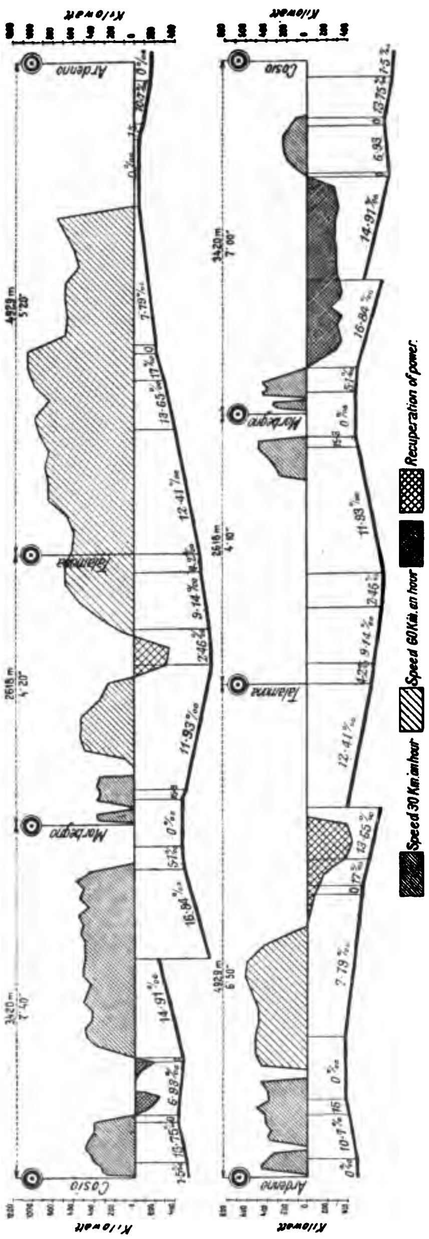

FIG. 60. Power curves on the Valtellina Railway.

The whole problem, however, is an extremely difficult one, and it is obvious that no solution can be anything more than a suggestion based on an imperfect knowledge of all the facts of the case.

An example of a somewhat similar problem is that of the Valtellina Railway, and the curves of power supply for this line are shewn in figure 60. In these curves the use of regenerative braking in descending grades is well shewn. In the system suggested above the same possibility is provided for, but with three speeds instead of two.

The advantages of the three-phase system are manifest. In the first place the necessity for distributing at low tension is avoided; in the second place the commutator is abolished; and in the third place the possibility of regenerative braking is introduced.

Of these three advantages, the first is without doubt the most important. The manufacture of direct current motors has been developed to such an extent that there remains now only one drawback to the use of a commutator, viz. the liability to flashing over. The third advantage cannot be claimed as the exclusive property of this system, for it can be obtained with direct currents by the use of shunt or compound motors.

The main disadvantage of the system lies in the necessity for at least two overhead wires for each track at different voltages. This branch of the subject has been already considered in Chapter 3 of this volume.

CHAPTER 5.

PART 1.

GENERAL THEORY OF SINGLE PHASE COMMUTATOR MOTORS AND DESIGN OF SINGLE PHASE SERIES MOTOR WITHOUT COMMUTATION POLES.

General. To all who have studied the problems of electric traction, the advantages presented by any system which employs alternating currents throughout are obvious. If for no other reason any alternating current system compels attention on account of the elimination of the rotary converter sub-stations and of low tension distribution. One or two considerations, however, are not sufficient on which to judge a system, since a large number of other considerations of minor importance in themselves may require to be taken into account.

For instance, in the three-phase system the main advantages are those presented by the high tension distribution of alternating currents. On the other hand there are undoubted disadvantages, such as the necessity of supporting two high tension overhead wires at different potentials and the inherent characteristic of the three-phase induction motor in operating economically only at constant speed.

Of recent years engineers have been endeavouring to evolve a motor suitable for traction work which shall combine the advantages of alternating current working with those of a variable speed characteristic, and a single overhead conductor.

These efforts have recently led to the introduction of commutator motors for use on a single phase alternating current supply; and although a comparatively short time has, as yet, elapsed since they were first used commercially, sufficient results have been obtained to justify the expectations of the engineers responsible for their employment.

Two types of motor. Of the many types of motor that were invented, broadly speaking only two have been put into commercial operation for traction purposes. These two are the series motor and the compensated repulsion motor. The time is not yet ripe for any prediction as to whether one of these two types will survive to the exclusion of the other, and it may even be that some other type which has not yet emerged from the experimental stage may come to the front.

Although many proposals have been made by inventors and designers for motors which are not included in the two types mentioned above, the probability of their survival remains sufficiently remote to justify their exclusion from consideration here. The records of the patent office and the proceedings of the various technical institutions furnish ample information on these inventions.

The inherent characteristics of both types. Both motors are possessed of a commutator, and the necessity of this feature is probably the most important factor in the design. Similarly both motors are bound to have alternating magnetic fields, and as a matter of design in both motors these fields are produced by series windings.

As a consequence, in both cases the speed-torque characteristic is similar to that of a direct current series motor, and not to that of a shunt motor or three-phase induction motor.

The fields being alternating, the magnets are composed of laminations to guard against extensive eddy currents, and the enclosing case does not form part of the magnetic circuit.

The armature is likewise built of stampings and is wound with conductors in the same way as a direct current motor armature, the windings being similarly connected to the commutator.

Put briefly, the chief feature of both motors is the use of a direct current armature for alternating currents in an alternating magnetic field; and it will be advisable to study the effect of this combination before considering the motors individually.

Transformer voltage. The main difficulty experienced in designing a single phase commutator motor is the existence of an electromotive force induced by the alternating field in those armature coils which are temporarily short circuited by the brushes. These coils are, in effect, the secondary of a transformer of which the primary is the field exciting windings. This E.M.F. would, unless prevented, give rise to large circulating currents in the closed coils, and as the armature revolves this current would be broken every time a commutator part leaves the brush. This would naturally give rise to excessive sparking, which would in a short time damage the commutator and the brushes to such an extent as to necessitate immediate renewal.

This induced E.M.F. is generally spoken of as the "transformer voltage," and it is probably the most important element in the design.

As an illustration, consider a 500 volt traction motor rated at 40 H.P. as a direct current motor. Such a motor would have 4 poles, and an armature wound with a two-circuit winding, the number of conductors being such that each brush would cover a little over three commutator parts. The flux per pole would probably be about $3{\cdot}5 \times 10^6$ C.G.S. lines. Suppose now that this field was an alternating field of 25 frequency.

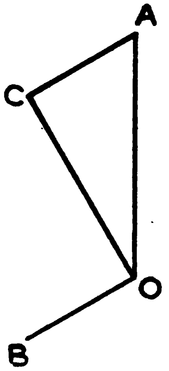

FIG. 61. Vector diagram shewing demagnetisation due to circulating current.

Then between consecutive commutator parts there would be two complete armature turns (the winding being a two-circuit winding). Therefore the voltage induced in these two turns would be

$$3{\cdot}5 \times 10^6 \times 2\pi \times 25 \times 2 \times 10^{-8} = 11 \text{ volts.}$$

That is to say, the transformer voltage between consecutive commutator parts would be 11 volts. Now, in the worst case, the brush would touch four commutator parts, and in this case the induced voltage between the front and rear edges of the brush would be

$$3 \times 11 \text{ volts} = 33 \text{ volts.}$$

It can easily be imagined that such an E.M.F., if unchecked, would cause destructive sparking, and would make the motor commercially impossible.

Effects of the circulating current caused by the transformer voltage. These effects are two, viz. sparking and demagnetisation.

(1) *Sparking.* This effect is obvious, being due to the continual breaking of the circulating current.

(2) *Demagnetisation.* This effect is precisely the same as that in a static transformer, in which the primary current is fixed In ordinary transformers, the primary voltage is fixed, and the result of a secondary current being allowed to flow is an alteration in the primary current, the magnetic field remaining practically constant. If however the primary current is fixed, any current induced thereby in the secondary circuit tends to change the phase of the magnetic field and also to reduce it. Thus, consider the diagram in figure 61. Let OA represent the fixed primary current. Then if there be no secondary current, the magnetic field will be due entirely to OA. Suppose, however, that a current OB be allowed to flow in a non-inductive secondary circuit. Then the magnetic field will be due to the resultant of OA and OB, that is OC, OC and OB being at right angles.

The effect of the circulating current is, therefore, to diminish the field and to cause it to lag.

Practical treatment of the transformer voltage. It may be safely said that there is no possibility of entirely eliminating the transformer voltage. It is possible and commercially practicable to eliminate it when a motor is revolving at a sufficient speed, as will be seen hereafter; but at starting, when the motor is at rest, this transformer effect is inherent and cannot be overcome.

This being recognised, it remains to examine the methods whereby the effects produced can be minimised. A number of proposals have been made and many experiments tried, notably with narrow brushes, and special commutators in which alternate segments only are connected to the armature windings.

Only one method, however, has so far stood the test of practice, and that consists in the insertion of high resistance leads between the armature windings and the segments of the commutator. These high resistance leads do not form part of the armature circuit except when the corresponding commutator parts are under the brushes, and consequently they do not give rise to a very great loss, and moreover they can be made relatively small for the value of the current which they carry. Figure 62 shews the position of these leads, AA being the armature windings, RR the high resistance commutator connections, and C and B the commutator and brush.

Now the most important point in the design of a motor is the limit which may be permitted to this transformer effect. As already explained there are two quantities which must be taken into account, viz. the transformer voltage and the circulating current produced thereby. If the limits for these two quantities be fixed the broad outlines of the design are settled at once.

In such a question theory can provide very little assistance. The values which may be worked to without causing excessive depreciation due to sparking must rest upon an experimental basis.

Now in a closed armature coil with high resistance leads and narrow brush contacts included in the circuit, the resistance predominates over the inductance. In such a circuit it is reasonable to assume that the energy expended in the arc formed when the circuit is opened by sliding the brush off the commutator part is proportional to the product of the current in the circuit by the voltage across the opening; in other words the energy is proportional to the transformer voltage between the brush tips and the circulating current.

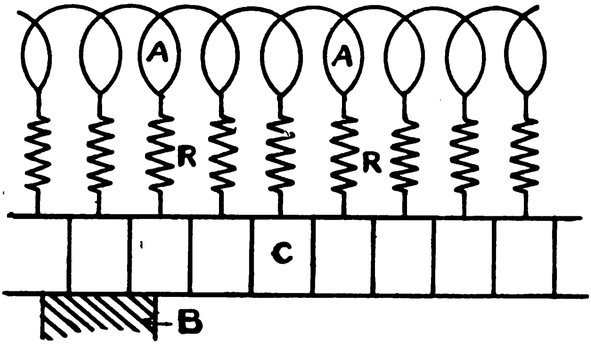

FIG. 62. Diagram shewing high resistance commutator connections in single phase commutator motors.

It is, as yet, premature to lay down any fixed rule as to the limits to be assumed, but it may be taken provisionally that a circulating current equal to twice the current in any conductor, and a voltage not exceeding 16 volts, may be allowed. These provisional limits deal only with the sparking effect when the motor is at rest.

The effect of armature rotation on the circulating current. When the motor is running the phenomenon occurring under the brushes is somewhat different.

Consider in the first place the effect of the rotation on the circulating current set up in the short circuited coils by an E.M.F. E in each coil. Now, as the armature is rotated successive coils are constantly coming under the brushes, taking the place of other coils which have just passed on. The question therefore arises, to what extent has the circulating current time to rise to its full value before the short circuit is opened by the commutator segment leaving the brush? This question, although of great importance, is very difficult to answer. The problem is scarcely susceptible of precise mathematical treatment owing to the complicated conditions such as the varying resistance of the brush contact, the varying distribution of the conductors in slots, and so on. The difficulty must be met in some way, however, and the following method is suggested as, at all events, possessing some relation to the facts of the case.

The rise of current due to a constant E.M.F. E in a circuit in which the resistance is R and the inductance L is given by the formula

$$\mathsf{C} = \frac{\mathsf{E}}{\mathsf{R}}\left[1 - e^{-\frac{\mathsf{RT}}{\mathsf{L}}}\right],$$

and the average circulating current during the time T is

$$\frac{\mathsf{E}}{\mathsf{R}}\left[1 - \frac{\mathsf{L}}{\mathsf{RT}}\left(1 - e^{-\frac{\mathsf{RT}}{\mathsf{L}}}\right)\right].$$

If therefore, in the case of an armature coil, the armature is standing still, the value of T will be infinite, and the circulating current will be simply $\frac{\mathsf{E}}{\mathsf{R}}$.

Now, in the usual case, several coils are undergoing commutation simultaneously. These coils have resistance, self inductance, and mutual inductance. Two cases may arise—(1) that in which the mutual inductance is equal to the self inductance as with two or more coils in the same slot; (2) that in which the mutual inductance is zero as is approximately true of two coils in different slots.

In the first case a little consideration will shew that as each fresh coil comes under the brush the circulating current in that coil leaps instantaneously to a definite value, and does not rise according to an exponential law, the mutual induction from the other short circuited coils cancelling the self induction in the coil in question*.

If, therefore, the case be imagined of an armature in which all the conductors are in complete inductive relation, the rotation of the armature would have no effect on the circulating current.

In the second case where there is no mutual induction between neighbouring coils, the circulating current in each coil must rise according to an exponential law, and will continue to rise, as long as the coil is short circuited, towards the full value.

Now the actual case lies somewhere between these two. There are in practice a limited number of coils in the same slot and a large number of slots. It is, therefore, proposed to use the formulae given above for circulating current as determined by the exponential law, by giving suitable values to the constants R, L, and T.

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* This point is best seen by considering the case where a brush covers an exact number of commutator segments, say 3. Then there will be always exactly 3 coils short circuited, and since self and mutual inductances are equal, the total inductance of the short circuited coils will remain constant. As soon, therefore, as the circulating current has reached its full value, it will be maintained at the same value independently of the rotation of the armature.

For the value of R take the same value as at standstill, so that $\frac{E}{R}$ is the total circulating current in that condition;

for the value of T take the time in seconds during which a brush short circuits any of the armature coils in a single slot; and

for L take a value due to the average number of conductors in one slot undergoing commutation simultaneously.

Thus suppose the brush has the width of m segments and there are n armature coils per slot, the time T will be that in which $n + m - 1$ segments pass a fixed point.

[*E.g.* say $m = 3$ and $n = 5$ as in figure 63, then the brush moves during T from the full line to the dotted line position, and $n + m - 1 = 7$.]

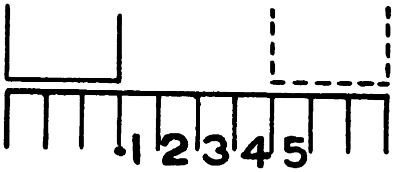

FIG. 63. Diagram of commutator and brush illustrating the period of short circuit of an armature coil.

Considering the line T as made up of $n + m - 1$ intervals, the average number of conductors required for obtaining the value of L can be calculated thus:

in interval 1, one conductor;

in interval 2, two conductors, and so on;

then the average number will be the sum of the conductors divided by the number of intervals.

[*E.g.* taking $m = 3$, $n = 5$, the sum of the conductors will be $1 + 2 + 3 + 3 + 3 + 2 + 1 = 15$, and the average number $\frac{15}{7} = 2\frac{1}{7}$.]

If, however, some of the conductors undergoing commutation under the negative brush are in the same slot as those undergoing commutation under the positive brush, these must be taken into account in computing the average. Thus supposing in the above example the armature winding was an exact pitch winding and the number of slots divisible by the number of poles, the average number of conductors would be $4\frac{2}{7}$.

The factor $\left[1 - \frac{\mathrm{L}}{\mathrm{RT}}\left(1 - e^{-\frac{\mathrm{RT}}{\mathrm{L}}}\right)\right]$ may be termed the **time-constant correction.**

The effect of reactance voltage on the circulating current. So far the circulating current has been considered independently of the main armature current. At standstill, very little error will be introduced by considering them separately, but with a rotating armature an additional circulating current is set up due to reactance voltage.

Now this voltage is not quite of the same nature as the transformer voltage, since it depends on the change of current in the armature coils during commutation. If the reactance voltage be calculated in the ordinary way and the resulting circulating current be obtained from it in the way described above, it will be found that under certain conditions the average circulating current will be greater than the current in a conductor. This of course is impossible. Such conditions, however, will very seldom occur with the motors under consideration, and any attempt to allow for this tendency would probably introduce fresh possibilities of error. In the examples of motor calculations given below, therefore, the ordinary method of obtaining the reactance voltage will be employed.

Total circulating current. The total circulating current will be due to the resultant of any impressed E.M.F. and the reactance voltage. This impressed E.M.F. may be simply the transformer voltage, or it may be the combination of this with other voltages*. If the armature conductors undergoing commutation are revolving in a magnetic field, however produced, there will be an additional circulating E.M.F. due to this cause. In particular, such an auxiliary magnetic field may be produced by means of a commutation pole specially wound to produce an E.M.F. equal and opposite to the resultant of the transformer and reactance voltages at a definite speed.

Calling $\mathrm{E_T}$ the transformer voltage, lagging 90° behind the main field, $\mathrm{E_R}$ the reactance voltage, in phase with the armature current, and e the E.M.F. produced by rotation in the auxiliary field, the total circulating current will be

$$\frac{\text{vector sum of } \mathrm{E_T},\ \mathrm{E_R}, \text{ and } e}{\mathrm{R}}\left[1 - \frac{\mathrm{L}}{\mathrm{RT}}\left(1 - e^{-\frac{\mathrm{RT}}{\mathrm{L}}}\right)\right].$$

The relative direction of this current can be determined from the considerations that the current due to the reactance voltage tends to increase the main field in a series motor, and the current due to the transformer voltage lags 90° behind the main field.

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* This total circulating voltage was first suggested by Punga in his book on single phase commutator motors as a criterion for sparking. In that book the circulating current produced thereby is not dealt with so fully as here.

The direction of the circulating current affects only the phase of the main field ; it is quite immaterial so far as any question of sparking is concerned.

Sparking. Making the assumption that the sparking in any particular case can be measured as the product of the circulating voltage into the circulating current per unit length of brush, a comparison may be made between the behaviour of a single phase and a direct current traction motor.

In a direct current traction motor the circulating voltage is partly due to rotation in the cross field produced by the armature. This voltage is simply added to the reactance voltage, the sum being the total circulating voltage. In such motors there are no high resistance commutator connections, and the circulating current will be determined by the brush contact resistance.

As a rough estimate the circulating voltage might be taken as from three to four volts for a large direct current railway motor. This is obviously much lower than the value of 16 mentioned above as the limit for alternate current motors; but against this must be put the following considerations: (1) in the alternating current motor the circulating current due to all causes is limited to a value not exceeding twice the current per conductor, whereas in the direct current motor the reactance voltage alone is sufficient to produce the same circulating current; (2) in the direct current motor, as a rule, the brushes are worked harder than in an alternating current motor, so that for the same value of circulating current, the value per unit length of brush is less in the latter.

In actual practice single phase commutator motors with or without commutating poles seem to spark much more at the start than when rotating (even at a very low speed). This is probably due to the concentration of the current at a few points of the brush tips, such concentration increasing as the resistance at those points falls owing to the heat generated there. As the armature begins to rotate this effect rapidly diminishes.

Most economical value of resistance in commutator connections. So far the circulating current has been regarded chiefly from the point of view of the limits allowable. It does not follow, however, that it is necessarily advantageous to adopt the maximum value; in all cases the losses occasioned thereby must be considered.

The loss in the commutator connections may be regarded as having two parts, viz.

(1) that due to the passage of the armature current through the resistances,

(2) that due to the circulating current.

The first of these is proportional to C^2R, where C is the armature current and R the resistance of each connection, and the second is proportional to $\frac{E^2}{R}$, where E is the resultant circulating voltage. To put it in the form of an equation

$$W \text{ (watts)} = k \, . \, C^2R + l \, . \, \frac{E^2}{R}.$$

The value of R should be adjusted so that W is a minimum, provided the limits already given are not exceeded.

Strictly speaking, average values of C^2 and of E^2 should be taken for any service which the motor is required to perform; but this, of course, is impracticable unless the service is very simple, as, for instance, a one hour run at the rated load.

For intermittent railway service of the usual type, however, it may probably be sufficient to adjust the value of the resistance with regard to the one hour rating.

There will be two cases, therefore, viz. motors in which the circulating current is unchecked by any commutation poles or auxiliary fields, and motors in which the circulating current is partially or completely suppressed by these means.

In the latter case the resistances are chiefly settled by the limits, whereas in the former case it will generally prove advantageous to keep the circulating current considerably below the limit.

Brush resistance. This resistance is a variable quantity and depends upon the current density and upon the quality of the carbon. The resistance is best approximately expressed by the equation

$$\text{drop} = A + B \times \text{current per sq. cm.}$$

In the examples worked out in this chapter, the "ordinary" quality of brush is assumed, having constants

$$A = \cdot65 \quad \text{and} \quad B = \cdot084.$$

Thus for a current density of 5 amperes per sq. cm.

$$\text{the drop} = \cdot65 + 5 \times \cdot084 = 1\cdot07 \text{ volts.}$$

In calculating the circulating current this peculiarity of brush resistance is allowed for by first deducting 1·3 (= 2 × ·65) volts from the circulating voltage, and then dividing by the resistance obtained by means of the constant ·084.

The single phase series motor. The single phase series motor may be said to be simply an adaptation of the direct current series motor to the conditions imposed by working it with alternating currents.

The various points in the adaptation are as follows:

(1) Lamination of the entire magnetic circuit.

(2) Small air gap and weak field to reduce as far as possible the E.M.F. of self induction.

(3) Suitable winding on the armature to keep the transformer voltage low (such as the use of low voltage windings with single turn coils).

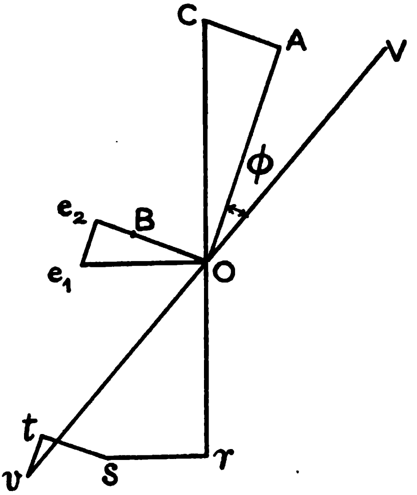

FIG. 64. Vector diagram of single phase series motor without commutator poles.

(4) Means for preventing armature distortion; such as compensation windings in the pole faces. This precaution is taken from two points of view, the prevention of sparking, and the reduction of the armature self induction.

(5) Provision of high resistance connections between the armature windings and the commutator.

(6) In some types of this motor, the provision of commutating poles specially wound to counteract the transformer voltage and the reactance voltage*.

---

* Such motors are built in England by Messrs Siemens Bros. Dynamo Works, Ltd., and in Germany by Messrs Siemens-Schuckert, and the Oerlikon Co.

Series motor diagram. In constructing the diagram for the series motor, it is best to begin with the main field. Let OC, figure 64, represent this field or rather the ampere turns per pole required to produce it. From the flux per pole deduce the transformer voltage Oe_1 and let $e_1 e_2$ represent the reactance voltage. Then the resultant circulating voltage will be Oe_2. Let OB represent the circulating ampere turns per pole produced by Oe_2. Then OA, equal and parallel to BC, will be the ampere turns per pole produced by the current in the field coils. Dividing OA by the turns per coil the motor current is obtained. It remains to obtain the terminal voltage.

The total internal voltage Ov is composed of four parts :

Or the E.M.F. produced by the rotation of the armature in the main field, opposite in phase to OC; rs the E.M.F. induced in the field coils lagging 90° behind OC; st the E.M.F. due to the leakage flux surrounding the armature and compensation windings lagging 90° behind OA; and tv the internal drop due to resistance, opposite in phase to OA.

The terminal voltage OV is equal and opposite to Ov.

Design of a single phase series motor without commutation poles.

Preliminary considerations. If the direct current series motor be regarded as the ideal of a railway motor (in the matter of design), and since many disadvantages are experienced by the introduction of alternating currents, it is obvious that the nearer the conditions approach those obtaining with direct currents the better. In other words, difficulties of design are minimised by keeping the frequency low. After the first attempts to make motors to operate at 50 frequency or 100 frequency, success was reached by using 15 frequency. Although there is much to be said for this plan, it is not necessary; and latterly the majority of manufacturers have adopted a frequency of 25.

The speed of the motors should be kept as high as is practicable; approximate maximum values may be taken at 30 metres per second for the armature, and 25 metres per second for the commutator periphery. Higher speeds are, of course, possible with special construction.

As with three-phase motors, the air gap or clearance must be kept as small as possible.

The excitation is, of course, in series with the armature. If the armature be wound with a multiple winding, it is necessary that the poles should be kept of equal strength. This may be done by putting the excitation coils in parallel with one another, or it may be done by means of equalising connections on the armature. The latter method is perhaps preferable, as, in the former method, if the armature drops a little out of centre one of the field coils may overheat. If the armature has a two-circuit winding, there is not the same necessity for equalising the poles.

Saturation of the magnetic circuit is not so easy to deal with as in the case of direct currents. This, of course, is no reason for avoiding saturation, the degree of which must be considered on its merits. It may be taken as sufficiently accurate that the flux follows a true sine curve. Consequently, if the iron is saturated with the maximum flux, the current will be very far from following a sine curve, and it is not at once obvious what value of current should correspond to any R.M.S. value of flux. The following method is proposed. It has been found by calculation in many cases that for all cases likely to occur in single phase railway motors, the R.M.S. value of the magnetising current is the mean between two values, the first of which corresponds to the R.M.S. flux, and the second to $\frac{1}{\sqrt{2}}$ times the value which corresponds to the maximum flux*. Thus, take as an example,

R.M.S. flux $3 \cdot 75 \times 10^6$; corresponding ampere turns 1380,

Maximum flux $\sqrt{2} \times 3 \cdot 75 \times 10^6$ or $5 \cdot 3 \times 10^6$; corresponding ampere turns 3700.

Then the R.M.S. ampere turns will be

$$\tfrac{1}{2}\left(1380 + \frac{1}{\sqrt{2}} \,.\, 3700\right) = \tfrac{1}{2}(1380 + 2620) = 2000.$$

For extreme saturation, this method is slightly incorrect, the error amounting to about 2 per cent. when the total ampere turns for the maximum flux are twice those required for the air gap alone. This error is however insignificant in comparison with other possible errors.

The method adopted is, therefore, to assume a sine wave current with this for its R.M.S. value.

The estimation of iron loss cannot be dealt with exactly. At standstill the loss may be regarded in the same way as in a transformer, and can be calculated from known hysteresis and eddy current constants and from the densities in the various parts of the magnetic circuit. In addition to this loss there will be a further waste of energy due to magnetic leakage. This may be regarded as being proportional to the square of the main current, and allowance made for it by adding something to the total resistance of the motor. When the motor is revolving, the cycle of magnetisation in the armature differs somewhat from the simple magnetic cycle due to the periodic reversals. For example, at synchronous speed, various parts of the armature do not undergo reversal at all, but only a variation of magnetism from zero to the maximum. This variation, however, considered as a cycle in itself has twice the frequency of the supply circuit; it does not therefore imply necessarily any diminution in the total iron loss. On the other hand, at speeds above synchronism the frequency of the magnetic cycles in the armature is greater than twice the fundamental frequency, and, all things considered, it is only reasonable to expect that the iron loss should increase as the speed rises, the strength of the field being kept constant.

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* The authors believe this method to be novel. It is distinguished from other methods in that it gives the value for the current that would be indicated by an ammeter.

Further, it is usual to regard iron loss in direct current motors as equivalent to a retarding force, in the same way as bearing friction. In a transformer, however, iron loss is supplied from the electrical source, and produces its own effect on the power factor. In a way, an alternating current commutator motor is a combination of these two types of apparatus. The following method is therefore proposed:

Calculate the total iron loss at standstill, and add it to the watts lost in resistance, in order to arrive at the internal drop. Assume that the iron loss remains appreciably constant at all speeds below synchronism, and for speeds above synchronism add to the total losses in the motor, but not to the loss from which the internal drop is calculated, an additional amount based on the assumption that the armature iron loss varies as the 1·5th power of the speed.

Thus the additional loss will be

$$\left\{\left(\frac{\text{speed}}{\text{synchronous speed}}\right)^{1\cdot5}-1\right\}\times\text{armature iron loss.}$$

Specification No. 3.

Single phase series wound motor.

Rating. 100 H.P. for 1 hour, 75° C. rise. 250 volts; about 750 r.p.m.; 380 amperes; 25 frequency.

Armature.

External diameter	500 mm.
Internal diameter	290 „
Gross length	330 „
Air ducts	3—10 „
Nett length	300 „
Percentage of iron	86 °/ₒ
Length of iron	258 mm.
Number of slots	66
Width of each slot	13·5 mm.
Depth of each slot	40 „
Opening of slot	13·5 „
Type of winding	multiple
Number of poles	4
Conductors per slot	8

Scheme of connections	1 ╲ ╱ 3 130
Size of conductor	12 mm. × 2·3 mm.
Total resistance (hot)	·021 ohm
Leakage reactance	·052 ohm
Weight of copper	105 kgms.
Weight of iron	190 kgms.
Number of cross connections	12
Section of cross connections	25 sq. mm.

Commutator.

Diameter	420 mm.	Number of segments	264
Length	300 „	Peripheral speed	16·5 m. per sec.

Commutator connections.

Material	"Eureka" metal	Length of each connection	700 mm.
Cross-section	4 mm. × 2·3 mm.	Resistance „ „	·036 ohm

Brushes.

Number of brush arms	4	Length of each brush	
Brushes per arm	5	Quality of brush	"ordinary"
Section of each brush	51 mm. × 15 mm.	Type of brush holder	

Field.

Bore	504 mm.	Length of pole radially	50 mm.
External diameter of stampings	734 „	Length of pole circumferentially	300 „
Gross length	330 „	Slots per pole	12
Air ducts	3—10 „	Width of each slot	13·5 mm.
Nett length	300 „	Depth of each slot	50 „
Percentage of iron	86 °/o	Opening of slot	1 „
Length of iron	258 mm.	Weight of stampings	342 kgms.

Compensation winding.

Conductors per slot	2	Type of compensation	forced
Size of conductor	40 mm. × 5 mm.	Total resistance (hot)	·0096 ohm
Size of end connections	50 mm. × 3·5 mm.	Leakage reactance	·054 ohm
		Weight of copper	140 kgms.

Excitation winding.

Turns per coil	3	Total resistance (hot)	·0023 ohm
Connection of coils	series	Leakage reactance	·0075 ohm
Size of conductor	40 mm. × 5 mm.	Weight of copper	39 kgms.

Miscellaneous.

Length between axle centres	18·6″	Diameter of driving wheel	38″
Gear ratio	19 : 74	Vertical distance between centres	54 mm. = $2\frac{1}{8}$″
Diameter of pinion	7·6″	Clearance between motor and rails	6″
Diameter of gear wheel	29·6″	Miles per hour at rated speed	21·9
Gear velocity at rated speed	1490 ft. per minute		

The general arrangement of this motor is shewn in the sketch, figure 65, and the details of the armature and field laminations in figure 66. From the specification and the sketch it will be seen that the motor is a four-pole motor, with a four-circuit armature winding and four sets

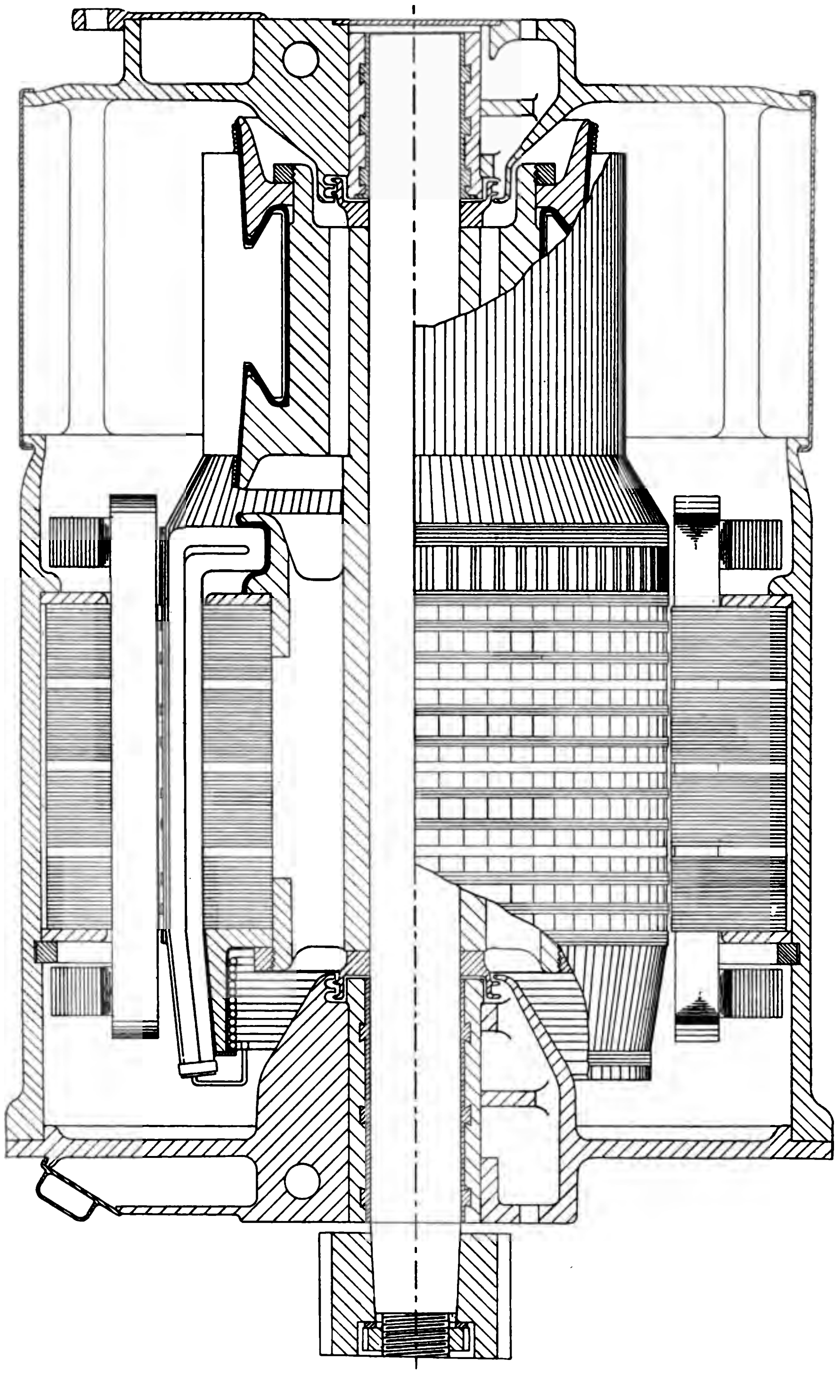

Fig. 65. General arrangement of single phase series motor without commutation poles.

of brushes. The armature is similar to that of a standard traction direct current motor having open slots and former wound coils. The joints of these coils are at the end remote from the commutator, the end connections at the commutator being bent down toward the centre of the armature. High resistance commutator connections of "Eureka" metal are wound up with the armature coils and extend from the joints, through the slots to the commutator segments.

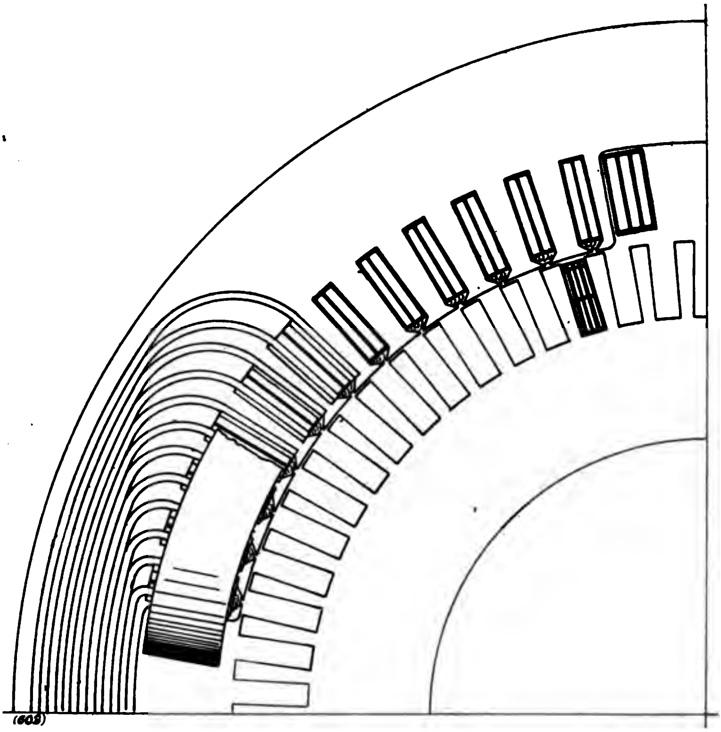

FIG. 66. Armature and field laminations and conductors for single phase series motor without commutation poles.

The field or stator is composed of laminations, and has four projecting poles with fairly deep recesses between. In these recesses are inserted the excitation windings, embracing the poles. In each pole there are 12 slots, each slot containing two conductors; these conductors are in series with the armature and form the compensation winding.

Method of calculation. The method of calculating the performance curves is to assume a definite speed, and for this speed assume a set of values for the flux per pole. For each value work through the diagram as explained on p. 126.

Only one point need be noticed. The reactance voltage must be assumed in magnitude and direction at the outset. Having made the assumption, the phase and magnitude of the motor current can be obtained, and the assumed value of the reactance voltage must be checked. The method is one of successive approximation; but the process is not at all lengthy.

From these calculations a set of curves can be plotted, shewing terminal voltage, cos ϕ, efficiency, B.H.P. and torque in terms of current for the speed chosen.

Similar sets of curves can be obtained at other speeds; and from all these curves a set of performance curves can be constructed, shewing speed, cos ϕ, efficiency, B.H.P. and torque in terms of current for a series of constant terminal voltages.

For the purpose of illustration, sheets of calculations are given for standstill, 200 r.p.m., and 800 r.p.m. Similar calculations have been made for 400, 600, 1000, and 1200 r.p.m., and a set of curves plotted as explained above; from these a second set of curves (figure 68) have been constructed for terminal voltages of 250, 225, 200, 175, 150, 125 and 100 volts.

Calculations for Specification No. 3.

Saturation and iron loss curves.

(a) *For steady currents.*

			Armature		Field		
		Air gap	Teeth	Core	Yoke	Teeth	Total
Section per pole, sq. cms.			362	167	167	443	
Length per pole, cms.			4	27·9	52·5	5	
Total volume, c.cms.			5780	18600	35000	8860	
Total weight, kgms.			45	145	273	69	
Effective section per pole, sq. cms.		475	243	450	450	410	
Effective length per pole, cms.		·2	4	11	19	5	
					30		
Induction		3000	5850		3160	3500	
		6000	11700		6350	6950	
		8000	15600		8450	9300	
		10000	19600		10600	11600	
		11000	21500		11600	12700	
	Flux 10^6						
Ampere turns per pole	1·425	480	4		20	3	510
	2·85	960	14		36	6	1020
	3·8	1280	80		52	10	1420
	4·75	1600	720		75	18	2410
	5·22	1750	1600		105	28	3480

Iron loss, watts per kgm. at 25 frequency	1·425	·6	·2	·2	
	2·85	1·8	·6	·7	
	3·8	2·8	1·0	1·2	
	4·75	4·0	1·5	1·7	
	5·22	4·75	1·7	2·0	
Total iron loss at 25 frequency	1·425	27	83	14	124
	2·85	81	250	48	380
	3·8	126	418	83	630
	4·75	180	630	118	930
	5·22	214	710	138	1060

(*b*) *For alternating currents.*

R.M.S. flux per pole, $10^6 \times$	2·12	2·47	2·82	3·1	3·32	3·53	3·67
R.M.S. amp. turns per pole	750	890	1050	1230	1415	1680	1850
R.M.S. current	250	296	350	410	472	560	617
Iron loss at 25 frequency	410	530	670	800	900	1000	1080

Figure 67 shows the curves of saturation and iron loss for this motor.

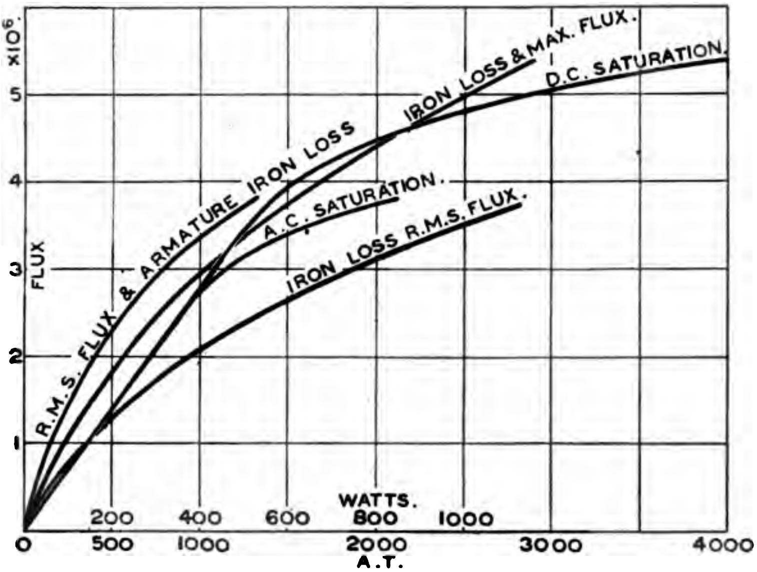

FIG. 67. Curves of saturation and iron loss for single phase series motor without commutation poles.

Calculation of Circulating Current.

Transformer voltage.

Width of brush 15 mm.

Pitch of commutator segments 5 mm.

∴ Number of armature coils short circuited by a brush is 3.

∴ Transformer voltage $=$ Flux per pole $\times 3 \times 2\pi \times 25 \times 10^{-8}$

$=$ Flux per pole $\times$ **471** $\times$ **10^{-8}**.

Reactance voltage.

Assuming 3 coils short circuited by the brush, and an average of 3½ bars in any one slot undergoing commutation simultaneously,

the number of lines surrounding each bar will be

$$[3{\cdot}5 \times 4 \times 30 + {\cdot}8 \times 50] = 460 \text{ (allowing constants 4 and } {\cdot}8\text{)};$$

the time of commutation at 100 r.p.m. will be

$$\frac{1{\cdot}5}{\frac{\pi \times 42 \times 100}{60}} = \frac{1}{147} \text{ second.}$$

Hence the reactance voltage in 3 coils or 6 bars

$$= 6 \times 460 \times 2 \times 147 \times \frac{C}{4} \times 10^{-8} \text{ where } C = \text{armature current}$$

$$= \mathbf{20} \times \mathbf{10^{-4}} \text{ volts per ampere at 100 r.p.m.}$$

Circulating ampere turns.

Consider two brush positions.

(1) The brush touching 3 commutator parts.

Allowing for ·8 mm. mica between segments, the width of contact between the brush and each commutator part = 4·2 mm.

$$\text{Resistance of such contact} = \frac{{\cdot}084}{5 \times 5{\cdot}1 \times {\cdot}42} \text{ ohm per brush arm}$$

$$= {\cdot}0078 \text{ ohm.}$$

This is in series with an Eureka resistance ·036 ohm.

∴ The total resistance in the path of the circulating current

$$= 2\,({\cdot}0078 + {\cdot}036) = \mathbf{{\cdot}0876}.$$

(2) Brush touching 4 segments symmetrically.

$$\text{Width of the outer contacts } \frac{15 - 10{\cdot}8}{2} \text{ mm.} = 2{\cdot}1 \text{ mm.}$$

Resistance of each outer contact = ·0156 ohm.

Total resistance in the path of the circulating current

$$2\,[{\cdot}0156 + {\cdot}036] = \mathbf{{\cdot}103 \text{ ohm.}}$$

Total resistance of each inner path = **·0876 ohm.**

It is difficult to combine all these figures into a single resultant; it may be, perhaps, justifiable to take a particular average case, and utilise the result in general.

Take a circulating voltage in 3 coils of 14·3 volts.

In case (1) in which only 2 coils are in circuit,

$$\text{circulating current} = \frac{\frac{2}{3} \times 14{\cdot}3 - 1{\cdot}3}{{\cdot}0876} \text{ amperes}$$

$$= 94 \text{ amperes per brush arm.}$$

$$\text{In case (2) outer circulating current} = \frac{14{\cdot}3 - 1{\cdot}3}{{\cdot}103} = 126 \text{ amperes in 3 coils,}$$

$$\text{inner circulating current} = \frac{\frac{1}{3}.\,14{\cdot}3 - 1{\cdot}3}{{\cdot}0876} = 40 \text{ amperes in 1 coil.}$$

In case (1) circulating A.T. per pole = 188.

(2) ,, ,, ,, ,, = 378 + 40 = 418.

If an average be taken at say 300 ampere turns in 2½ turns, the circulating current will be 120 amperes.

This will give an equivalent resistance

$$= \frac{14{\cdot}3 - 1{\cdot}3}{120} = \frac{1}{9{\cdot}25} \text{ ohm.}$$

Time constant correction. Assuming average number of bars per slot undergoing commutation simultaneously as 3·5,

$$\mathsf{L}\text{ for 6 bars} = [3{\cdot}5 \times 4 \times 30 + {\cdot}8 \times 50] \times 6 \times 10^{-8} = {\cdot}276 \times 10^{-4},$$

$$\mathsf{R} = \frac{1}{9{\cdot}25},$$

t at 100 r.p.m. (allowing for the width of $4+3-1=6$ segments)

$$= \frac{6 \times {\cdot}5}{\dfrac{\pi \times 42 \times 100}{60}} = {\cdot}0137\text{ second.}$$

Therefore at 100 r.p.m.

$$x = \frac{\mathsf{R}t}{\mathsf{L}} = \frac{\dfrac{1}{9{\cdot}25} \times {\cdot}0137}{{\cdot}276 \times 10^{-4}} = 53{\cdot}6.$$

It will be seen from the calculation sheet below, that in this case the circulating current is limited to a value considerably less than the maximum allowable. The resistances might have been made a little greater; but the diminution of the losses would be small. On the other hand, if the resistances had been such that the circulating current was the maximum permissible, the watts lost at the rated load would have been greater to the extent of 2 to 2½ kw.

Data for calculating Performance Curves.

Series resistance.

Armature	·021 ohm.
Commutator connections $\frac{2}{7} \times {\cdot}036$	·0103
Brushes $\dfrac{2 \times {\cdot}084}{10 \times 5{\cdot}1 \times 1{\cdot}5}$	·0021
Compensation	·0096
Excitation	·0023
Add to allow for miscellaneous losses proportional to (current)^2	·008
Total	·0533

Say **·054 ohm.**

Leakage reactance. $\cdot052 + {\cdot}054 + {\cdot}0075 = {\cdot}113$ **ohm.**

Brush friction. Allow a pressure of ·1 kgm. per square cent. and a coefficient of friction of ·3.

$$\text{At 100 r.p.m. friction watts} = \frac{\pi \times {\cdot}42 \times 100}{60} \times 20 \times 5{\cdot}1 \times 1{\cdot}5 \times {\cdot}1 \times {\cdot}3 \times 9{\cdot}81$$

$$= \mathbf{96\ watts.}$$

Bearing friction and windage. Say 90 watts at 100 r.p.m. the loss varying with the 1·5th power of the speed.

Extra iron loss due to rotation. For speeds above synchronism, *i.e.* above 750 r.p.m. assume the extra iron loss as

$$\left\{\left(\frac{\text{r.p.m.}}{750}\right)^{1{\cdot}5} - 1\right\} \times \text{armature iron loss at 25 frequency.}$$

Characteristics at standstill.

Flux per pole, $10^6 \times$	2·7	3·0	3·3	3·6
Transformer voltage	12 7	14·1	15·5	17·0
Circulating current, amps.	117	130	143	157
Circulating A.T. per pole	292	325	358	393
A.T. per pole required to produce flux	980	1150	1400	1750
A.T. per pole in excitation coils	1020	1200	1450	1790
Motor current, amps.	340	400	483	596
Series C^2R ($= C^2 \times {\cdot}054$), watts	6250	8640	12600	19100
Iron loss at 25 frequency, watts	620	740	890	1040

Drop $\left(=\frac{\text{iron loss}+C^2R}{\text{current}}+1{\cdot}3\right)=tv$		21·5	24·7	29·2	35·1
E.M.F. induced by main flux $= rs$		51	56·5	62·3	68
Leakage E.M.F. $(= C \times {\cdot}113) = st$		38·4	45·1	54·5	67·3
Terminal voltage $=$ OV		92	104·5	120·5	140
Power factor or cos ϕ		·425	·41	·395	·38
Cos θ (θ = angle between magnetic field and current)		·96	·96	·965	·98
Torque due to magnetic field and armature current		546	715	950	1300
Torque due to static friction of commutator and bearings*		76	76	76	76
Nett torque, lbs. at 1 ft. radius		470	640	870	1220
Losses	Current × drop	7300	9900	14100	20900
	4 × circulating current × circulating volts	6000	7300	8900	10700
	Total, kw.	13·3	17·2	23	31·6

Characteristics at 200 r.p.m.

Time constant correction.

$$x=\frac{Rt}{L}=\frac{53{\cdot}6}{2}=26{\cdot}8.$$

$$y=\left\{1-\frac{1}{26{\cdot}8}\left(1-\frac{1}{e^{26{\cdot}8}}\right)\right\}={\cdot}964.$$

Friction. Brushes $96 \times 2 = 192$

Bearings $90 \times 2^{1{\cdot}5} = 250$

Total **440** say.

Extra iron loss. Nil.

Flux per pole		1·8	2·5	2·9	3·2	3·4	3·6
Transformer voltage		8·5	11·8	13·7	15·1	16·0	17·0
Reactance voltage		·86	1·2	1·44	1·74	2·0	2·32
Resultant circulating voltage		8·35	11·55	13·4	14·8	15·8	16·7
Circulating current		62·8	91	108	120	129	137
Circulating A.T. per pole		157	227	270	300	323	343
A.T. per pole required to produce flux		640	900	1080	1290	1500	1750
A.T. per pole in excitation coils		645	905	1085	1300	1500	1740
Motor current		215	302	362	434	500	580
Series C^2R ($C^2 \times {\cdot}054$)		2500	4920	7070	10200	13500	18100
Iron loss at 25 frequency		320	540	700	840	940	1040
Drop $\left(=\frac{\text{iron loss}+C^2R}{\text{current}}+1{\cdot}3\right)=tv$		13·1	19·4	22·7	26·7	30·2	34·3
E.M.F. generated by rotation $=$ Or		31·6	44	51	56·3	59·8	63·3
E.M.F. induced by main flux $= rs$		33·9	47	54·6	60·3	64	68
Leakage E.M.F. $(= C \times {\cdot}113) = st$		24·3	34	41	49	56·5	65·5
Terminal voltage $=$ OV		71	100	117	134	147	163
Power factor or cos ϕ		·73	·73	·73	·71	·696	·68
Input in kw.		10·2	22	31	40·5	51	64·5
Losses	Main current × drop	2820	5860	8220	11600	15100	19900
	4 × circulating current × circulating voltage	2100	4200	5800	7100	8150	9200
	Friction	440	440	440	440	440	440
	Total, kw.	5·36	10·5	14·46	19·14	23·7	29·54
Output in kw.		4·84	11·5	16·54	21·36	27·3	34·96
Efficiency, °/o		47·5	52·3	53·4	52·8	53·5	54
B.H.P.		6·49	15·4	22·2	28·6	36·6	46·9
Torque in lbs. at 1 ft. radius		170	404	583	751	961	1230

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* Allowing for armature complete weighing 1650 lbs., journals 4 ins. in diameter, and ·25 coefficient of static friction.

Characteristics at 800 r.p.m.

Time constant correction.

$$x=\frac{Rt}{L}=\frac{53\cdot6}{8}=6\cdot7.$$

$$y=\left\{1-\frac{1}{6\cdot7}\left(1-\frac{1}{e^{6\cdot7}}\right)\right\}=\cdot85.$$

Friction. Brushes $96\times8=768$
Bearings $90\times8^{1\cdot5}=2040$

Total **2810** say.

Extra iron loss. $\left[\left(\frac{800}{750}\right)^{1\cdot5}-1\right]\times$ armature iron loss
$=\cdot1\times$ armature iron loss.

Flux per pole, $10^6\times$		1·0	1·5	2·0	2·4	2·7	3·0	3·2
Transformer voltage		4·7	7·06	9·4	11·3	12·7	14·1	15·1
Reactance voltage		1·8	2·76	3·62	4·32	4·9	5·75	6·4
Resultant circulating voltage		4·7	7·0	9·4	11·2	12·6	14	15·1
Circulating current		26·7	44·8	63·6	77·5	89	100	108
Circulating A.T. per pole		67	112	159	194	222	250	271
A.T. per pole required to produce flux		360	550	720	860	980	1150	1290
A.T. per pole in excitation coils		340	520	680	810	920	1080	1200
Motor current		113	173	227	270	306	360	400
Series C^2R ($C^2\times\cdot054$)		690	1620	2770	3940	5060	7000	8640
Iron loss at 25 frequency		140	250	380	500	620	740	840
Drop $\left(=\frac{\text{iron loss}+C^2R}{\text{current}}+1\cdot3\right)=tv$		7·4	10·8	13·9	16·4	19·9	22·8	25
E.M.F. generated by rotation $=Or$		70·5	106	141	169	190	211	225
E.M.F. induced by main flux $=rs$		18·8	28·3	37·7	45·3	51	56·5	60·4
Leakage E.M.F. ($=C\times\cdot113$) $=st$		12·8	19·6	25·6	30·5	34·6	40·7	45·2
Terminal voltage $=OV$		82	123	163	195	221	246	264
Power factor or $\cos\phi$		·972	·977	·98	·98	·98	·98	·98
Input in kw.		9·04	20·8	36·2	51·5	66·3	86·5	103
Losses	Main current × drop	830	1870	3150	4430	6100	8200	10000
	4 × circulating current × circulating voltage	500	1250	2390	3480	4480	5600	6520
	Extra iron loss	10	10	20	20	30	30	40
	Friction	2810	2810	2810	2810	2810	2810	2810
	Total, kw.	4·15	5·94	8·37	10·74	13·44	16·64	19·37
Output in kw.		4·89	14·86	27·83	40·76	52·86	69·86	83·63
Efficiency, °/₀		54	71·5	77	79·2	79·8	80·8	81·2
B.H.P.		6·55	19·9	37·3	54·6	70·8	93·6	112
Torque in lbs. at 1 ft. radius		43	130	245	358	465	615	735

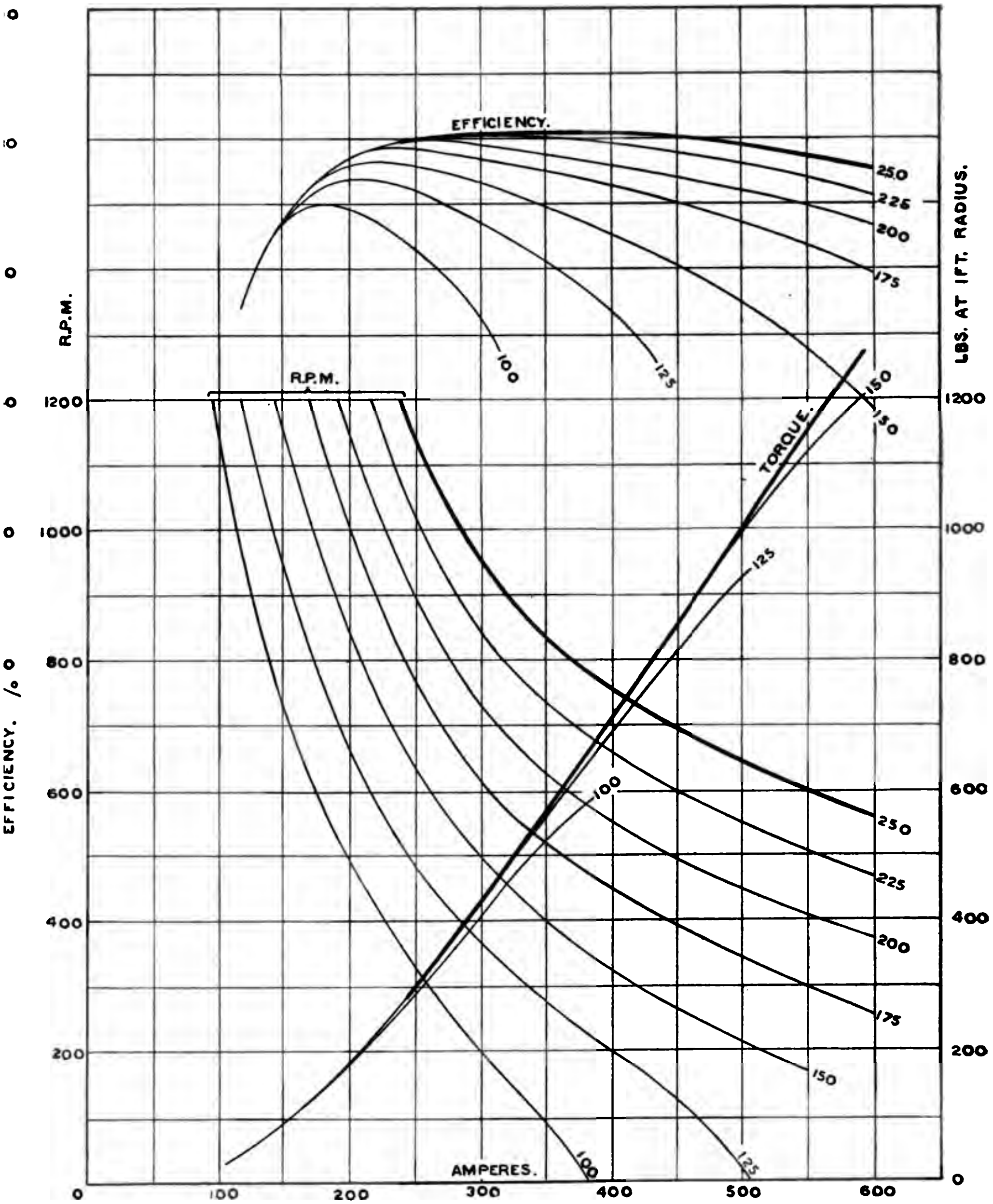
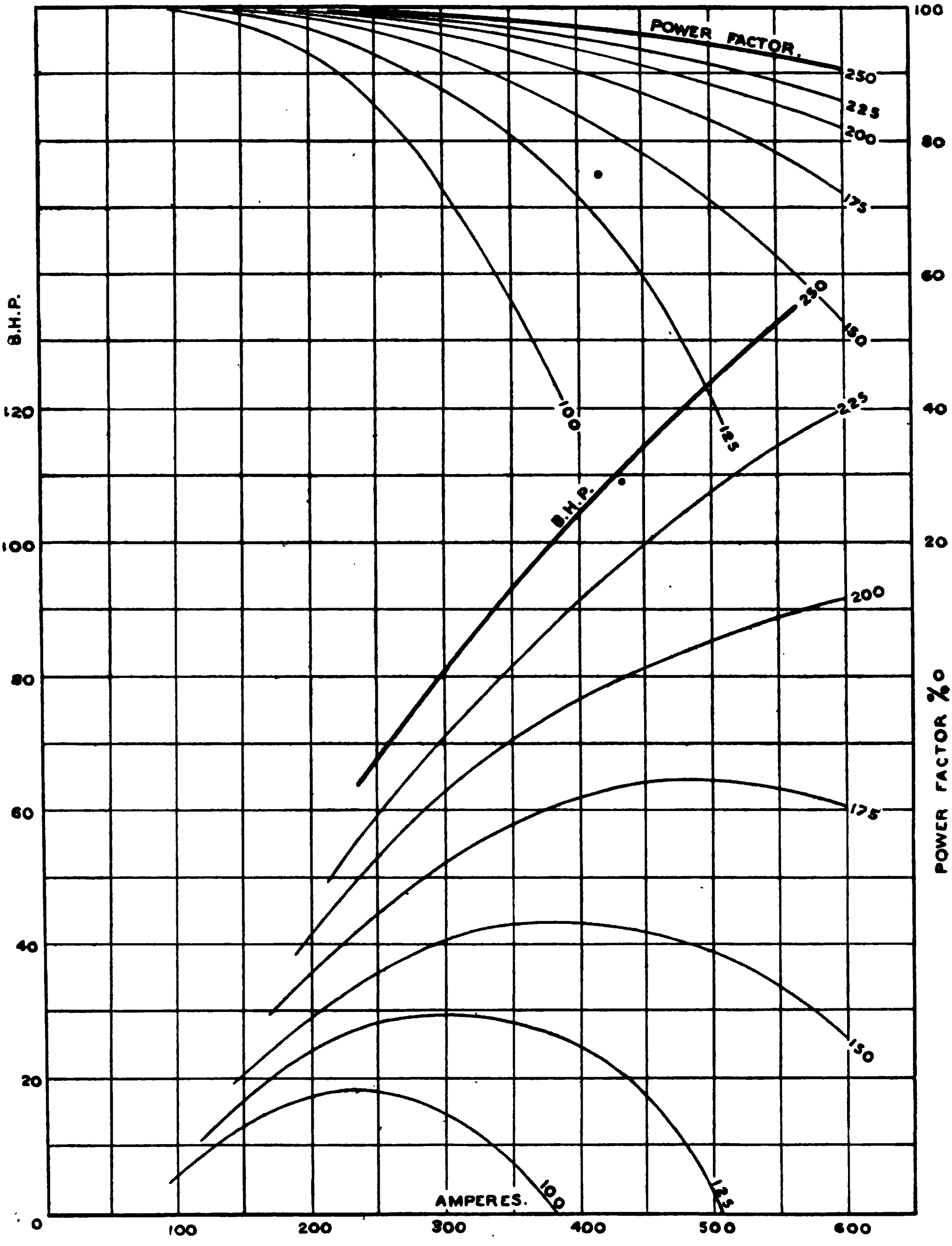


FIG. 68. Performance curves of 100 .P. single



phase series motor without commutation poles.

PART 2.

DESIGN AND METHOD OF CALCULATION FOR SINGLE PHASE SERIES MOTOR WITH COMMUTATION POLES.

Single phase series motor with commutation poles. The provision of commutation poles makes several important differences in the design and calculation of the single phase series motor.

The chief difference, and that one which may be taken as the basis of the other differences, is that the transformer effect is eliminated entirely for a certain definite speed and with more or less completeness at other speeds. Naturally it produces no effect when the motor is at rest, as it depends for its usefulness upon the rotation of the armature. It cannot, therefore, be claimed that it entirely eliminates the transformer voltage, as it leaves it quite unaffected at that moment when it is perhaps most harmful. The partial elimination is, however, of considerable value, as the succeeding calculations will make manifest.

Effect of commutation pole on the diagram. The presence of the commutation pole will affect the diagram in two ways, firstly in the partial or complete suppression of the demagnetisation due to circulating currents under the brushes, and secondly in slight alterations produced by the self induction of the poles themselves. Both of these effects will depend on the method of excitation of the commutation poles.

Methods of excitation of commutation poles. On the assumption that at a certain speed the commutation is perfect, the commutating or auxiliary field must possess two components of the proper magnitude and phase.

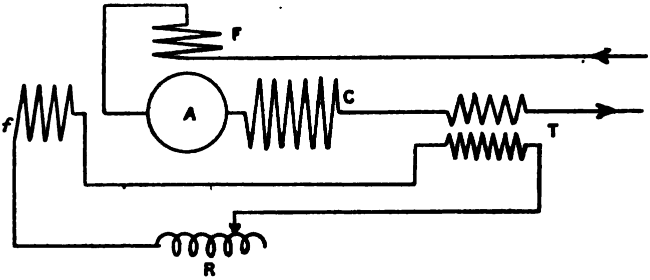
The component *a* will serve to neutralise the reactance voltage and the component *b* will serve to neutralise the transformer voltage.

Component *a* will be in phase with the armature current and will be proportional to the reactance voltage at the given speed.

Component *b* will be at right angles to the main field, and will be proportional to the transformer voltage under the given conditions.

Various methods have been proposed for obtaining the proper excitation of the auxiliary poles. Of these methods there are two principal ones which are adopted by different manufacturers.

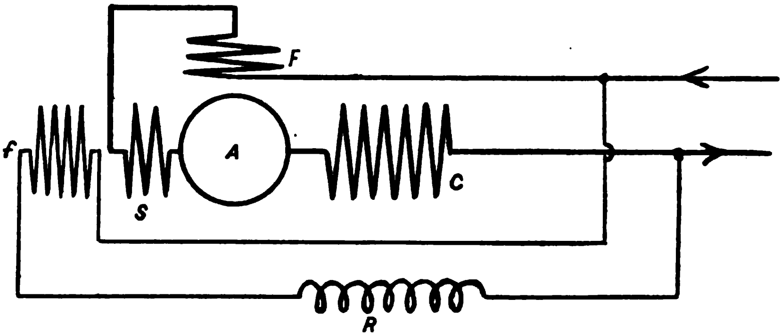
Oerlikon method. This consists in obtaining the excitation from a small series transformer, the primary of which is in series with the main current, while the secondary is connected through an adjustable non-inductive resistance to the excitation windings of the auxiliary poles. By suitable adjustment of the transformer, resistance and excitation windings, any desired magnitude and phase of the auxiliary field can be obtained. See figure 69. This method is adopted by the Oerlikon Company *.



A Armature. F Main field winding. C Compensation winding. T Series transformer. R Adjustable resistance. *f* Commutation field winding.

FIG. 69. Diagram of Oerlikon method of exciting the commutation poles in single phase motors.

Siemens method. This consists in winding the auxiliary poles with coils connected through a resistance to the terminals of the motor or to suitable points on the secondary of the transformer supplying the motor. Series coils are also wound on the auxiliary poles connected in the main circuit of the motor, partly to complete the compensation of the armature and partly to supply the component *b* of the auxiliary field. The latter coils can be arranged to counteract precisely the



A Armature. F Field winding. C Compensation winding. S Series winding on commutation pole. *f* Shunt winding on commutation pole. R Resistance.

FIG. 70. Diagram of Siemens method of producing commutation field.

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* See English patent 28968 of 1904, also *Electrische Bahnen und Betrieb*, January 24, 1906.

reactance voltage in exactly the same way as in direct current machines; and the former can be adjusted by means of the resistance to give the other component required. See figure 70.

Diagram for auxiliary field. In figure 71 let OC represent the main field. Then OB lagging 90° behind OC will represent the transformer voltage, and on the assumption that the transformer voltage and the reactance voltage are both exactly neutralised OA in phase with OC will represent the reactance voltage. The resultant of these is OF with which the auxiliary field must be in phase.

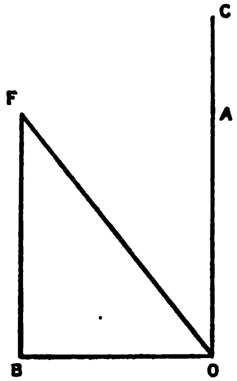


FIG. 71. Vector diagram for excitation of commutation pole.

Consider now the excitation of the auxiliary poles to produce this field. The total ampere turns giving rise to the field will be the resultant of those on the auxiliary poles and those on the armature which are not compensated by the conductors embedded in the main poles. In other words, the uncompensated ampere turns on the armature must first be neutralised, and then, in addition, ampere turns must be provided to produce the desired commutating field.

Consider first the Oerlikon method, and suppose that the winding on each commutation pole consists of a single coil, the current in which is obtained from the series transformer. In the diagram in figure 72, Og represents those armature ampere turns which are not compensated by the compensation windings in the main poles; these must be counterbalanced by an equal number of ampere turns on the commutation poles, represented by Og'. In addition, ampere turns necessary to produce the commutating field must be provided; let Of, proportional to and in phase with OF in figure 71, represent these additional ampere turns. Then the total will be the resultant of Og' and Of, that is Ok. The commutating field will produce a back E.M.F. Ol in the auxiliary circuit, and the resistance in that circuit will cause a drop *lm*. Hence the voltage which must be applied to the auxiliary winding is equal and opposite to O*m*.

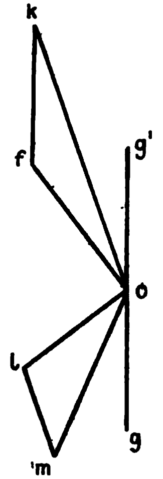


FIG. 72. Vector diagram of Oerlikon method of exciting commutation poles.

A similar diagram can be constructed for the Siemens method. Taking the same vector OF (figure 71) to represent the commutating field required, another diagram (figure 73) can be constructed for the

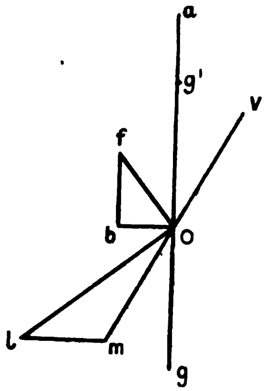


FIG. 73. Vector diagram of Siemens method of producing commutation field.

currents and voltages involved. Let O*g'* as before represent the counteracting ampere turns, supplied in this case by the series turns on the commutation poles, and *g'a* the ampere turns for counteracting the reactance voltage, supplied also by the series turns. Then O*b* will represent the ampere turns supplied by the shunt winding for the purpose of counteracting the transformer voltage, and must be such that the resultant of $g'a$ and Ob is Of proportional to and in phase with OF. The internal voltage in the shunt circuit will be the resultant of Ol at right angles to Of, and lm parallel to Ob and proportional to the total resistance in the shunt circuit; hence the voltage that must be applied to this circuit is equal and opposite to Om, and the resistance and winding must be adjusted so that this voltage is in phase with the terminal voltage of the motor and either equal to it or to a suitable fraction of it.

Specification No. 4.

Single phase series motor with commutating poles.

Rating. 100 H.P. for 1 hour, 75° C. rise. 320 volts; 750 r.p.m.; 280 amperes; 25 frequency.

Armature.

External diameter	500 mm.
Internal diameter	316 „
Gross length	330 „
Air ducts	2—12 „
Nett length	306 „
Percentage iron	88 °/o
Length of iron	270 mm.
Number of slots	79
Width per slot	10·5 mm.
Depth per slot	42 „
Opening of slot	2 „
Type of winding	2-circuit double
Number of poles	6

Conductors per slot	8
Scheme of connections	1 \ / 213 106
Size of conductors	14 mm. × 1·6 mm.
Total resistance (hot)	·0259 ohm
Total leakage reactance	·079 ohm
Weight of copper	92 kgms.
Weight of iron	174 kgms.
Peripheral speed	19·6 m.sec.

Commutator.

Diameter	400 mm.
Length	246 „

Number of segments	316
Peripheral speed	15·7 m.sec.

Commutator connections.

Material	German silver
Cross section	3·8 mm. × 1·5 mm.

Length of one connection	750 mm.
Resistance of one connection	·0284 ohm

Brushes.

Number of brush arms	4
Brushes per arm	6
Section of each brush	34 mm. × 12 mm.

Length of each brush	
Quality of carbon	ordinary
Type of brush-holder	box

Field.

Bore	504 mm.	Number of poles	6
External diameter	704 „	Number of slots	54 + 12
Gross length	330 „	Width per slot	16·5 mm. and 8 mm.
Air ducts	2—12 „		
Nett length	306 „	Depth per slot	50 „
Percentage iron	88 °/₀	Opening of slot	2 „
Length of iron	270 mm.	Weight of iron	293 kgms.

Field windings.

	Compensation	Excitation	Commutation
Conductors per slot	6 and 2	3	28 (4 × 7)
Section of conductor, mm.	20 × 4	20 × 4	2·4 × 3·1 bare 2·8 × 3·5 covered
Section of end connections, mm.	50 × 1·6	20 × 4	ditto
Total resistance (hot), ohm	·016	·00765	·6
Total leakage reactance, ohm	·083	·023	·7
Weight of copper, kgms.	145	20·2	13·8

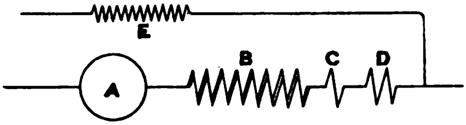
Miscellaneous.

Length between axle centres	17·8″	Vertical distance between centres	42 mm. = 1·65″
Gear ratio	18 : 71	Clearance between motor and rails	6″
Diameter of pinion	7·2″		
Diameter of gear wheel	28·4″	Diameter of driving wheel	38″
Gear velocity at rated speed	1410 ft. per minute	Miles per hour at rated speed	21·6

The general design of this motor may be seen from the foregoing specification and the sketch in figure 74. It will be seen that the motor is wound for six poles, and that the armature has a two-circuit double (or series parallel) winding; the commutator connections are inserted in the slots, and the general arrangement of the armature is similar to that shewn in figure 65.

The field or stator, however, is quite different, in that there are no recesses opposite the points of commutation. These recesses are filled up with the commutation poles, which are contained between the special open slots. Each commutation pole contains two small slots in which are conductors carrying the main current, and which serve to compensate for the main current in the armature conductors under the commutation pole. This arrangement should assist in securing a uniformly distributed commutating field under all conditions of commutation. Surrounding these poles are the shunt coils placed in the open slots as shewn, the same slots containing the excitation windings of the main poles.

The compensation winding consists of conductors uniform with the excitation winding, embedded in the slots in the pole face. These slots, however, contain six conductors each, these conductors being connected two in parallel, whereas the compensation windings in the commutation poles and also the main excitation windings are arranged in a single circuit. These connections are shewn diagrammatically in figure 74.



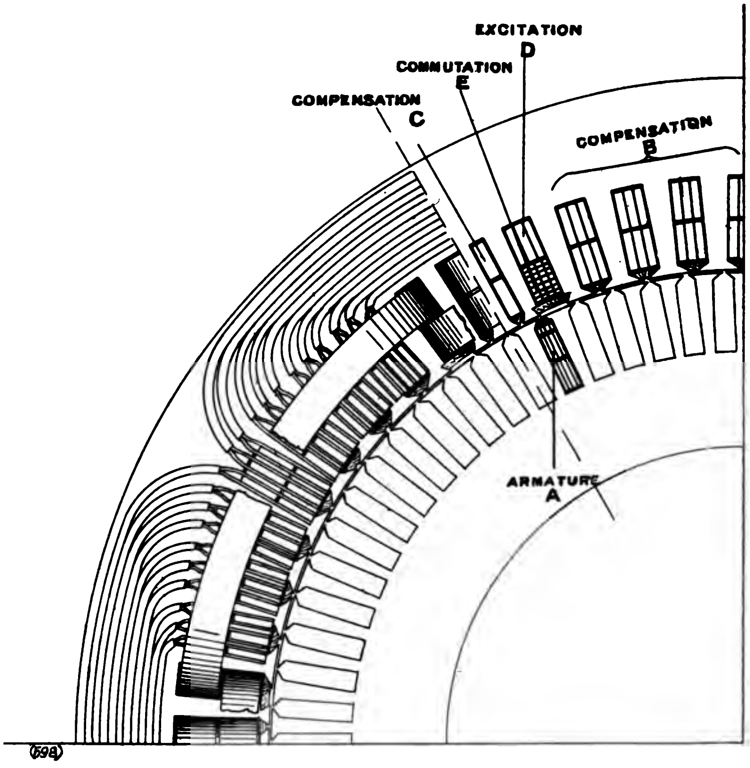


FIG. 74. Armature and field laminations and conductors for single phase series motor with commutation poles.

Calculation of performance curves. The method of calculating the performance curves only differs slightly from that described in connection with the plain series single phase motor, the difference being due to the presence of the commutation pole. This factor makes itself evident in the calculation of the magnitude and phase of the circulating current. The method is as follows, see figure 75: Assume a certain motor speed and a definite value of the flux per pole; draw the vector OC for the ampere turns per pole required to produce this flux (obtained from the saturation curve). From this flux per pole set off, at right angles thereto, the transformer voltage Oe_1; assume the reactance voltage e_1e_2 in phase and magnitude, and assume also the phase of the terminal voltage OV and hence the phase and magnitude of the voltage e_2e_3 produced by rotation in the commutation field. The resultant of these three voltages is the circulating voltage Oe_3, from which the circulating ampere turns per pole OB are deduced. These ampere turns are then compounded with those required to produce the main flux in order to obtain the ampere turns per pole OA in the main excitation coils. Dividing the number so found by the turns per pole, the motor current is obtained. From this point onwards the construction of the diagram is quite straightforward and follows the same course as with the plain series single phase motor, the final result being the terminal voltage OV in phase and magnitude. At this point it is necessary to check the assumed value and phase of the reactance voltage, and the assumed phase of the terminal voltage, and if necessary to repeat the calculation with the corrected values and phases.

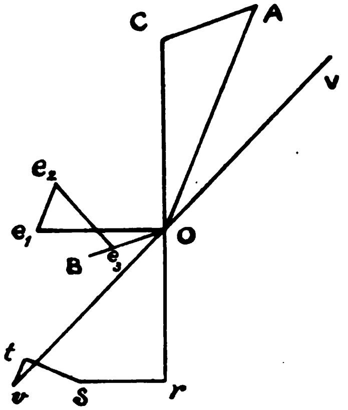


FIG. 75. Vector diagram for single phase series motor with commutation poles.

The process is one of trial and error, and at first sight seems complicated and tedious; a very little practice, however, will enable the designer to make assumptions which are either immediately correct or near enough to need only one approximation.

Having thus prepared the diagram, the rest of the calculation consists simply in finding the input, the losses, the output and so on, which are plainly set out in the calculation sheets below and need no special explanation.

Curves have been calculated for standstill, 200, 400, 600, 800, 1000 and 1200 r.p.m., and from these the performance curves in figure 77 have been deduced. The calculation sheets for standstill, 200 and 800 r.p.m., are given in the text, the others are similar.

Calculations for Specification No. 4.

Saturation and iron loss curves.

(*a*) *For steady currents.*

	Flux 10^6	Air gap	Armature Teeth	Armature Core	Field Yoke	Field Teeth	Com. pole teeth	Total
Section per pole, sq. cms.		550	205	270	270	253	73	
Length per pole, cms.			4·2	9·6	17·1	5	5	
Total volume, c.cms.			6800	15550	27800	7580	2190	
Total weight, kgms.			53	121	217	59	17	
Effective section per pole, sq. cms.		550	193	380	380	223		
Effective length per pole, cms.		·2	4·2	9	14	5		
					23			
Induction		3000	8550		4350	7400		
		5000	14200		7250	12300		
		6000	17100		8700	14800		
		7000	20000		10200	17200		
		7500	21400		10900	1850		
Amp. turns per pole	1·65	478	7		25	8		520
	2·75	800	42		32	22		900
	3·3	960	180		40	70		1250
	3·85	1115	880		54	220		2270
	4·12	1200	1630		65	450		3340
Iron loss, watts per kgm. at 25 frequency	1·65		1·02		·4	·85		
	2·75		2·4		·8	1·9		
	3·3		3·2		1·1	2·6		
	3·85		4·2		1·4	3·1		
	4·12		4·7		1·6	3·7		
Total iron loss at 25 frequency	1·65		40		135	50		225
	2·75		96		270	112		480
	3·3		128		370	150		650
	3·85		170		470	180		820
	4·12		190		540	220		950

(*b*) *For alternating currents.*

R.M.S. flux per pole, $10^6 \times$	2·9	2·76	2·62	2·33	2·12
R.M.S. amp. turns per pole	1635	1300	1095	818	704
R.M.S. current	545	433	365	273	235
Iron loss at 25 frequency	920	840	760	640	560

Figure 76 shews the curves of saturation and iron loss for this motor.

Calculation of circulating current and commutation field. In this case the calculation of circulating current is not quite so simple as in the former case owing to the difference in the scheme of armature winding.

Transformer voltage. If N be the flux per pole, the transformer voltage will be that induced in three armature coils (since there are six poles and a wave winding, and the brushes are not wide enough to cover more than three commutator parts, alternate parts only being connected to the same armature winding).

Hence $\qquad E_T = 3 \times \mathsf{N} \times 2\pi \times 25 \times 10^{-8} = 4{\cdot}71 \times \mathsf{N} \times 10^{-8}.$

Circulating currents. The resistance in the short circuit varies with the position of the brushes.

(*a*) Brush touching three segments only. The width of the contact between the brush and each of the outer of the three segments will be 3·5 mm., since the pitch of the commutator parts is 4 mm. and the thickness of the mica may be taken in this case as ·5 mm., hence the resistance will be

$$2 \times {\cdot}0284 + \frac{2 \times {\cdot}084}{6 \times 3{\cdot}4 \times {\cdot}35}$$
$$= {\cdot}0568 + {\cdot}0236 = \mathbf{{\cdot}08\ ohm.}$$

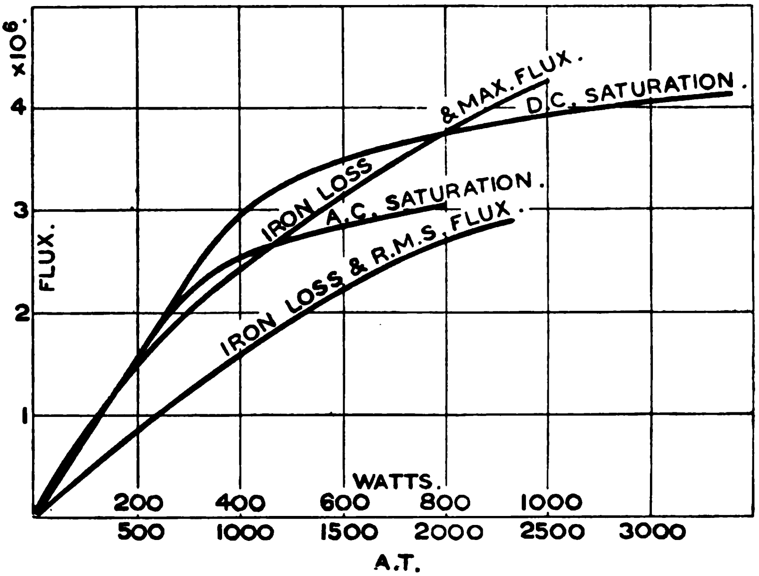


FIG. 76. Curves of saturation and iron loss for single phase series motor with commutation poles.

(*b*) Brush touching four segments, resistance will be

$$2 \times {\cdot}0284 + \frac{2 \times {\cdot}084}{6 \times 3{\cdot}4 \times {\cdot}175}$$
$$= {\cdot}0568 + {\cdot}047 = \mathbf{{\cdot}114\ ohm.}$$

In addition to these resistances, the back E.M.F. of 1·3 volts must be taken into account in each case.

The circulating current due to E_T will, therefore, be

$$\frac{E_T - 1{\cdot}3}{{\cdot}08} \text{ and } \frac{E_T - 1{\cdot}3}{{\cdot}114} \text{ respectively.}$$

The former of these flows in one turn per pole, and the latter in two turns per pole in parallel. Now since there are two sets of brushes for each polarity, it may be taken that each set occupies one of the two positions mentioned above, and since the brushes are in parallel, the total ampere turns per pole will be

$$\frac{E_T - 1{\cdot}3}{{\cdot}08} + 2\,\frac{E_T - 1{\cdot}3}{{\cdot}114} \text{ or } 30\,(E_T - 1{\cdot}3).$$

This must be corrected for an additional circulating current flowing through the lead connecting the two sets of brushes of the same polarity, which in this case may be estimated as an extra 10 %; making the total

$$33 \times (E_T - 1{\cdot}3) \text{ ampere turns per pole.}$$

Reactance voltage. Taking three as the average number of bars in any one slot undergoing commutation simultaneously, the lines of force attached to each bar corresponding to 1 ampere in the armature or $\frac{1}{4}$ ampere in each bar, will be

$$\tfrac{1}{4}(3 \times 4{\cdot}5 \times 30{\cdot}6 + {\cdot}8 \times 37) = \tfrac{1}{4} \times 444 = 111 \text{ c.g.s. lines.}$$

Now, if there were two sets of brushes, the time of commutation would be that corresponding to the width of the brush less the width of one commutator segment, since alternate segments only are connected to the same winding. But as there are four sets of brushes, the period of commutation is lengthened.

The width of a brush is 12 mm., the pitch of commutator segments is 4 mm. If, therefore, there were only two sets of brushes the time of commutation would correspond to a width of 8 mm., as it is only to alternate commutator parts that each of the armature windings is connected. To allow for the other sets of brushes, assume a width of 10 mm.

Then the reactance voltage per bar per ampere at 100 r.p.m.

$$= 111 \times 2 \times \frac{\pi \times 40 \times 100}{1{\cdot}0 \times 60} \times 10^{-8} = 4{\cdot}65 \times 10^{-4},$$

or for 6 bars in series $= 6 \times 4{\cdot}65 \times 10^{-4} = 28 \times 10^{-4}$.

Time constant correction.

$$L \text{ for 6 bars} = 444 \times 6 \times 10^{-8} = {\cdot}266 \times 10^{-4}.$$

$$R \text{ for 2 sets of brushes} = \frac{1}{\dfrac{1}{{\cdot}08} + \dfrac{1}{{\cdot}114}} = \frac{1}{21}.$$

t at 100 r.p.m. (allowing for $3 + 4 - 1 = 6$ segments at 4 mm. each)

$$= \frac{2{\cdot}4}{\pi \times 40 \times \frac{100}{60}}$$

$$= {\cdot}0115 \text{ second.}$$

$$\therefore\ x = \frac{Rt}{L} \text{ for 100 r.p.m.} = \frac{\frac{1}{21} \times {\cdot}0115}{{\cdot}266 \times 10^{-4}} = 20{\cdot}5.$$

Shunt excitation of commutation pole. The two components of the commutating field at the rated load will correspond to the transformer voltage and the reactance voltage respectively.

The transformer will be approximately

$$2{\cdot}35 \times 10^{6} \times 2\pi \times 25 \times 3 \times 10^{-8} \text{ volts in 6 bars}$$

$$= 11{\cdot}1 \text{ volts.}$$

The reactance voltage will be approximately

$$28 \times 10^{-4} \times 280 \times \tfrac{750}{100} \text{ volts in 6 bars}$$

$$= 5{\cdot}85 \text{ volts.}$$

Now if the commutation pole is so adjusted in strength and phase that both these voltages are cancelled, the armature will have no magnetising or demagnetising effect on the main field; consequently the main field will be in phase with the armature current and the transformer voltage will be at right angles to the reactance voltage. The resultant of the two voltages will therefore be

$$\sqrt{11{\cdot}1^2 + 5{\cdot}85^2} = 12{\cdot}5 \text{ volts.}$$

If B represent the strength of the commutating field in the air gap

$$B \times 31 \times 19{\cdot}7 \times 6 \times 10^{-8} = 12{\cdot}5 \text{ volts}$$

or

$$B = 3400.$$

The ampere turns per pole required for excitation will be

$$\frac{3400 \times 2}{12{\cdot}57} \text{ for the gap alone} = 540$$

or to allow for the iron say **600**.

In this case, the armature ampere turns are almost exactly neutralised by the compensation on the field; thus with a motor current of 280 amperes

$$\text{Compensation ampere turns per pole} = 280\,(3{\cdot}5 \times 3 + 2) = 3500$$

$$\text{Armature} \quad ,, \quad ,, \quad ,, \quad ,, \quad = \tfrac{280}{4}\left(\tfrac{312}{6}\right) = 3640.$$

(The 312 is used instead of the full number of armature turns, 316, because of the fact that 8 armature turns are short circuited by the brushes, in each of which half the normal current per bar may be allowed.) There remains therefore 140 ampere turns per pole to be supplied by the shunt coil in addition to the 600 ampere turns required for excitation.

The resultant of these two currents will be about 660 amperes on the basis of the main power factor being ·93.

Shunt current $= \tfrac{660}{28} = 23{\cdot}6$ amperes.

Commutation flux per pole.

The gap area $= 4 \times 31 = 124$ sq. cms.

Flux per pole $= 124 \times 3400 = {\cdot}42 \times 10^6$.

Back E.M.F. $= 6 \times 28 \times 2\pi \times 25 \times {\cdot}42 \times 10^6 \times 10^{-8}$ volts

$= 111$ volts, all 6 coils in series.

Leakage E.M.F. $= 23{\cdot}6$ multiplied by leakage reactance

$= 23{\cdot}6 \times {\cdot}7 = 16{\cdot}5$ volts.

Resistance drop $= 23{\cdot}6 \times {\cdot}6 = 14{\cdot}2$ volts.

Voltage to be applied to the shunt coils. Strictly speaking the commutation field will vary very slightly in phase and strength as the main current varies, due to slight changes in the shunt current and hence to slight differences in the leakage E.M.F. and the resistance drop.

It will be sufficiently accurate, however, to assume a constant value of the commutation field, the strength being 3400 and the phase lagging behind the terminal voltage by about 82°.

The voltage to be applied to the shunt coils in order to produce the required strength of field can easily be calculated and is about 135 volts.

[NOTE. In working out performance curves it will be found a considerable assistance to cut out in cardboard an angle of 82° in order to set off the commutating field lagging by this angle behind the terminal voltage.]

Data for calculating performance curves.

Series resistance.

Armature	·0259 ohm.
Commutator connections $\tfrac{2}{7} \times {\cdot}0284$	·0081 ,,
Brushes $\dfrac{2 \times {\cdot}084}{12 \times 3{\cdot}4 \times 1{\cdot}2}$	·0034 ,,
Compensation	·016 ,,
Excitation...	·0077 ,,
Add to allow for miscellaneous losses proportional to the (current)^2	·009 ,,
	·0701

Say **·07 ohm.**

Brush friction. Allow a pressure of ·1 kgm. per sq. cm. and a coefficient of friction ·3.

$$\text{Friction watts at 100 r.p.m.} = \frac{\pi \times \cdot 4 \times 100}{60} \times \cdot 1 \times 24 \times 3\cdot 4 \times 1\cdot 2 \times \cdot 3 \times 9\cdot 81 \text{ watts}$$

$$= \mathbf{60 \text{ watts}} \text{ at 100 r.p.m.}$$

Bearing friction and windage. Say 90 watts at 100 r.p.m., the loss varying as the 1·5th power of the speed.

Shunt loss. Assume this loss constant, except for low power factors as at standstill,

$$C^2R = 23\cdot 6^2 \times \cdot 6 = 350$$

$$\text{Iron loss say} = 50$$

$$\overline{\mathbf{400 \text{ watts.}}}$$

Extra iron loss due to rotation. For speeds above synchronism, *i.e.* above 500 r.p.m., assume the extra iron loss as

$$\left(\frac{\text{r.p.m.}}{500}\right)^{1\cdot 5} - 1 \times \text{armature iron loss at 25 frequency.}$$

Standstill characteristic.

	Flux per pole, $10^6 \times$	1·5	2·0	2·3	2·5	2·7	2·9
	Transformer voltage in 6 bars	7·06	9·42	10·86	11·8	12·75	13·7
	Circulating A.T. per pole	190	269	316	346	378	410
	A.T. per pole required to produce flux	475	670	800	970	1220	1630
	A.T. per pole in excitation coils	510	723	860	1030	1280	1680
	Main motor current, amps.	170	241	287	343	427	560
	Series C^2R ($C^2 \times \cdot 07$), watts	2030	4100	5800	8200	12700	22000
	Iron loss at 25 frequency, watts	370	520	620	710	800	920
	Drop $\left(= \frac{C^2R + \text{iron loss}}{\text{current}} + 1\cdot 3\right) = tv$, volts	15·4	20·4	23·6	27·2	32·9	42·3
	E.M.F. induced by main flux $= rs$, volts	42·4	56·6	65	71	76·5	82
	Leakage induced E.M.F. ($C \times \cdot 185$) $= st$, volts	31·4	44·5	53	63·5	79	104
	Terminal volts = OV	77	105	123	140	162	193
	Cos ϕ	·404	·395	·385	·364	·343	·324
	Cos θ (θ = angle between main field and armature current)	·93	·928	·93	·94	·955	·973
	Torque in lbs. at 1 ft. radius (gross)	264	496	680	900	1220	1750
	Torque due to static friction in bearings and on commutator *	73	73	73	73	73	73
	Nett torque, lbs. feet	191	423	607	827	1147	1677
Losses	Main motor current × drop	2620	4900	6770	9340	14000	23700
	2 × transformer voltage × circulating current	2680	5100	6860	8160	9660	11300
	Shunt loss	400	430	470	500	530	600
	Total losses, kw.	5·7	10·4	14·1	18	24·2	35·6

Characteristic at 200 r.p.m.

Time constant correction.

$$x = \frac{Rt}{L} = 10\cdot 25 \, ; \quad y = 1 - \frac{1}{10\cdot 25} = \cdot 9.$$

Friction. Brushes 120 watts

Bearings, etc. 250 ,, $(= 90 \times 2^{1\cdot 5})$

Total **370**

Extra iron loss due to rotation. Nil.

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* Allowing for armature complete weighing 1650 lbs., journals 4 inches in diameter, and ·25 coefficient of static friction.

Flux per pole, $10^6 \times$		2·0	2·3	2·5	2·7	2·9
Transformer voltage in 6 bars		9·42	10·86	11·8	12·75	13·7
Reactance voltage ,,		1·37	1·62	1·94	2·37	3·1
Commutating voltage ,,		3·33	3·33	3·33	3·33	3·33
Resultant circulating voltage		7·8	9·15	10	10·87	11·8
Circulating A.T. per pole		193	234	259	284	312
A.T. per pole required to produce flux		670	800	970	1220	1630
A.T. per pole in excitation coils		740	870	1032	1270	1660
Main motor current, amps.		246	290	344	423	554
Series C^2R ($C^2 \times \cdot07$), watts		4250	5900	8300	12500	21500
Iron loss at 25 frequency, watts		520	620	710	800	920
Drop $\left(=\frac{\text{iron loss}+C^2R}{\text{current}}+1\cdot3\right)=tv$, volts		20·7	23·8	27·4	32·7	41·8
E.M.F. induced by main flux $= rs$, volts		56·5	65	70·8	76·2	82·2
Leakage induced E.M.F. ($C \times \cdot185$) $= st$, volts		45·5	53·5	63·7	78·5	102·5
E.M.F. generated by rotation $= Or$, volts		63	72·6	79	85·4	92
Terminal volts $= OV$, volts		128	148	166	188	222
Cos ϕ (excluding shunt current)		·754	·75	·735	·706	·667
Input (including shunt loss), kw.		24·1	32·6	42·4	56·7	82·4
Losses	Main current × drop	5100	6900	9440	13800	23200
	2 × circulating voltage × circulating current	3020	4300	5180	6180	7350
	Friction loss	370	370	370	370	370
	Shunt loss	400	400	400	400	400
	Total, kw.	8·9	12	15·4	20·75	31·3
Output, B.H.P.		20·3	27·5	36·1	48	68·2
Efficiency, %		63	63	63·5	63·5	62
Torque, lbs. at 1 ft. radius		535	725	950	1260	1800

Characteristic at 800 r.p.m.

Time constant correction.

$$x=\frac{Rt}{L}=2\cdot56\ ;\quad y=\cdot64.$$

Friction.	Brushes	480 watts
	Bearings, etc. ($90 \times 8^{1\cdot5}$)	2020 ,,
	Total	**2500**

Extra iron loss due to rotation.

$\left\{\left(\frac{800}{500}\right)^{1\cdot5}-1\right\}=1\cdot02$ times armature iron loss at 25 frequency.

Flux per pole, $10^6 \times$	1·0	1·5	2·0	2·3
Transformer voltage in 6 bars	4·71	7·06	9·42	10·86
Reactance voltage ,,	2·9	3·85	5·08	5·98
Commutating voltage ,,	13·33	13·33	13·33	13·33
Resultant circulating voltage	7·1	4·85	2·4	1·0
Circulating A.T. per pole	122	75	23	0
A.T. per pole required to produce flux	320	475	670	800
A.T. per pole in excitation coils	390	510	680	800
Main motor current, amps.	130	170	227	267
Series C^2R ($C^2 \times \cdot07$), watts	1180	2030	3600	5000
Iron loss at 25 frequency, watts	240	370	520	620
Drop $\left(=\frac{\text{iron loss}+C^2R}{\text{current}}+1\cdot3\right)=tv$, volts	12·2	15·4	19·4	22·3
E.M.F. induced by main flux $= rs$, volts	28·2	42·4	56·5	65
Leakage induced E.M.F. $= st$, volts	24·1	31·4	42	49·4
E.M.F. generated by rotation $= Or$, volts	126	190	253	291
Terminal voltage $= OV$, volts	153	221	290	334
Cos ϕ (excluding shunt current)	·82	·89	·933	·94
Input (including shunt loss), kw.	16·7	33·9	61·8	84·4

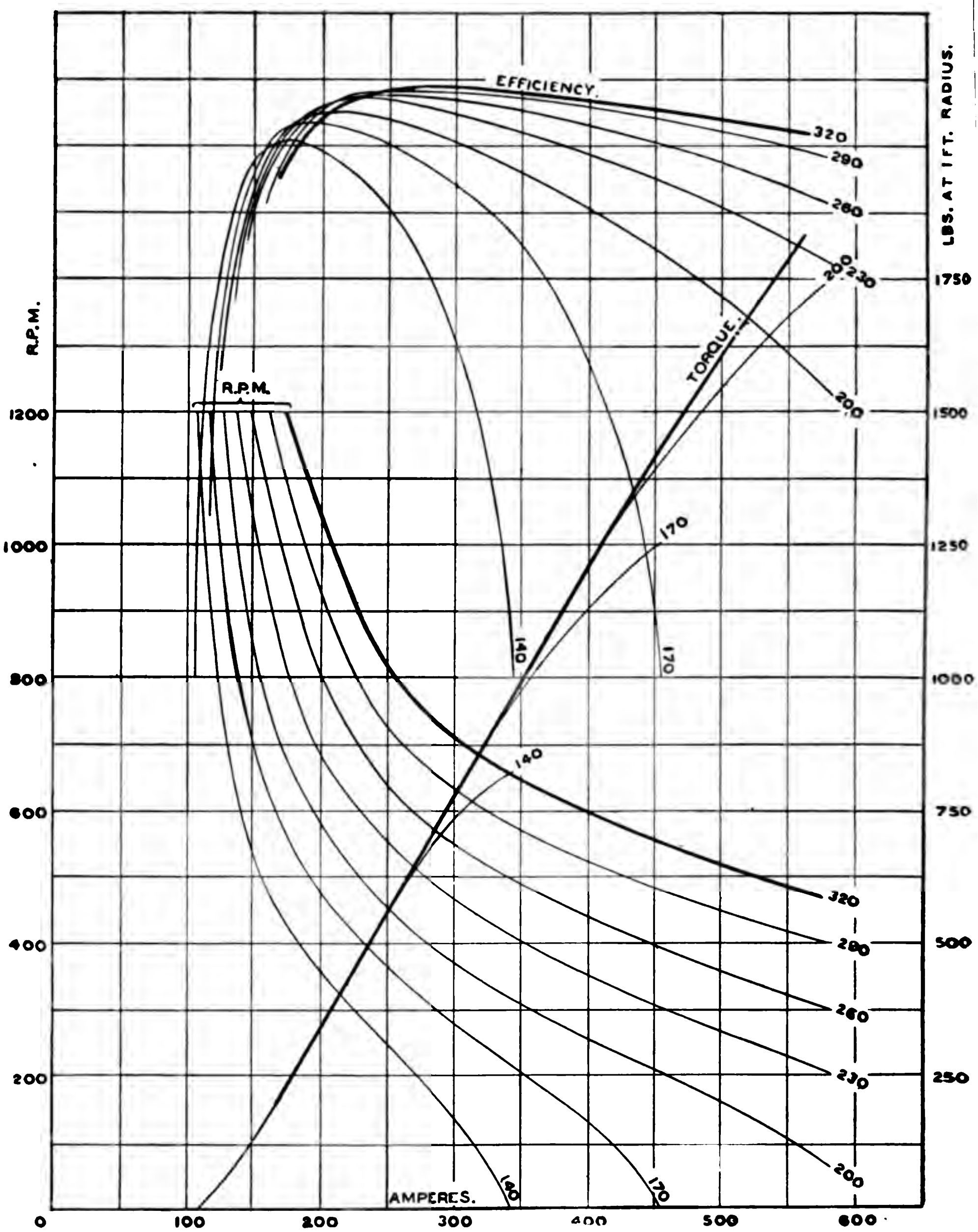
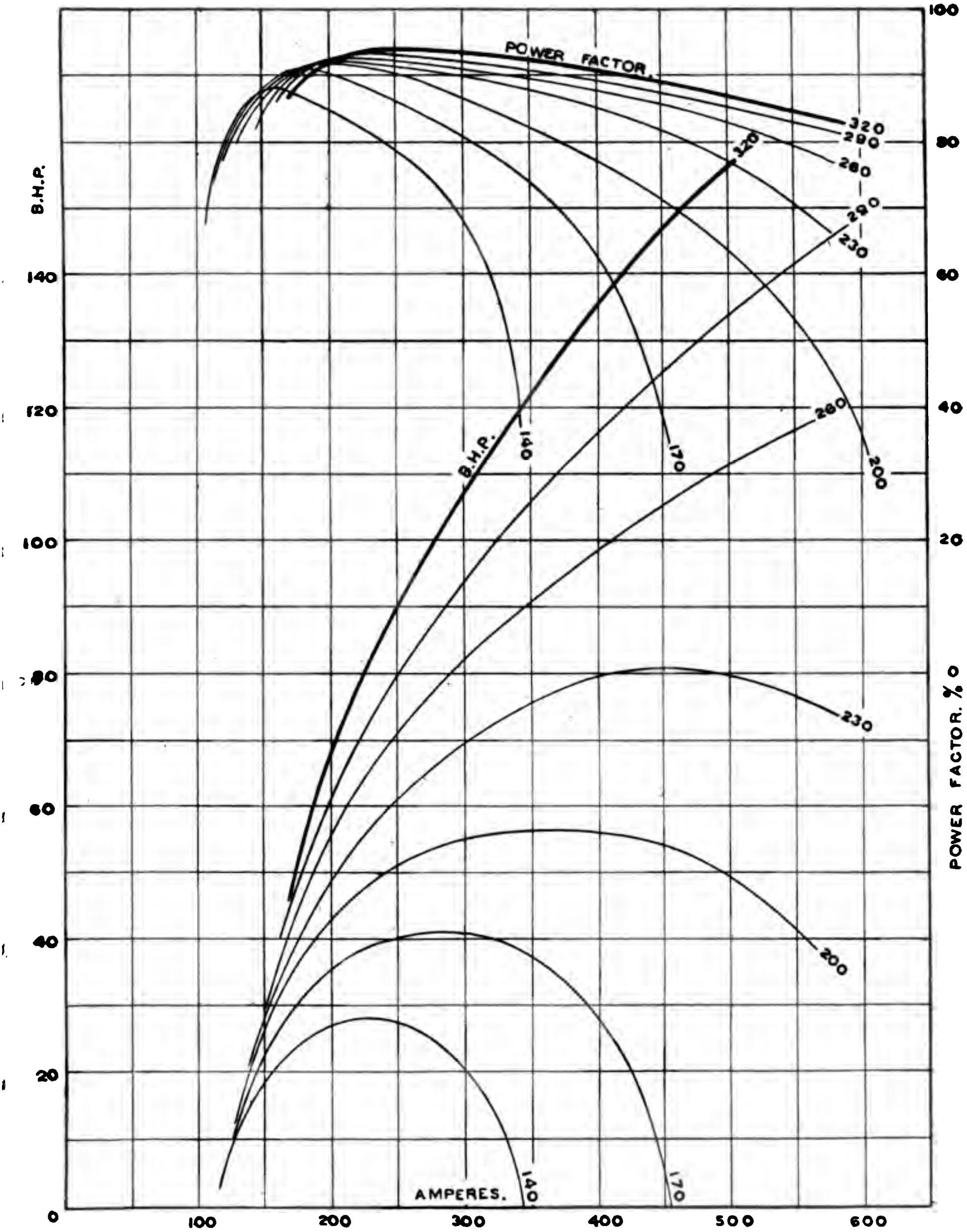


Fig. 77. Performance curves for 100 H.P. single



phase series motor with commutation poles.

Losses	Main current × drop	1590	2620	4300	5950
	2 × circulating volts × circulating current	1730	725	110	0
	Friction loss	2500	2500	2500	2500
	Extra iron loss due to rotation	240	380	530	630
	Shunt loss	400	400	400	400
	Total, kw.	6·46	6·62	7·84	9·48
Output, B.H.P.		13·5	36·5	72	100
Efficiency, °/₀		60·5	80·5	87·5	88·8
Torque, lbs. at 1 ft. radius		89	240	480	660

PART 3.

DESIGN AND METHOD OF CALCULATION FOR THE COMPENSATED REPULSION MOTOR.

The compensated repulsion motor. This motor partakes in a general way of the characteristics of a series wound single phase motor. As already mentioned (page 116) it possesses an armature wound in precisely the same way as are direct current armatures, and similarly connected to a commutator. This armature revolves in a magnetic field, which by acting upon the armature conductors produces the required torque.

So far there is no difference between the two types of single phase motor. In the method of obtaining these interacting elements there are important differences. In the series motor the armature conductors form part of the main circuit, and the main current of the motor flows through them; in the compensated repulsion motor the armature currents are produced by electromagnetic induction; the armature conductors forming the secondary of a transformer, the primary of which is wound on the stator.

Again, as regards the main field; this is produced by series wound field coils in the case of the series motor, whereas in the case of the compensated repulsion motor the main field is produced by a second system of currents in the armature conductors.

Figure 78 illustrates these differences and shews diagrammatically the connections of the various parts of the motor. FF represents the field or the stator with its winding W_F. AA represents the armature with its windings W_A, connected in the usual way to the commutator C. On the commutator are two sets of brushes B_1B_1 and B_2B_2. The first set are directly connected together and carry the main armature current. The second set B_2B_2 supply to the armature the current required for producing the main field. The method of connection is obvious from the diagram. The source of alternating currents LL is connected through the series transformer T to the terminals of the stator winding W_F. This winding forms the primary of a transformer of which the secondary consists of the armature winding W_A together with the short circuited brush system B_1B_1. The secondary of the series transformer T supplies the current for producing the main field to the armature through the brushes B_2B_2. By varying the ratio of this transformer T the working of the motor can be regulated within wide limits.

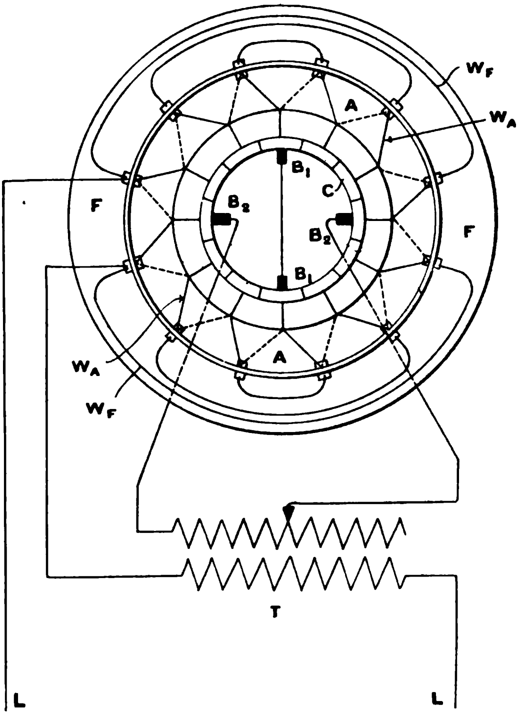


FIG. 78. Diagrammatic representation of compensated repulsion motor.

It will be evident from this description that the total magnetic field in the air gap of the motor will have two components, viz. (1) the *transformer field* produced by the stator winding W_F, which induces the main or working currents in the armature conductors, and (2) the *main field* produced by the excitation currents in the armature supplied by B_2B_2.

As will be seen when the diagram of this motor is constructed, these two components are approximately 90° apart in phase, and as their axes are 90° apart in space distribution, their resultant partakes of the nature of a revolving field. The action of this revolving field on the excitation currents in the armature is such as to eliminate partially or completely the self induction, so that under certain conditions the power factor of the motor reaches 100 per cent.

Diagram for the compensated repulsion motor*. For the sake of clearness it will be best to consider first the diagram for an ideal motor in which the resistances and leakage fields are negligible. In these circumstances the diagram is simply that of a transformer without leakage or resistance, but with an E.M.F. imposed upon the secondary.

Suppose for the moment that there are two windings on the armature, one for the main current and one for the excitation current. Then there will be two E.M.F.s in each of these windings.

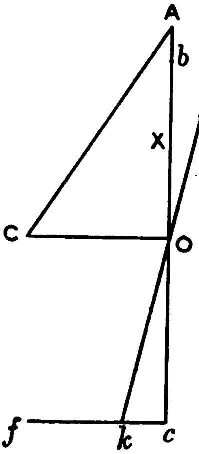


FIG. 79. Vector diagram of compensated repulsion motor without resistance and leakage reactance.

1*a*. In the main winding—the E.M.F. due to the rate of change of the transformer field.

1*b*. In the main winding—the E.M.F. due to rotation in the main field.

2*a*. In the excitation winding—the E.M.F. of self induction due to the rate of change of the main field.

2*b*. In the excitation winding—the E.M.F. due to rotation in the transformer field.

In figure 79 let OA represent the stator current. This current or a current proportional to it, as determined by the ratio of the series transformer, produces the main field, which is therefore in phase with

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* To a certain extent the authors follow the method set out in *Single phase Commutator Motors* by Punga. Various points of difference will be seen on comparison.

OA. Let OX represent the main field. The armature revolving in this field gives rise to an E.M.F. O*b*, in phase with OA and OX. On the assumption, already made, that the resistances and leakage fields are negligible, the total E.M.F. in the armature main winding must be zero; and since the total E.M.F. consists of the two components 1*a* and 1*b* it follows that 1*a* must be equal and opposite to 1*b*. Let O*c* represent the induced E.M.F. 1*a*; then this must be produced by the combined action of the stator winding and the main armature winding. If therefore OC at right angles to OA represents this resultant, AC will be the main armature current. O*c* will also represent on a suitable scale the back E.M.F. in the stator winding, and it only remains to add the back E.M.F. in the series transformer to obtain the total back E.M.F. equal and opposite to the terminal voltage.

Neglecting losses, etc. in the series transformer, its back E.M.F. will be the resultant of the two E.M.F.s in the excitation winding on the armature. The E.M.F. 2*a* of self induction will lag 90° behind the main field OX, and may be shewn as *cf*, and the E.M.F. 2*b* will be opposite in phase to the transformer field OC and is drawn as *fk*. Thus the total back E.M.F. will be O*k* equal and opposite to the terminal voltage OV.

It is thus evident that if *fk* is equal to *cf* the terminal volts will be in phase with the current OA and the power factor will be unity, whereas if *fk* is greater or less than *cf* the power factor will be less than unity, the current leading in one case and lagging in the other.

Unfortunately, commercial motors are not ideal, and it is necessary to take account of resistance, leakage reactance and circulating currents under the brushes.

The resistances and leakage reactance in the stator circuit require no comment. In the armature circuits there are two superposed currents in the same conductors; but, inasmuch as they are distributed by means of two sets of brushes equally spaced round the commutator, it can easily be shewn that the two currents do not interfere in any way with each other. That is to say, each current produces its own resistance drop and its own reactance voltage, as if the other current were not present.

As regards the circulating currents under the brushes, distinction must be made between the two sets. Those under the excitation brushes B_2B_2 may be disregarded; as at starting the transformer field is quite small, and at full speed, when the transformer field is of normal strength, the voltage produced thereby is neutralised by rotation in the main field. Under the other system the transformer action of the main field has its full effect at starting; but as the speed rises this action gradually decreases until it may be entirely neutralised.

The general diagram for the motor is given in figure 80, which is merely an extension of the previous diagram for the ideal motor. Proceeding as before OA represents the stator current; but in this case the main field OX differs in phase from OA by the angle a (which may be positive or negative, it being as shewn in the figure when the motor is at rest). Ob, as before, represents the voltage induced by rotation in the main field OX. In this case the total voltage in the main armature circuit which must be zero is composed of four parts, the voltage of rotation Ob, the voltage induced by the transformer field Oc, and the two components of the impedance drop, viz. the resistance drop and the leakage reactance drop.

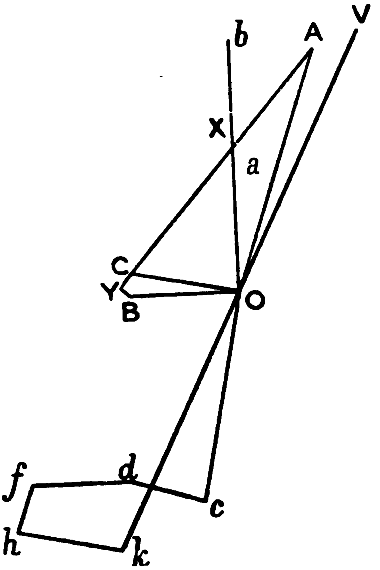


FIG. 80. Vector diagram for compensated repulsion motor.

Hence, the resultant of the ampere turns on the stator as represented by OA, and the ampere turns due to the induced current in the armature as represented by AC, must be equal to the resultant of three components, viz. that required to produce in the armature an E.M.F. equal and opposite to Ob, that required to produce an E.M.F. equal and opposite to the resistance drop, and that required to produce an E.M.F. equal and opposite to the leakage reactance drop. If the resultant of OA and AC fulfil this requirement, the total voltage in the short circuited armature circuit is zero. The components of this resultant are shewn as OB, BY and YC; thus OC represents the magnetising component of the transformer field.

Having thus obtained the magnitudes and phases of the various fields and currents, the construction of the diagram for the terminal voltage is quite straightforward. Thus O*c* (to a suitable scale) represents the volts induced in the stator winding (the primary), *cd*, at right angles to OA, the leakage reactance volts in the primary (including stator and magnetising transformer), *df*, at right angles to OX, the volts of self induction in the magnetising circuit of the armature (as transmitted by the transformer), *fh*, parallel to OA, the total resistance drop in the primary, and *hk*, parallel to CO, the volts in the magnetising circuit of the armature generated by its rotation in the transformer field. OV, equal and opposite to the resultant O*k*, will be the terminal volts.

Saturation in the magnetic circuit. In the compensated repulsion motor, the question of magnetic saturation is important for two reasons.

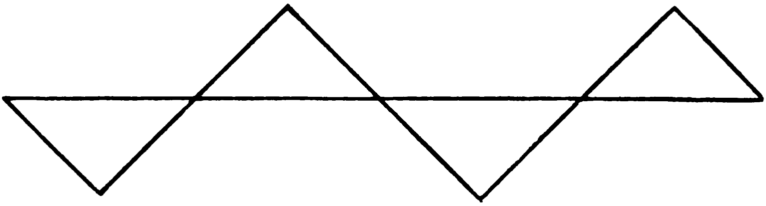


FIG. 81. Distribution of induction in the air gap of a compensated repulsion motor with magnetic circuit unsaturated.

In the series single phase motor the alternating flux is distributed more or less uniformly over the pole, whereas in the motor under consideration the induction at all stages of the period is graduated, being at a maximum in the centre of the pole and varying to zero at the extremities. If, therefore, the magnetic circuit be so proportioned that there is no saturation anywhere, it follows that, on the average, the induction in the iron must be very low. In other words, the material is not being used to the best advantage.

Secondly, when running in the neighbourhood of synchronism, the power factor of the motor does not depend very appreciably on the strength of the main field, and it is not so important in this case to keep down to a minimum the ampere turns required for magnetisation.

In the absence of any regular practice, the actual degree of saturation must be settled by the judgment of the designer; but the following method of treatment may possibly assist him in arriving at a decision.

As a preliminary, it must be understood that both on the stator and on the rotor, the magneto-motive force is due to a uniformly distributed winding; in other words, in considering any pole, the magneto-motive force varies *uniformly* from zero at the extremity to a maximum at the centre; and if the effect of the iron were inappreciable the curve of induction in the air gap would be a set of isosceles triangles, as in figure 81. When the field is produced by armature currents, the centre of a pole occurs at the position of those conductors in which the current is undergoing commutation.

In order to consider the effect of saturation proceed as follows:

A. Construct a saturation curve just as if the motor were a direct current motor with all the excitation ampere turns surrounding the entire pole. Thus, the area of the air gap belonging to each pole will be required, and the corresponding section of iron per pole in the teeth and core and yoke. With the equivalent length of each element of the magnetic circuit, the saturation curve can be calculated. Plot induction in the air gap vertically, and ampere turns per pole horizontally. See curve A, figure 83, page 170.

Now, this curve will represent also the distribution of induction over the pole face corresponding to any given maximum value of ampere turns per pole, by imagining a suitable horizontal scale. That is to say, the scale must be so chosen that the number of divisions which originally represented the maximum ampere turns selected will now represent half the circumferential length of the pole. For example, suppose 3000 ampere turns be selected and be represented by 3 divisions, and suppose also that the half length of the pole be 9 inches, then for this condition the horizontal scale will be such that each division represents 3 inches.

B. Integrate curve A, or find for each value of the abscissae the corresponding average value of the ordinates of curve A from zero to the point selected. Then if each average value be multiplied by the polar area of the air gap, the curve obtained by plotting the results against ampere turns will represent the total flux per pole corresponding to any given value of the ampere turns per pole due to the distributed winding. See figure 83, curve B.

C. Construct the curve of equivalent flux. This equivalent flux is defined as that which would produce the same E.M.F. of induction, if it acted on all the turns, as does the actual distributed flux on the distributed turns. Thus suppose the inductive E.M.F. actually produced is E. Let n be the turns per pole in series, t the time in which the ampere turns vary from $(\mathrm{AT})_1$ to $(\mathrm{AT})_2$ (supposed uniformly) and F_1, F_2 the equivalent fluxes for these values of the ampere turns, then

$$\mathrm{E} = \frac{n \times (\mathrm{F}_1 - \mathrm{F}_2)}{t} \times 10^{-8} \text{ volts.}$$

This curve is obtained approximately in the following way.

Take as an example maximum AT = 3000.

Divide the abscissa into 10 equal parts of 300 each, and suppose that each of these parts represents one-twentieth of the polar area and contains one conductor, so that there are in all 20 conductors or 10 turns. Then if h_{3000} be the maximum ordinate of the induction curve and A the area of the air gap, the flux included

$$\text{in 10 turns will be } \frac{A}{20} \times h_{3000},$$

$$\text{in 9 turns will be } \frac{A}{10} \times h_{2700}, \text{ and so on.}$$

The sum of these terms is

$$\frac{A}{20} \times 10 \times h_{3000} + \frac{A}{10} \times 9 \times h_{2700} + \dots;$$

hence the equivalent flux

$$= \frac{1}{10}\left[\frac{A}{20} \times 10 \times h_{3000} + \frac{A}{10} \times 9 \times h_{2700} \dots\right].$$

By making similar calculations for various values of the ampere turns a curve of equivalent flux can be constructed. Then the rate of change of the ordinates of this curve, multiplied by the number of turns in series, gives the induced E.M.F.

These three curves A, B and C will enable the designer to calculate the current required to produce any desired E.M.F.

Now, it must be noticed that curve B gives the E.M.F. generated by rotation, and curve C that produced by induction, and it does not follow for high degrees of saturation that if the current varies in such a way as to make the equivalent flux follow a sine law, the total flux per pole will likewise do so. In all cases, however, the determining condition is that the total internal voltage shall be equal and opposite to, and therefore of the same wave form as the applied potential difference.

Now, at standstill, this condition requires that the rate of change of the equivalent flux shall approximately follow a sine law, as there are no generated E.M.F.S. In this case the same method can be used, as has already been described in dealing with the saturation of the series motor; which consists in assuming that the R.M.S. current is the mean between that value which corresponds to $\frac{1}{\sqrt{2}}$ times the maximum flux and $\frac{1}{\sqrt{2}}$ times the value corresponding to the maximum flux.

When the motor is rotating, however, the total internal E.M.F. is made up of induced voltages and generated voltages. In this case no general rule can be laid down as to the variation of the fluxes ; but it will probably be found in most cases in practice, that if the equivalent flux be made to vary according to a sine law, the departure of the total flux curve from a sine wave will not be very marked. The wave form of the current will, of course, vary considerably from the sine law ; but this can be allowed for by taking a R.M.S. value as described above.

As an example, in the curves in figure 83, if a maximum value of 5500 ampere turns be taken, and the equivalent flux be made to vary according to a sine law, the corresponding curve for the total flux nowhere departs more than 3 per cent. from a true sine wave ; but the maximum value of this sine wave is about 3 per cent. less than the apparent value, and the maximum current is 1·7 times the R.M.S. value.

Thus for this condition it would be quite sufficiently accurate to assume that all the E.M.F.S follow a sine law with values as given above, and for practical purposes the curve of current might also be assumed to be a sine wave.

Thus from the curves B and C two effective curves *b* and *c* can be constructed, giving R.M.S. values of total flux per pole and equivalent flux per pole against either R.M.S. ampere turns or simply R.M.S. current as may be found most convenient.

Magnetisation due to circulating currents. So far the fields have been considered as due entirely to uniformly distributed windings. The circulating currents, however, are concentrated, and practically speaking embrace the whole of the polar area. Neglecting saturation, therefore, these currents will produce a uniform field, the strength of the field being proportional to the circulating ampere turns per pole.

Comparing the two types of field, still neglecting saturation, it will be evident that if the concentrated ampere turns per pole have the same value as the total ampere turns per pole in the distributed case, the flux per pole in the former will be twice that in the latter case, and the equivalent flux in the former will be to the equivalent flux in the latter case in the ratio of $\frac{1}{2}$ to $\frac{2}{3} \times \frac{1}{2}$, that is, in the ratio of 1·5 to 1.

In order to use the saturation curves already described in connection with the distributed winding the effective ampere turns per pole due to the circulating currents may be taken as twice the actual circulating ampere turns per pole, since a distributed winding is half as effective as a concentrated winding.

Therefore the total field will be due to the resultant of two sets of ampere turns as stated above, and the induced E.M.F. will be due to an equivalent flux produced by the distributed ampere turns compounded with $1\frac{1}{2}$ times the actual circulating ampere turns, or $\frac{3}{4}$ of the effective circulating ampere turns.

Saturation will tend to affect the accuracy of the results obtained as above, but it would be quite impracticable to examine every case, specially as magnitude of ampere turns, degree of saturation, and phase difference of ampere turns all vary independently. It is sufficiently accurate, however, to use the curves of total flux and equivalent flux as described above, since they take account of saturation and since saturation affects both types of field in the same direction.

Calculation of the iron loss in the compensated repulsion motor. As a preliminary this question may be considered in two different conditions, viz. at standstill and at synchronous speed.

(1) At standstill. In this case the transformer field is quite small, and may with sufficient accuracy be disregarded. The motor field is pulsating and not rotating, and for the purpose of calculating the iron loss it will be approximately correct to assume that the flux per pole is distributed uniformly so far as the teeth are concerned, and also the core and the yoke by assuming an effective section greater than the actual section between poles.

(2) At synchronous speed. In this case the two fields are practically equal, and are displaced one quarter period in space and approximately the same in time.

If the phase difference in time were exactly a quarter period the resultant field would rotate with constant magnitude. If the phase is not exact it is simple to resolve one of the fields into two components, one of which will combine with the second field and the other will continue as a pulsating field.

The field due to a uniformly distributed winding, such as the armature winding, can be analysed into a series of harmonics, with the following magnitudes (neglecting saturation),

1st harmonic or fundamental ... ... $h\frac{8}{\pi^2}$,

3rd harmonic ... ... $-h\frac{8}{(3\pi)^2}$,

5th harmonic ... ... $+h\frac{8}{(5\pi)^2}$, and so on,

where h is the maximum field strength produced by the winding.

The fields due to two similar windings displaced a quarter period, and with alternating currents differing in time by a quarter period, have the same maximum values as above and rotate uniformly in the same direction with relative speeds

$$1,\ \tfrac{1}{3},\ \tfrac{1}{5},\ \text{etc.}$$

Now, using these figures, the maximum strength of field as it rotates will be h, and the minimum

$$h\frac{8}{\pi^2}\left(1-\frac{1}{9}+\frac{1}{25}-\frac{1}{49}\dots\right)$$
$$=\mathbf{\cdot 7h}.$$

Therefore, assume an effective field strength of $\cdot 85h$ revolving uniformly.

[If saturation is at all marked, take the field strength corresponding to a point $\frac{l}{8}$ from the maximum, l being the pole pitch.] This strength of field will apply to all the stator teeth, and similarly ·85 times the maximum flux per pole will apply to the whole of the yoke, reckoning the density on the basis of the actual cross section.

In addition there will be the loss due to the pulsating component.

In the armature the fundamental rotating field will produce no loss, and the other harmonics will produce losses due to

$-h\frac{8}{(3\pi)^2}$ with $\frac{1}{3}$ pole pitch, rotating relative to armature at $\frac{2}{3}$ synchr. speed,

$+h\frac{8}{(5\pi)^2}$ with $\frac{1}{5}$ pole pitch, rotating relative to armature at $\frac{4}{5}$ synchr. speed, and so on ;

or $-h\frac{8}{(3\pi)^2}$ with frequency twice the fundamental

$+h\frac{8}{(5\pi)^2}$ with frequency 4 times fundamental, and so on.

Supposing the loss increases with the 1·5th power of the frequency the components will be

$$h\frac{8}{\pi^2}\left[\frac{1}{9}\times 2^{1\cdot 5}+\frac{1}{25}\times 4^{1\cdot 5}+\dots\right]$$
$$=\text{say } h\frac{8}{\pi^2}\left[\cdot 314+\cdot 32+\dots\right].$$

This result if applied to all the armature teeth and the core would probably be an over-estimate ; it is only possible to make a rough guess at the proper figure, but for the purposes of calculation it may perhaps be assumed at $h\frac{8}{\pi^2}\times\cdot 5$ or say $\mathbf{\cdot 4h}$.

There will also be in the armature the further loss due to the pulsating component.

By these methods two points can be obtained on the curve of iron loss against speed, and in order to ascertain roughly the general shape of the curve additional points may be calculated for 50 per cent. and 150 per cent. of synchronous speed.

Effect on the design of the method of control. The control of the compensated repulsion motor will be discussed in detail in the next chapter, but it must be referred to here as it has an effect on the design of the motor.

It is obvious that, if there is no fixed relation between the strength of the main field and the current in the armature, as in the case of the series motor, there are two ways of obtaining the large torque generally required for starting. The field may be kept at its normal value and the armature current increased, or *vice versâ*. In the series motor, the necessary variation takes place mostly in the current. In some forms of control for the compensated repulsion motor the variation is confined almost entirely to the field.

The result of this is largely to increase the transformer voltage, and since it is necessary to proportion the motor so that the maximum transformer voltage is kept within proper bounds, the design of the motor in view of its normal operation is made more difficult.

Probably the most favourable method of starting and regulating the motor from this point of view is to keep the ratio of the series transformer invariable and to vary the voltage on the motor as a whole; if the motor is supplied through a main step down transformer, the secondary voltage can be so chosen as to avoid the necessity for any series transformer for excitation, as in the example below.

Specification No. 5.

Single phase compensated repulsion motor.

Rating. 100 H.P. for 1 hour, 75° C. rise. 800 volts; 700 r.p.m; 113 amperes; 25 frequency.

Armature.

External diameter	500 mm.
Internal diameter	264 „
Gross length	350 „
Air ducts	3—10 „
Nett length	320 „
Percentage iron	90 °/₀
Length of iron	288 mm.
Number of slots	86
Width per slot	9·5 mm.
Depth „	42 „
Opening „	9·5 „
Type of winding	2 circuit double
Number of poles	4

Conductors per slot	6
Scheme of connections	$\begin{array}{ccccccccc}1&&&&257&&&&513\\&\diagdown&&\diagup&&\diagdown&&\diagup&\\&&128&&&&384&&\end{array}$
Size of conductor	16 × 2 sq. mm.
Total resistance (hot)	·019 ohm
Total reactance (leakage)	·041 ohm
Weight of copper	131 kgms.
Weight of iron	227 „
Peripheral speed	18·3 m. per sec.

Commutator.

Diameter	420 mm.
Length	315 „
Number of segments	258
Peripheral speed	15·4 m. per sec.

Commutator connections.

Material	German silver
Cross section	3 mm. × 2 mm.
Length of one connection	750 mm.
Resistance „	·0264 ohm

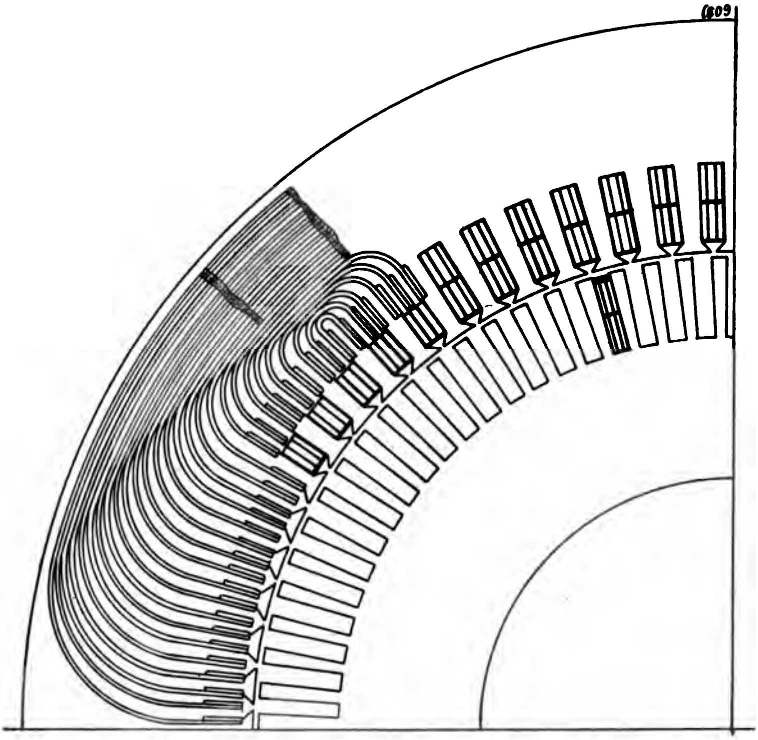


FIG. 82. Armature and field laminations and conductors for compensated repulsion motor.

Brushes.

Main		Excitation	
Number of brush arms	4	Number of brush arms	2
Brushes per arm	6	Brushes per arm	4
Section per brush	45 mm. × 12·5 mm.	Section per brush	45 mm. × 12·5 mm.
Length per brush		Length per brush	
Quality of brushes	"ordinary"	Quality of brushes	"ordinary"
Type of holder	box	Type of holder	box

Field.

Bore	506 mm.
External diameter	750 „
Gross length	350 „
Air ducts	3—10 „
Nett length	320 „
Percentage iron	90 °/ₒ
Length of iron	288 mm.
Number of slots	72
Width per slot	14 mm.
Depth „	46
Opening	2 mm.
Type of winding	single circuit
Number of poles	4

Conductors per slot	6
Scheme of connections :— single circuit as in figure 82.	
Size of conductor	18 mm. × 3·3 mm.
Size of end connections	50 mm. × 1·2 mm.
Total resistance (hot)	·144 ohm
Total reactance (leakage)	·725 ohm
Weight of copper	214 kgms.
Weight of iron	368 kgms.

Regulating transformer. None.

Ratio of transformation.

The sketch in figure 82 gives the details of the armature and field laminations and conductors.

Calculations for Specification No. 5.

Single phase compensated repulsion motor.

Saturation curves.

Curve A.

		Armature		Field		
	Gap	Teeth	Core	Yoke	Teeth	Totals
Average section per pole, sq. cms.	970	450	217	217	512	
Length per pole, cms.	·3	4·2	13·4	26·5	4·6	
Total volume, cubic cms.		7560	23200	46000	9250	
Total weight, kgms.		59	180	360	72	
Effective section per pole, sq. cms.	970	416	say 600	600	500	
Effective length per pole, cms.	·3	4·2	say 12	say 20	4·6	
			32			
Induction	3000	7000	4800		5800	
	5000	11700	8100		9700	
	7000	16300	11300		13600	
	8000	18600	12900		15500	
	9000	say 20500	14500		17500	
	9500	say 21500	15300		18500	
Ampere turns per pole	716	6	32		5	760
	1200	15	50		10	1275
	1670	114	100		37	1920
	1910	420	185		88	2600
	2150	1140	380		230	3900
	2270	1700	540		440	4950

Curves B *and* C.

Ampere turns per pole	500	1000	1500	2000	2500	3000	3500	4000
Flux per pole (B)	·97	1·94	2·85	3·72	4·44	5·01	5·5	$5{\cdot}86 \times 10^6$
Equiv. flux per pole (C)	·65	1·3	1·91	2·44	2·89	3·22	3·46	$3{\cdot}66 \times 10^6$

[Curves A, B and C are shewn in figure 83.]

Curves b and c.

Ampere turns per pole	1110	1810	2280	2680	3230
Flux per pole, R.M.S. (*b*)	2·12	3·3	3·87	4·28	$4{\cdot}66\times10^6$
Equiv. flux per pole, R.M.S. (*c*)	1·41	2·12	2·48	2·68	$2{\cdot}9\times10^6$

[Curves *b* and *c* are shewn in figure 84.]

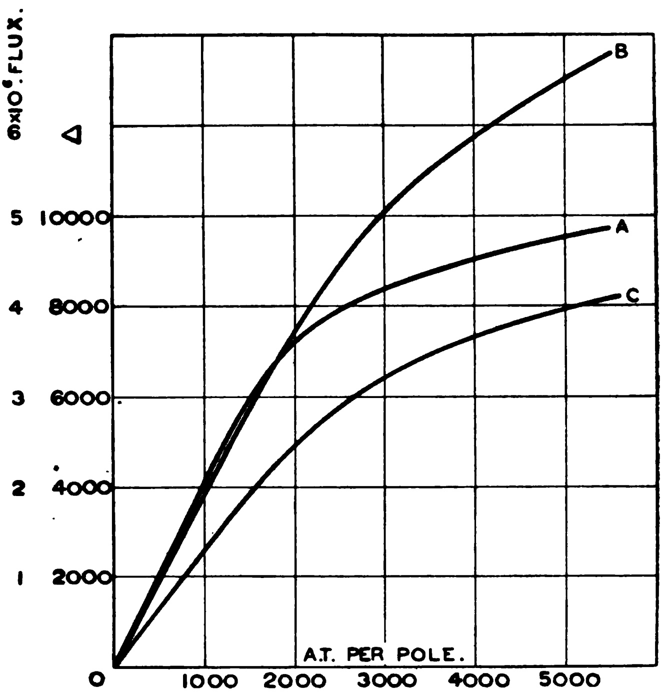


FIG. 83. Saturation curves for compensated repulsion motor with steady currents.

Data for calculating performance curves.

Resistance of primary circuit, including stator winding and the path of the excitation current through the armature.

Stator winding	·144 ohm.
Excitation brushes $\frac{2\times\cdot084}{4\times4\cdot5\times1\cdot25}$	·0075 „
Commutator connections $\frac{2}{2\cdot5}\times\cdot0264$	·021 „
Armature resistance	·019 „
Total	**·1915 ohm.**

Say **·21 ohm** to include miscellaneous losses.

Leakage reactance of primary circuit.

Stator	·725 ohm.
Armature	·041 „
Total	**·766 ohm.**

Resistance of secondary circuit.

Armature winding	·019 ohm.
Commutator connections $\frac{2}{5} \times \cdot 0264$	·0106 ,,
Short circuit brushes $\frac{2 \times \cdot 084}{12 \times 4{\cdot}5 \times 1{\cdot}25}$ ...	·0025 ,,
Total	·032 ohm.

Say ·035 to include miscellaneous losses.

Leakage reactance of secondary circuit ·041 ohm.

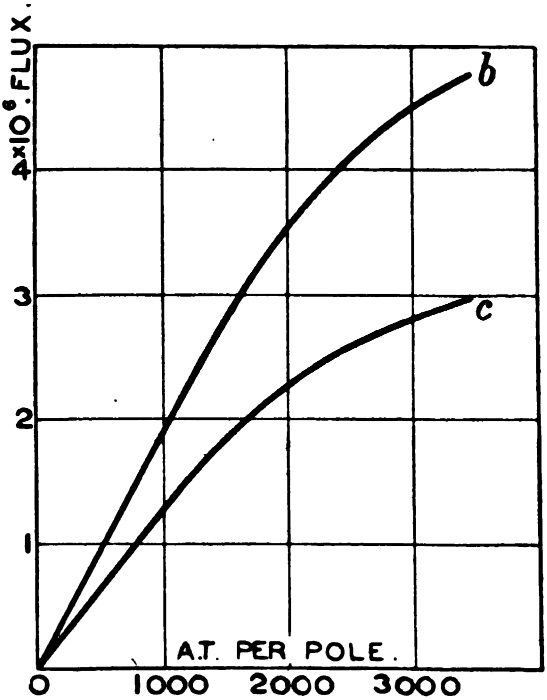


Fig. 84. Saturation curves for compensated repulsion motor with alternating currents.

Transformer voltage.

Pitch of commutator segments	5·1 mm.
Width of brush...	12·5 mm.

Therefore number of armature coils in series short circuited by any brush $= 2$.

Therefore transformer voltage $=$ motor flux per pole $\times 2 \times 2\pi \times 25 \times 10^{-8}$,

$= $ motor flux $\times 314 \times 10^{-8}$

Reactance voltage. For one ampere in the short circuit, the current per conductor is ¼ ampere.

Time of commutation at 100 r.p.m.

$$= \frac{10}{\dfrac{\pi \times 420 \times 100}{60}} = \frac{1}{220} \text{ second}$$

(allowing equivalent width of brush = 10 mm., see page 150).

Inductance due to 4 bars with $\frac{1}{4}$ amp. in each

$$= 4 \times \tfrac{1}{4} \times [3 \times 32 \times 4 + 43 \times \cdot 8]$$

lines, assuming 4 as the average number of bars in the same slot undergoing commutation simultaneously = 415 lines.

Reactance voltage per amp. at 100 r.p.m.

$$= 2 \times 415 \times 220 \times 10^{-8} = \mathbf{1{\cdot}83 \times 10^{-3}}.$$

Time-constant correction at 100 r.p.m.

$\mathsf{L} = 10 \times 415 \times 4 = 16600.$

$\mathsf{R} = \cdot 04$ ohm or 40×10^6.

t = time for $(3 + 3\frac{1}{2} - 1) = 5\frac{1}{2}$ segments to pass a fixed point

$$= \frac{5{\cdot}5 \times 5{\cdot}1}{\dfrac{\pi \,.\, 420 \,.\, 100}{60}} = \cdot 0217 \text{ second.}$$

$$x = \frac{\mathsf{R}t}{\mathsf{L}} = \frac{40 \times 10^6 \times \cdot 0127}{16600} = \mathbf{30{\cdot}6}.$$

Induced voltages.

a. Stator. Equivalent flux per pole $\times 2\pi \times 25 \times 54 \times 4 \times 10^{-8}$

$= \text{equiv. flux per pole} \times 339 \times 10^{-6}$.

b. Armature. Equivalent flux per pole $\times 2\pi \times 25 \times 64{\cdot}5 \times 10^{-8}$

$= \text{equiv. flux per pole} \times 101 \times 10^{-6}$.

$$\text{Ratio.} \quad \frac{\text{stator induced volts}}{\text{armature induced volts}} = \mathbf{3{\cdot}35}.$$

Circulating current (short circuited brushes only). With brush symmetrically situated the width of contact on the outer segments

$$= \{12{\cdot}5 - (2 \times 5{\cdot}1 + \cdot 8)\} \times \tfrac{1}{2}$$

(allowing ·8 mm. mica) $= \cdot 075$ centimetre.

$$\text{Resistance of outer contact} = \frac{\cdot 084}{6 \times 4{\cdot}5 \times \cdot 075} = \mathbf{\cdot 0415 \text{ ohm}}.$$

Resistance due to segment completely covered

$$= \frac{\cdot 084}{6 \times 4{\cdot}5 \times \cdot 43} = \mathbf{\cdot 0072 \text{ ohm}}.$$

Resistance of two commutator connections in series ·0528.

$$\text{Total} = \cdot 0415 + \cdot 0072 + \cdot 0528 = \mathbf{\cdot 1015 \text{ ohm}}.$$

Combined resistance of 2 brush arms in parallel = **·051 ohm** say.

To allow for cross connection between brush arms of the same polarity, assume an effective increase of circulating current of 25 per cent., *i.e.* assume an effective resistance of **·04 ohm**.

The circulating current due to this resistance will flow in two parallel circuits; *i.e.* the total circulating A.T. per pole will be

$$2 \times \frac{\text{circ. voltage} - 1{\cdot}3}{\cdot 04} \text{ or } \mathbf{50\ [\text{circ. voltage} - 1{\cdot}3]}.$$

Method of calculation of the performance curves. The method of calculating the performance curves for the compensated repulsion motor is very similar to that used for the series motor with commutating poles. The chief differences are:

1. The commutating field is not constant in strength and phase.
2. The main current in the armature is induced through the agency of the transformer field.
3. There are five separate components of the internal voltage, which is equal and opposite to the terminal voltage.

Refer now to figure 80 on page 160. Starting with the motor field OX, the generated E.M.F. O*b* is calculated directly therefrom, and hence OB which gives a very close first approximation to the transformer flux.

Knowing the flux OX, the transformer voltage is also known in magnitude and direction. The reactance voltage must be assumed, and the commutating voltage must be calculated from the speed and the transformer field. Thus, supposing the transformer flux per pole to be $3{\cdot}5 \times 10^6$, the periodic maximum will be $4{\cdot}95 \times 10^6$. On reference to saturation curves A and B this corresponds to a maximum density at the centre of the pole of 8300 C.G.S. lines per sq. cm. or a R.M.S. density of 5860. Then the commutating voltage in 4 bars, for a speed of say 800 r.p.m., will be

$$4 \times 5860 \times 970 \times \frac{800 \times 4}{60} \times 10^{-8} = 12{\cdot}1 \text{ volts}$$

(970 being the effective gap area per pole).

The resultant circulating voltage must be found as the resultant of these three components, and hence the circulating current and the effective circulating ampere turns per pole. Knowing the ampere turns required to produce the motor field the actual ampere turns provided by the excitation are found at once, and hence the primary or motor current.

The next step is to determine the main armature current or the secondary current, and the transformer flux. In figure 80, OB, BY, and YC represent ampere turns required to produce three components of the transformer flux, to the same scale as OA and AC represent ampere turns.

OB represents the ampere turns required to produce an equivalent flux per pole which will exactly neutralise the E.M.F. generated in the armature by rotation in the main field. OB is thus fixed in magnitude and direction and OA has already been found. Therefore AB is known and AYB is a right-angled triangle in which CY and YB have each a definite ratio to AC. These ratios are obtained thus: calling $\frac{\text{CY}}{\text{CA}}$ the leakage inductance factor, the leakage inductance volts produced by the current corresponding to AC ampere turns per pole

$$= \frac{\text{AC}}{16{\cdot}1} \times {\cdot}041,$$

the equivalent flux per pole to overcome this voltage

$$= \frac{\text{AC}}{16{\cdot}1} \times {\cdot}041 \times \frac{10^6}{101},$$

and, neglecting saturation, the ampere turns per pole required to produce this equivalent flux

$$= \frac{\text{AC}}{16{\cdot}1} \times {\cdot}041 \times \frac{10^6}{101} \times \frac{800}{10^6} = \text{AC} \times {\cdot}02,$$

i.e. the leakage inductance factor or $\frac{\text{CY}}{\text{CA}} = 2$ **per cent.**

Similarly, neglecting the variable resistance of the brushes, the resistance factor $\frac{\text{BY}}{\text{AC}}$ could be found. This assumption would involve only a very small error, and may be considered justifiable. Taking the secondary resistance as ·037 ohm,

the resistance factor = **1·8 per cent.**

AC is thus obtained and hence the secondary current, and also the transformer flux per pole. If these have already been assumed correctly, it remains only to find the terminal voltage, cos ϕ, efficiency, B.H.P., and torque.

One point should be noticed, viz., that the equivalent motor flux per pole is due to the excitation ampere turns compounded with ¾ the effective circulating ampere turns.

To make the process quite clear, it is summarised with reference to separate diagrams, figures 85 and 86, as follows:

1. Assume motor flux, and denote it by OX (figure 85), the ampere turns per pole required to produce it.

2. Calculate O*b* the secondary volts generated by rotation, and thence the equivalent transformer flux required to overcome it.

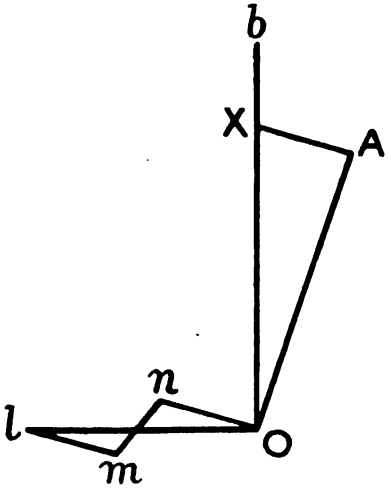


FIG. 85. Vector diagram of circulating voltage for compensated repulsion motor.

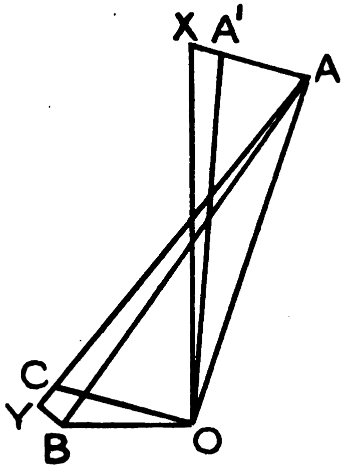


FIG. 86. Vector diagram of the secondary currents in the compensated repulsion motor.

3. Deduce O*l* the transformer voltage.
4. Assume *lm* the commutating voltage.
5. Assume *mn* the reactance voltage.

6. Find resultant circulating voltage O*n*.

7. Calculate effective circulating ampere turns per pole XA.

8. Hence find OA the ampere turns per pole due to excitation and thus the primary or motor current.

9. Draw OB (figure 86) the ampere turns per pole required to overcome O*b*.

10. Construct the triangle AYB, and thus find AC, and the secondary current. Now go back and check the assumptions in 4 and 5.

11. Draw OA′ where $AA' = \frac{2}{3} . XA$, and find saturation curve C the equivalent motor flux per pole.

12. Construct the total internal voltage.

Having calculated a series of performance curves at constant speeds, say at 200, 400, 600, 800, 1000, 1200 r.p.m. and at standstill, a fresh set of curves at constant terminal voltages can be drawn immediately without further calculations. The latter set of curves is shewn in figure 87.

Standstill characteristics.

	Motor flux per pole, $10^6 \times$	3·0	3·5	4·0	4·3	4·6
	Transformer voltage	9·42	11	12·57	13·5	14·4
	Circulating current	406	484	564	610	654
	Effective circulating A.T. per pole	812	968	1128	1220	1308
	A.T. per pole to produce motor flux	1640	2000	2400	2720	3150
	A.T. per pole due to excitation	1810	2220	2660	2980	3430
	Motor current	112	138	166	185	213
	A.T. per pole due to primary winding	6060	7440	8920	10000	11500
	A.T. per pole due to secondary winding	5880	7290	8740	9800	11270
	Secondary current	365	451	542	607	700
	Iron loss, watts	540	690	860	960	1060
	O*c**, Volts in primary due to transformer flux	66·5	82	98·5	110	127
	*cd**, Volts due to leakage flux	86	106	127	142	163
	Equivalent motor flux per pole, $10^6 \times$	1·97	2·26	2·53	2·70	2·87
	*df**, Volts induced by motor flux	199	228	256	273	290
	*fh**, Primary drop $\left(\frac{C^2R + \text{iron loss}}{C} + 1{\cdot}3\right)$	28·1	35·3	41·3	45·2	50·8
	Terminal volts	350	418	482	525	577
	Cos ϕ	·394	·395	·394	·386	·38
	Cos θ (θ = angle between motor field and secondary current)	·89	·9	·905	·91	·915
	Torque due to motor field and secondary current	590	855	1180	1440	1780
	Static torque due to commutator and bearing friction	80	80	80	80	80
	Nett torque, lbs. at 1 ft. radius	510	775	1100	1360	1700
Losses	Secondary current × drop	5150	7700	11000	13600	18000
	Primary current × drop	3160	4900	6900	8400	10800
	2 × circulating current × circ. volts	7650	10600	14200	16500	18800
	Total, kw.	16	23·2	32·1	38·5	47·6

Characteristics at 200 r.p.m.

Time-constant correction = ·93.

Friction.	Brushes	260
	Bearings $90 \times 2^{1\cdot5}$	250
	Total	**510**

Motor flux per pole, $10^6 \times$	2·0	3·0	3·5	4·0	4·3	4·6
E.M.F. generated in secondary	34·4	51·7	60·2	68·8	74	79·2
Equiv. transformer flux, $10^6 \times$	·42	·59	·69	·82	·89	·98
R.M.S. density of commutating field (B)	1240	1850	2120	2550	2740	3100

* See figure 80, page 160.

Commutating voltage ($B \times$ r.p.m. $\times 2{\cdot}58 \times 10^{-6}$)	·64	·96	1·09	1·32	1·41	1·6
Transformer voltage	6·28	9·42	11·0	12·6	13·5	14·5
Reactance voltage	·84	1·3	1·57	1·87	2·1	2·41
Resultant circulating voltage	5·4	8·0	9·35	10·7	11·4	12·2
Circulating current	190	310	375	440	470	505
Effective circulating A.T. per pole	380	620	750	880	940	1010
A.T. per pole to produce motor flux	1070	1640	2000	2400	2720	3140
A.T. per pole due to excitation	1100	1690	2050	2450	2760	3170
Motor current, C_1	68·3	105	127	152	172	197
A.T. per pole to produce transformer flux	320	486	550	640	700	780
A.T. per pole due to primary winding	3680	5670	6860	8200	9300	10650
A.T. per pole due to secondary winding	3700	5720	6900	8250	9300	10650
Secondary current, C_2	230	355	429	512	577	660
Iron loss, watts	210	385	500	620	710	800
Equivalent transformer flux, $10^6 \times$	·42	·59	·69	·82	·89	·98
Oc, Volts in primary due to transformer flux	142	200	234	278	302	332
cd, Volts due to leakage flux ($C_1 \times {\cdot}766$)	52·3	80·5	97·5	116	132	151
Equivalent motor flux, $10^6 \times$	1·37	1·95	2·26	2·50	2·68	2·85
df, Volts induced by motor flux	138	197	228	252	271	288
fh, Primary drop $\left(\frac{C_1^2R + \text{iron loss}}{C_1} + 1{\cdot}3\right)$	18·7	26·9	32	37·3	41·5	46·4
Total transformer flux, $10^6 \times$	·63	·90	1·03	1·23	1·33	1·47
hk, Volts generated by rotation in transformer field	10·8	15·5	17·7	21·2	23	25·3
Terminal volts	257	367	430	500	555	617
Cos ϕ	·762	·762	·762	·76	·75	·725
Losses						
Secondary current × drop	2140	4870	7000	9850	12400	16100
Primary current × drop	1280	2820	4100	5700	7100	9200
2 × circulating current × circulating voltage	2050	4950	7000	9400	10700	12300
Friction	510	510	510	510	510	510
Total, kw.	6	13·15	18·6	25·5	30·7	38·1
Input, kw.	13·4	29·4	41·6	57·8	71·6	88
Output, B.H.P.	9·9	21·7	30·8	43·2	54·7	66·7
Efficiency, °/o	55·3	55·3	55·3	55·9	57	56·7
Torque, lbs. at 1 ft. radius	260	570	810	1140	1440	1760

Characteristics at 800 r.p.m.

Time-constant correction = ·75.

Friction.	Brushes	1040
	Bearings $90 \times 8^{1\cdot5}$	2020
	Total	**3060**

Motor flux per pole, $10^6 \times$	1·0	1·5	2·0	2·4	2·8	3·2
E.M.F. generated in secondary	68·8	103	138	165	193	220
Equiv. transformer flux, $10^6 \times$	·71	1·06	1·41	1·67	1·98	2·24
R.M.S. density of commutating field (B)	2200	3300	4280	4950	5500	5830
Commutating voltage ($B \times$ r.p.m. $\times 2{\cdot}58 \times 10^{-6}$)	4·53	6·8	8·83	10·2	11·3	12
Transformer voltage	3·14	4·71	6·28	7·54	8·80	10·1
Reactance voltage	1·55	2·18	2·86	3·4	4·02	4·77
Resultant circulating voltage	2·0	2·95	3·75	4·15	4·7	4·9
Circulating current	26	62	92	107	128	135
Effective circulating A.T. per pole	52	124	184	214	256	270
A.T. per pole to produce motor flux	530	790	1070	1270	1500	1760
A.T. per pole due to excitation	500	725	965	1140	1330	1560
Motor current, C_1	31	45	60	70·5	82·5	97
A.T. per pole to produce transformer flux	560	840	1100	1350	1680	1950
A.T. per pole due to primary winding	1670	2430	3240	3820	4460	5230

A.T. per pole due to secondary winding		1680	2400	3150	3750	4450	5250
Secondary current, C_2		105	149	196	232	276	326
Iron loss, watts		140	240	360	480	620	770
Equiv. transformer flux, $10^6 \times$		·71	1·06	1·41	1·67	1·98	2·24
O*c*, Volts in primary due to transformer flux		240	359	478	566	671	760
cd, Volts due to leakage flux ($C_1 \times \cdot 766$)		24	34·5	46	54	63	74
Equiv. motor flux, $10^6 \times$		·67	·98	1·37	1·56	1·80	2·0
df, Volts induced by motor flux		68	99	139	158	182	202
fh, Primary drop $\left(\frac{C_1^2R + \text{iron loss}}{C_1} + 1\cdot 3\right)$		12·3	16·1	19·9	22·9	26·2	29·6
Total transformer flux, $10^6 \times$		1·06	1·59	2·10	2·53	3·02	3·46
hk, Volts generated by rotation in transformer flux		73	109	145	174	208	238
Terminal volts		265	393	525	615	725	822
Cos ϕ		·96	·964	·964	·968	·974	·98
Losses	Secondary current × drop	525	970	1600	2180	3040	4140
	Primary current × drop	380	725	1200	1610	2160	2870
	2 × circulating current × circulating voltage	105	365	520	890	1200	1320
	Friction	3060	3060	3060	3060	3060	3060
	Total, kw.	4·07	5·12	6·38	7·74	9·46	11·4
Input, kw.		7·9	17	30·3	42	58·2	78
Output, B.H.P.		5·1	17·2	31·9	45·8	65	89
Efficiency, °/₀		48·5	70	79	81·5	83·7	85·4
Torque, lbs. at 1 ft. radius		33	113	210	302	427	585

PART 4.

COMPARISON OF THE THREE TYPES OF SINGLE PHASE COMMUTATOR MOTOR, STRUCTURAL DETAILS AND EXAMPLES OF ACTUAL MOTORS.

The influence of the circulating current on the performance curves. A careful examination of the three sets of performance curves given above for the three types of single phase motor will bring out the marked influence of the circulating current.

In the first, or plain series, motor this influence makes itself felt in producing a high power factor and a relatively low efficiency, the high power factor being caused by the shifting of the phase of the main field relatively to the motor current. The authors are aware that this result as shewn in the performance curves is not in accordance with some curves that have been published for similar motors; but there is experimental evidence to shew that this action of the circulating current is actually produced in commercial motors. It is not always clearly stated whether published curves are based on calculations or on tests; if the former, the results may depend on the method of calculation, if the latter, on the method of test, which may be direct by means of a wattmeter, or indirect by means of measurements of the currents and voltages. On the other hand, the curves given in figure 92 are

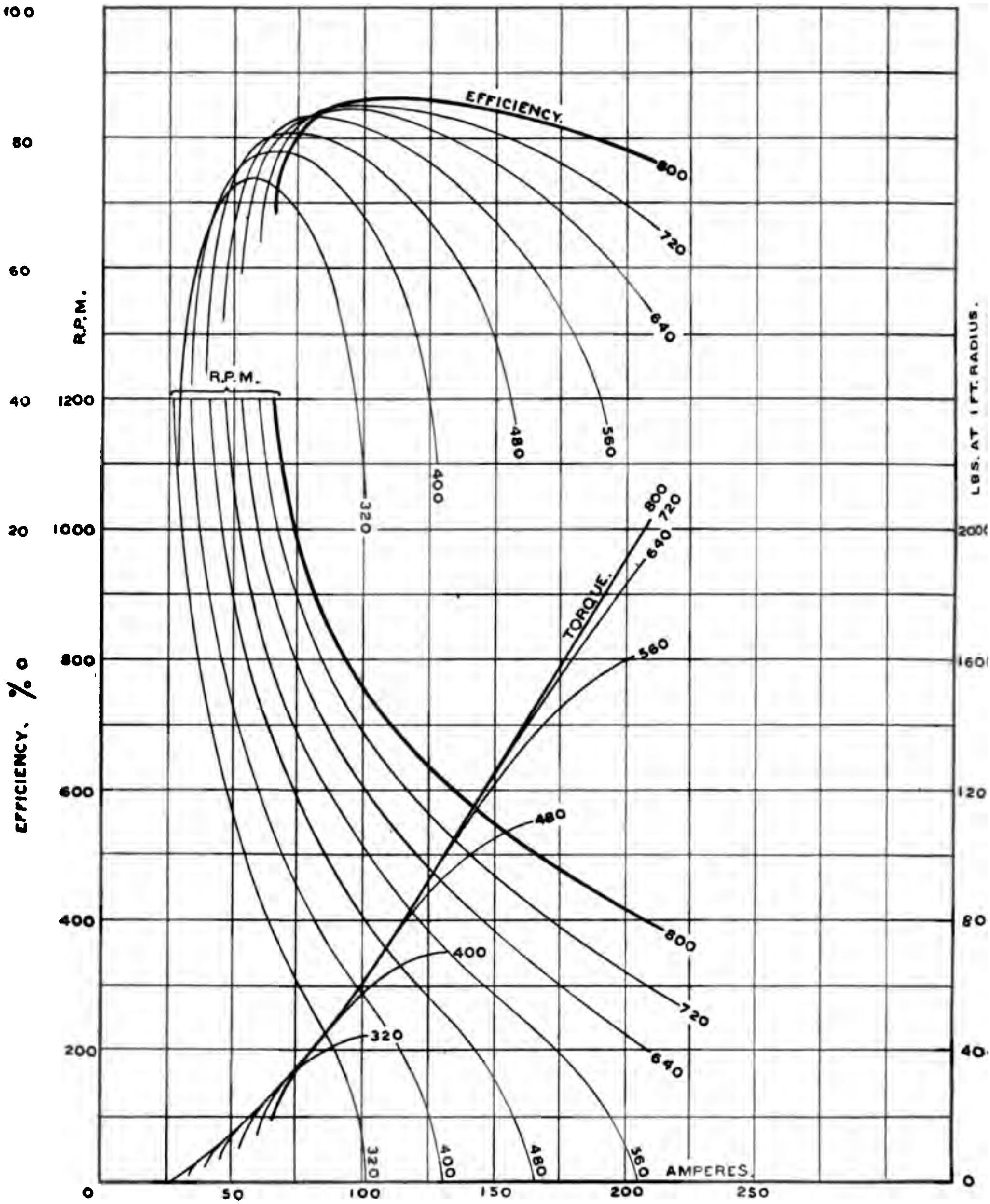
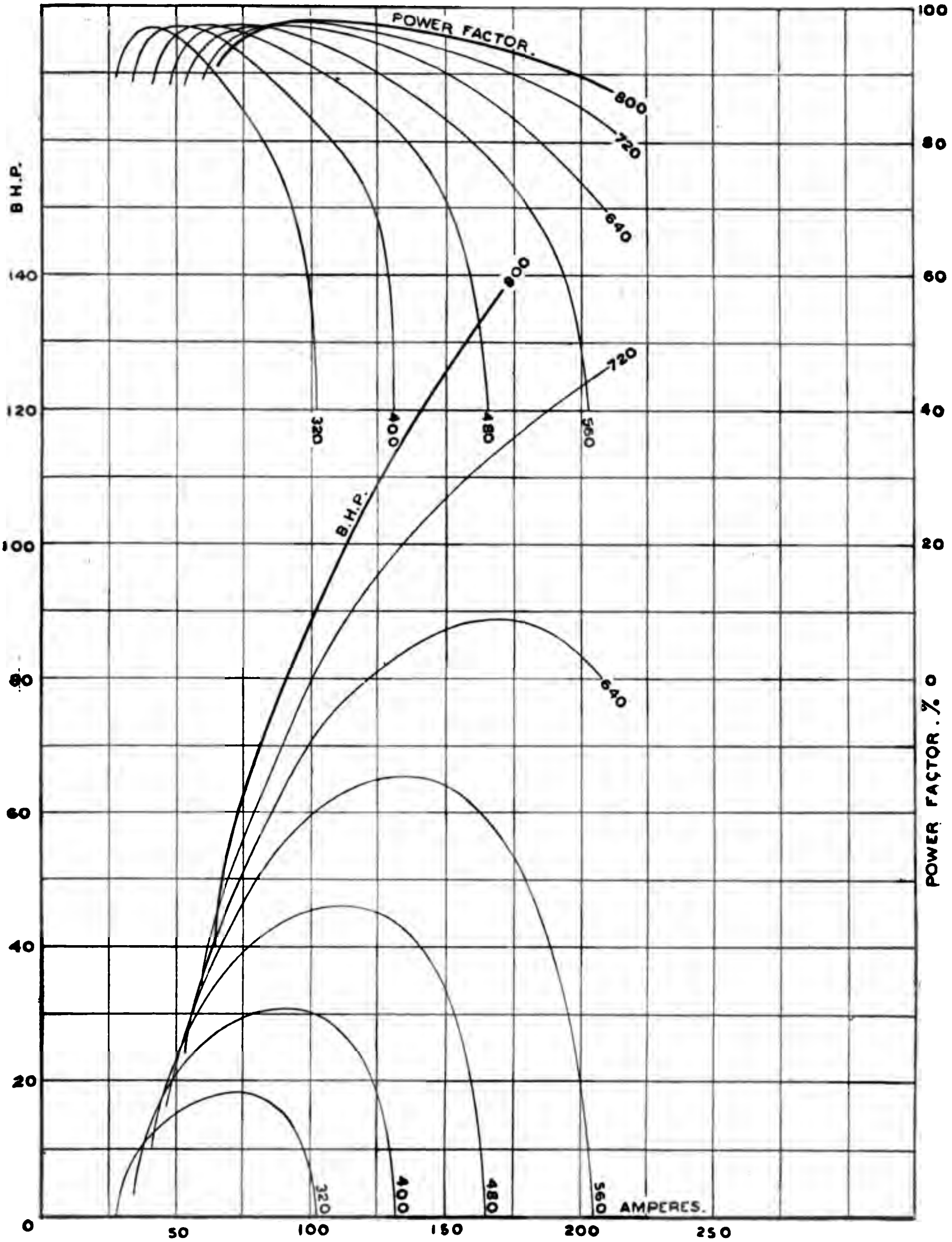


Fig. 87. Performance curves o



100 H.P. compensated repulsion motor.

in fair agreement with those calculated. The only satisfactory method of testing a single phase motor is by means of single phase currents and an alternating current wattmeter.

In the second motor, *i.e.* the series motor with commutating poles, the circulating current follows quite a different law owing to the presence of the commutating field. As has already been pointed out the circulating current is entirely suppressed at or about the nominal one hour load, and consequently in the neighbourhood of this load there is not only perfect commutation but high efficiency and a power factor determined solely by the self induction of the motor. Below this speed, and more particularly at standstill, the circulating current produces an effect similar to that experienced with the plain series single phase motor, namely a raising of the power factor combined with an increase in the total losses. Above the speed at which the commutation is perfect, the presence of the commutation poles makes itself evident in lowering the power factor; this is due to the fact that the strength of the commutating field is in excess of that required for perfect commutation, and in consequence a circulating current is produced in a direction opposed to that of the transformer voltage. The result of this reversed circulating current is that the main field lags behind the main current, so that in the example worked out there is actually a maximum value of the power factor for each terminal voltage; whereas in the plain series motor the power factor continually approaches unity as the current decreases and the speed rises. These calculated results are corroborated by the test curves of the Siemens-Schuckert motor in figure 97.

The influence of the commutation pole may, therefore, be summed up as, on the average, higher efficiency combined with lower power factor, and a fairly wide range of sparkless running.

In the third, or compensated repulsion motor, the transformer field plays the same part as the commutating field in the second motor. The power factor at the rated load in this case is not far from unity due to the compensating effect of the transformer field on the excitation current; but in other respects the effects are the same, namely high efficiency and sparkless running. Above the synchronous speed also a similar result is produced, viz. the circulating current is reversed. The consequence of this reversed circulating current is important as it may overpower the compensating effect on the excitation current. In the example worked out, the performance curves shew that the power factor reaches a maximum which is less than unity, which indicates that the angle of lag never reverses its sign, and that a leading current is never produced. The possibility of this effect is important, as many statements have been made that at speeds above synchronism, the current is in advance of the volts, and that this may raise the average power factor of a system which supplies a number of motors.

Comparison of the three motors. It will be seen at once that each of the three motors has some advantage peculiar to itself and lacking in the others. In the first motor, probably the chief feature is simplicity; it also has a high power factor but at the expense of efficiency; in the second motor, the circulating current is entirely suppressed at the rated load and partially at other loads, resulting in a high efficiency and a wide range of sparkless operation; in the third motor, the peculiar feature is chiefly the method of excitation resulting in a high power factor without any corresponding loss, and permitting a greater air gap than the other motors.

If the motors be regarded from the point of view of the rated load (100 H.P. for one hour with 75° C.) there seems no doubt that the first motor would not fulfil its requirements. The armature losses corresponding to 100 H.P. are much greater in this motor than in the others, owing to the presence of the circulating currents; and therefore, strictly speaking, they should not all be called 100 H.P. motors.

Railway motors, however, must be considered from the point of view of the service required of them, which is very seldom to perform the rated load for one hour. In actual service on an urban or suburban railway, the advantage possessed by the last two motors over the first is not so decided. The armature losses, apart from the circulating currents, would be practically the same, and the losses due to the circulating currents might not be far different, though very differently distributed in time. Thus at the start the first motor would have the advantage, as the circulating current is kept a good deal below the maximum, whereas at higher speeds the other motors would shew up better. It may be justifiable, therefore, to put the three motors on the same footing, though it is certainly illogical to call them 100 H.P. motors. The comparative performance of the three motors is examined in Chapter 8. As to what their rating should be is a matter of some difficulty. The rules laid down for finding the average working current of a D.C. tramway or railway motor (page 10, Vol. I.) are applicable to motors of a certain class. In such motors armature iron loss is of much greater importance than in those under consideration. So far as the authors are aware, no rules have ever been formulated which will give any clear indication as to the intermittent rating of these motors, and the only possible course is to argue from general considerations in the light of the practice with D.C. railway motors.

One thing is quite certain, that no conclusions can be drawn on the basis of the radiating surface of the whole motor. Each part requires separate consideration in regard to its own radiation, and to the influence upon it of the other parts.

Regarding the armature it seems clear that a greater allowance may be made for it in A.C. motors than in D.C. for two reasons, (1) the iron loss in the former is much smaller, (2) the excitation watts in the former are much smaller, and the losses in the compensation coils are not so important as those in the D.C. field coils, as the conductors are in intimate contact with field laminations which serve as an excellent vehicle for the transmission of heat to the outside air.

From these considerations it may be suggested that the armature C^2R watts should be limited to ·75 watts per square inch.

Even then, however, the matter is not very simple. The variation of the circulating currents with the speed is different for the three motors, so that a general statement can scarcely be made. Thus, for example, if the speed of the second motor be suitably regulated in relation to the current, the circulating current may be suppressed entirely. The only possible course is to assume a typical service, and for this service to find what R.M.S. value of the current will give rise to an armature C^2R loss equivalent to ·75 watts per square inch of the radiating surface. This point is investigated in a subsequent chapter (Chapter 8) in which the following results are obtained for the three single phase motors and a direct current motor when performing the same scheduled service:—

Motor type	Plain series single phase	Series single phase with commutation poles	Compensated repulsion	Direct current
Voltage of motor	250	320	800	500
R.M.S. current	186	140	52·5	93
Armature watts per sq. inch	1·16	·635	·85	·415

Taking these figures as representing the result of the same typical short run, the permissible R.M.S. current for each motor may be proportioned from these results so as to give ·75 watts per square inch for the single phase motors and ·4 for the direct current motor.

The result may be taken as the rating of each motor and these will be

Plain series single phase motor ...	150 amperes 250 volts.
Series single phase motor with commutation poles	152 amperes 320 volts.
Compensated repulsion motor ...	49 amperes 800 volts.
120 H.P. direct current motor ...	91·5 amperes 500 volts.

Relation between performance and speed. Speaking very approximately it may be said that in the case of the two types of series motor there is a fairly definite relation between peripheral speed and power factor, consistent with good design. Thus, for instance, a peripheral speed of 20 metres per second or 4000 feet per minute is a fairly usual figure for the one hour rating. If for any reason the rated speed is much below this value, there will be a corresponding fall in the power factor.

On the other hand, although in the general case the rated speed is limited to the above value so that the maximum speed in practice shall not exceed 6000 feet per minute or 30 metres per second, in some cases it may be possible to assume that the maximum speed is not greater than the rated speed. [For instance, a locomotive may be required to work intermittently on a definite load corresponding to the one hour rating, say for pushing heavy trains up a steep incline.] In this case, a higher power factor may be expected.

With the compensated repulsion motor, however, the power factor depends mainly upon the relation between any speed in question and the synchronous speed; and consequently by arranging that the rated speed shall coincide with the synchronous speed, high power factors can be obtained at comparatively low speeds.

This property of the compensated repulsion motor gives it an advantage over the series motors in certain cases, but the advantage is chiefly confined to the diminution of current to be transmitted, the size and weight of the motors being very little affected thereby.

Influence of the frequency. The influence of a low frequency on the design is obvious. In all three cases a frequency of 25 has been assumed, as this is by far the most frequently adopted. In some cases, however, a frequency of 15 has been chosen. The advantage of this is manifest so far as the design of the series motor is concerned. All the induced voltages including the transformer voltage are proportionately smaller. The compensated repulsion motor, however, being from one point of view a transformer is not affected so favourably. The synchronous speed of a four-pole motor on a 15 frequency circuit is 450 r.p.m. which is a comparatively slow speed for railway motors.

Constructional details. In many ways, the constructional details of single phase motors are identical with those of direct current railway motors. The motors of the two classes serve the same purpose in practically the same way, and, as a consequence, the same methods are employed in the two cases.

In most of the points in which single phase motors differ from direct current railway motors they approximate to three-phase railway motors, as for instance, in the lamination of the field, the necessity for a small air gap, and the winding of conductors in slots in the poles.

There is, perhaps, only one point in which there is a radical departure from precedent, viz. in the provision of high resistance commutator connections.

Electrically, these connections are quite straightforward; but mechanically they are not so simple. The most obvious place for them is between the armature and the commutator. This method, however, has two drawbacks, viz. the difficulty of supporting them rigidly, and inaccessibility for repair. It must be remembered that the heat capacity of these connections is very small, and there is always a danger that, if the motor from any cause does not start immediately, the connections may burn out, due to the heavy circulating current. Thus a German silver wire, when carrying current at a density of 10 amperes per square millimetre, will heat up at the rate of 6° C. per second, the rate of heating varying as the square of the current density. Taking, as an example, a density of 20 amperes per square millimetre, lasting for 20 seconds, the temperature rise would be 480° C. neglecting radiation.

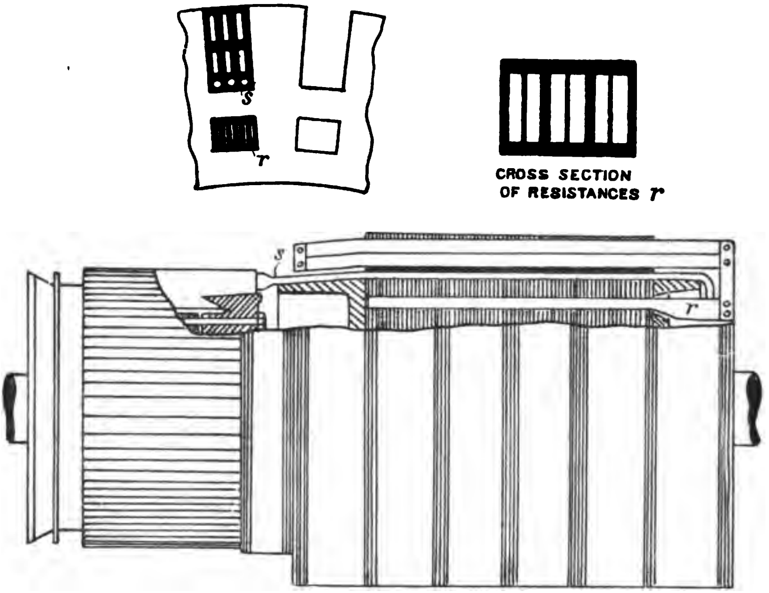


FIG. 88. Diagram of high resistance commutator connections. (British Thomson-Houston.)

Accessibility is, therefore, of considerable importance. In the motors manufactured by the Westinghouse Co., the high resistance connections are inserted in the slots*. The armature coils are connected up at the end remote from the commutator and from these joints the high resistance connections go through the slots to the commutator segments. In this way accessibility is secured, and a suitable means of support and insulation is provided.

A somewhat similar method has been patented† by the British Thomson-Houston Co., which consists in putting the resistances in square holes under the usual slots for the armature winding as in figure 88. To secure accessibility the resistances *r* are inserted from the pulley end and are joined up to the commutator by copper wires *s* running through the armature slots. By this means the tendency to overheat is lessened, as the high resistance strips are in intimate contact through mica insulation with the body of the armature core, which will absorb the heat almost as fast as it is generated.

A minor point in which single phase motors differ from direct current railway motors is in the number of brush arms. In the latter there are seldom more than two brush arms, whereas in the former the number may be any even number up to twelve for a big motor. This fact taken in conjunction with the inaccessibility of the brush gear makes it essential that the brush arms should not be fixed, but should be on a rocker which can be revolved so as to bring any arm under the inspection opening. This involves the provision of some means for connecting the brushes to fixed leads and of registering the rocker in the correct position after inspection.

Single phase motors on direct current circuits. Single phase series motors, inasmuch as they are identical in principle with direct current series motors, can obviously be used on direct current circuits, provided the voltage of the circuits is suitable. The normal voltage on direct current traction systems varies from 500 to 600 volts, whereas the voltage of single phase series motors varies from 200 to 350 volts. The simplest plan therefore, and that which is always adopted, is to connect two motors in series for direct current working and either in parallel or in series for alternate current working.

In the designs worked out in this chapter, it has been seen that the R.M.S. value of the magnetic field at the rated load has produced a degree of saturation in the iron considerably lower than is usual in direct current railway motors. When the motor is used on a direct current supply there is no reason for keeping down the saturation, and its capacity can be increased considerably by raising the excitation.

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* British patent 23,057 of 1903.

† British patent 10,897 of 1905.

This is generally effected by dividing the excitation winding of each motor into two equal parts which are connected in parallel for alternate current working and in series for direct current working. Thus in the latter case the excitation compared to the armature current is twice the value in the former case.

In motors designed for double working this should be borne in mind in proportioning the section of the excitation coils; otherwise in direct current working these coils will be liable to overheat.

In such motors there is, of course, no choice as to the type of compensation, the short circuited type being valueless on a direct current circuit.

In this connection two other points may be noticed, viz. (1) the absence of any tendency to flash over, and (2) the possibility of direct current working with shunted fields.

(1) As has been already mentioned, the phenomenon of flashing over in direct current railway motors is mainly due to the fact that the rise of the magnetic field when current is suddenly switched on is interfered with by eddy currents in the yoke and circulating currents in the coils short circuited by the brushes. Both these sources of interference are absent in single phase series motors and there is, therefore, no danger of flashing over. Apart from this the motor is designed for a low voltage, and the mean difference of potential between consecutive commutator parts is considerably less than is usual with 500 or 600 volt direct current railway motors.

(2) The provision of compensating windings, by neutralising the magnetising action of the armature winding, makes the sparking practically independent of the relative strengths of the field and the armature; whereas, in the normal direct current railway motor sparking becomes prohibitive if the strength of the field is much reduced.

Compensated repulsion motors are obviously unsuitable for D.C. working.

Pulsating torque. In all types of single phase commutator motor the torque is pulsating, varying between zero and a maximum which is approximately twice the mean. Now the limit of torque is generally settled by adhesion, and consequently if the average value is kept within the limit so determined, it might be supposed that half the effort of the motor would be wasted in skidding the wheels.

This point has been dealt with by G. Ossanna (*Elektrische Bahnen und Betrieb*, May 4th, 1906) who has shewn that a very minute amount of slipping in each half period is sufficient to cause the excess energy to be stored up by the rotational momentum of the various revolving parts. He calculates that, in the case of the single phase locomotive supplied by Messrs Siemens-Schuckert to the Swedish Government, the loss from this cause does not amount to one part in 1000.

On the other hand, experiments have been made by B. G. Bergman (see *Electrician*, Nov. 9, 1906, p. 144) which indicate that for the usual frequency of 25 the mean tractive effort at which the driving wheels will begin to slip is about 15 per cent. less with single phase commutator motors than with standard direct current motors. The suggestion is made, however, that this effect can easily be eliminated by introducing some slight flexibility in the mechanical connection between the motor armature and the driving wheel, whether the connection be direct or through gearing.

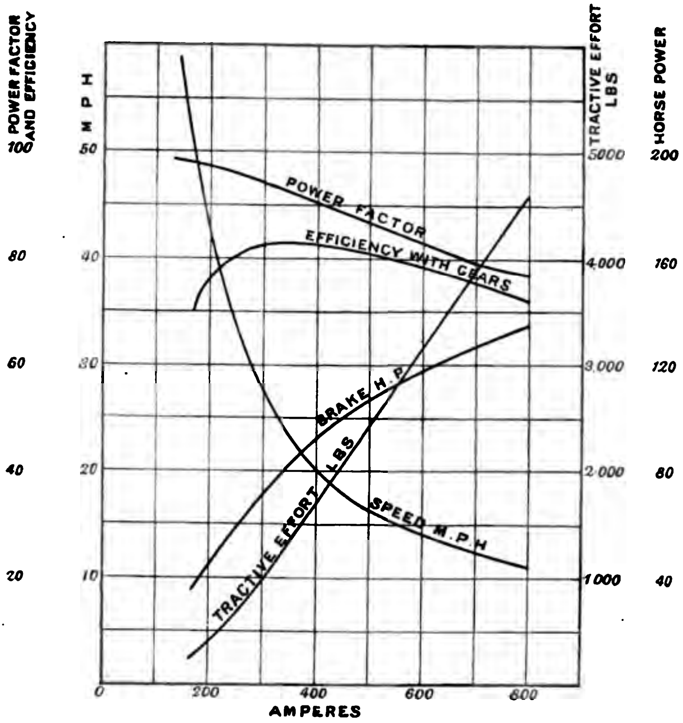

FIG. 89. Performance curves of Westinghouse 100 H.P. single phase geared railway motor.

Examples of single phase motors. A good many motors have already been built and set to work on single phase railway systems; but it would require too much space to refer to each individual case. A few have been selected for illustration, chiefly those which have been designed most recently, and represent the best results that have yet been obtained.

(*a*) Series motors without commutating poles.

These motors are manufactured chiefly by the Westinghouse and the General Electric Companies in America, and the British Westinghouse Company in England. The first motors were designed for a frequency of $16\frac{2}{3}$, but subsequently the standard frequency was raised to 25.

Figure 89 shews curves for the Westinghouse 100 H.P. motor. This is a 6-pole motor rated at 100 H.P. for one hour with 225 volts, 25 frequency; the continuous capacity is stated to be 45 H.P. at 200 volts, the power factor under these conditions being 96 per cent. There are 6 brush-holder arms mounted on a rocker ring. The weight of the motor complete with gears, gear case and axle bearings is 5200 lb.

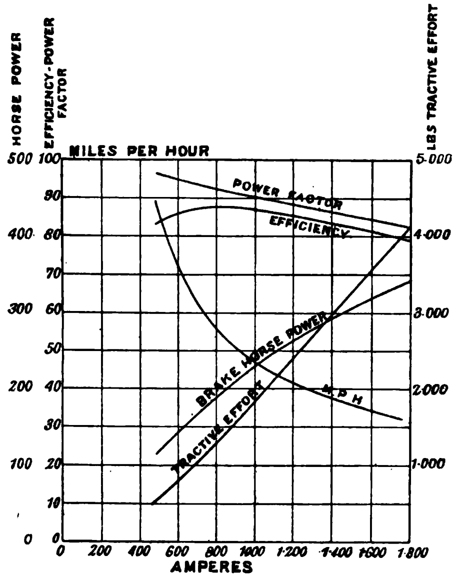

FIG. 90. Performance curves of Westinghouse 250 H.P. single phase gearless motor, 220 volts, 25 frequency, 62″ wheel diameter.

Figure 90 shews the performance curves of the 250 H.P. Westinghouse motor for the direct current single phase locomotives on the New York, New Haven and Hartford Railway. This motor possesses several points of interest both in itself and in its spring coupling. Being direct coupled to a 62″ driving wheel, the speed of the motor is necessarily slow. The armature is approximately 40″ in diameter and is wound with a 12-pole multiple circuit winding. A general view of the motor with its driving wheel is shewn in figure 91; the spring suspension having been already illustrated in Chapter 16, Vol. I.

In figures 92 and 93 are shewn the performance curves of a 75 H.P. single phase motor manufactured by the General Electric Company of America, from which the comparative working on alternating current and direct current circuits can be seen. In the former case the voltage is 250 and the frequency 25, the field coils being connected in two parallel circuits; in the latter case the field coils are all in series, the voltage being 250. The following are a few particulars relating to this motor:—

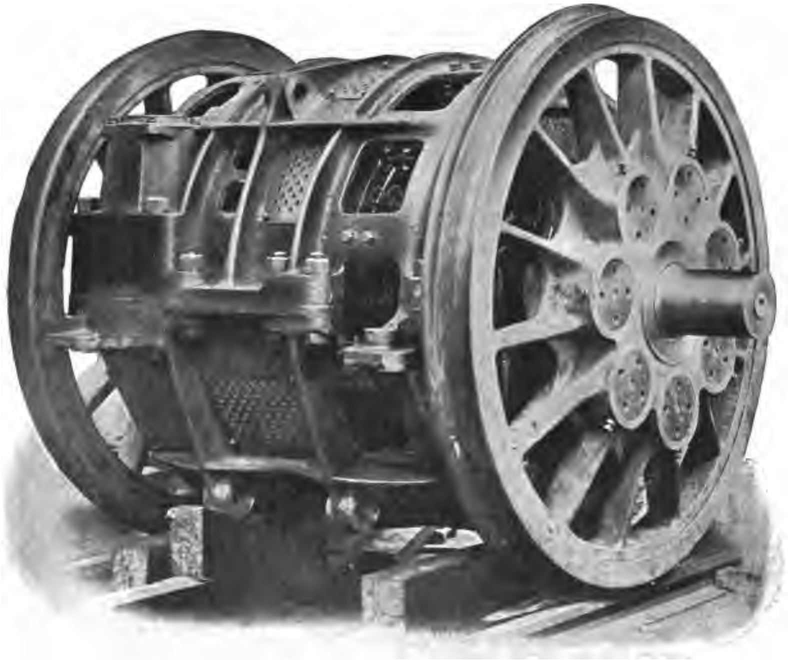

FIG. 91. View of Westinghouse 250 H.P. single phase gearless motor with driving wheels.

Weight complete with gear and gear case	4500 lb.
Weight without gear and gear case	4200 lb.
Weight of armature alone	1200 lb.
Clearance with 36-inch wheels	$4\frac{7}{8}''$
Maximum net efficiency with alternating currents	80 per cent.
Maximum net efficiency with direct currents	83·5 per cent.

Ten motor coaches have been supplied to the Toledo and Chicago Interurban Railway, each equipped with four of these motors.

(*b*) Series motors with commutating poles.

These motors are manufactured chiefly by the Oerlikon Company and by Messrs Siemens-Schuckert on the Continent, and by Messrs Siemens Brothers and Co. in England.

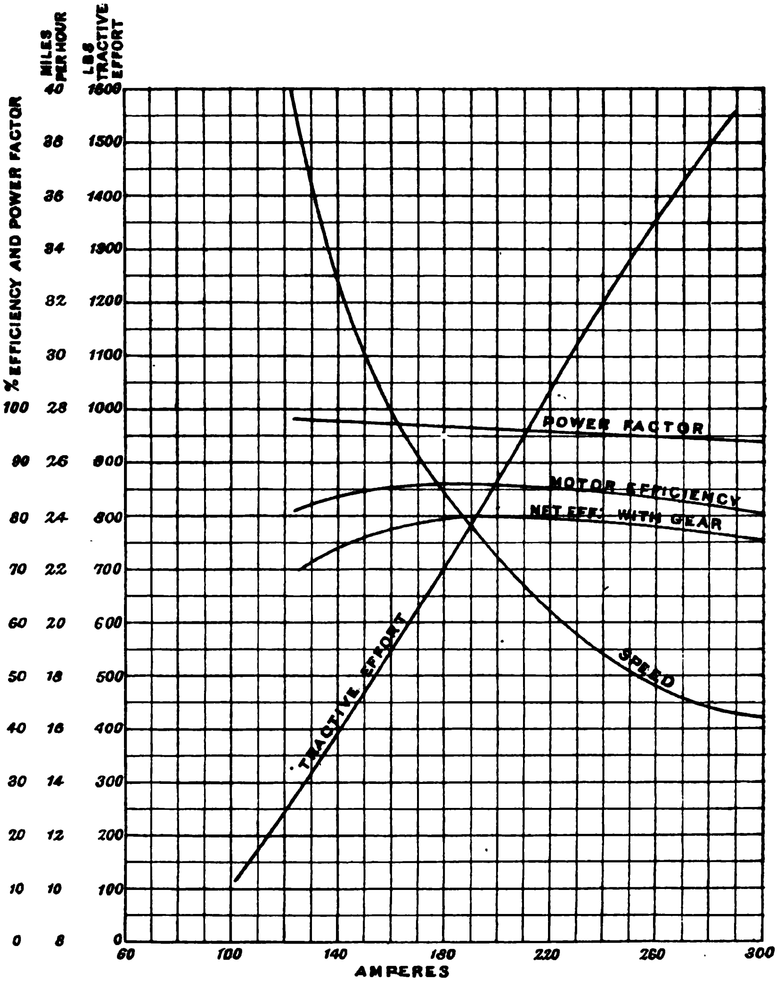

FIG. 92. Performance curves of General Electric 75 H.P. single phase geared motor with alternating currents.

The single phase motors used on the Vienna-Baden Railway are shewn in figure 94, and their performance curves with direct and with alternating currents are given in figures 95 and 96 respectively. These motors are constructed by the Austrian Siemens-Schuckert Company, which has equipped fourteen motor coaches and twelve trailers for this line. Each motor coach contains four 40 H.P. motors, of which two are always connected in series.

The motors are wound for 250 volts alternating at 15 frequency, and the armature windings are connected direct to the commutator without high resistance connections. In addition to the ordinary excitation winding, there are compensation windings and auxiliary windings for producing the commutation field. The commutator is suitable for three brushes per set, the brushes being $\frac{3}{8}''$ thick. The motor is wound for 6 poles, and is fitted with a brush rocker carrying 6 brush arms. For direct current operation at 500 volts the motors

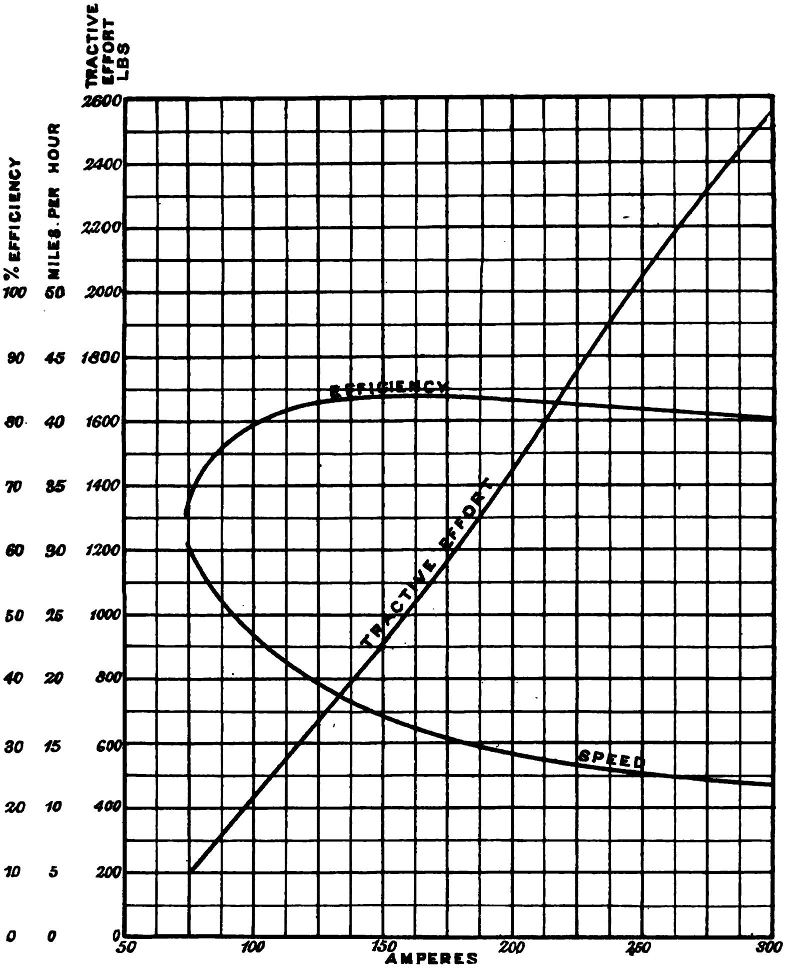

FIG. 93. Performance curves of General Electric 75 H.P. single phase geared motor with direct currents.

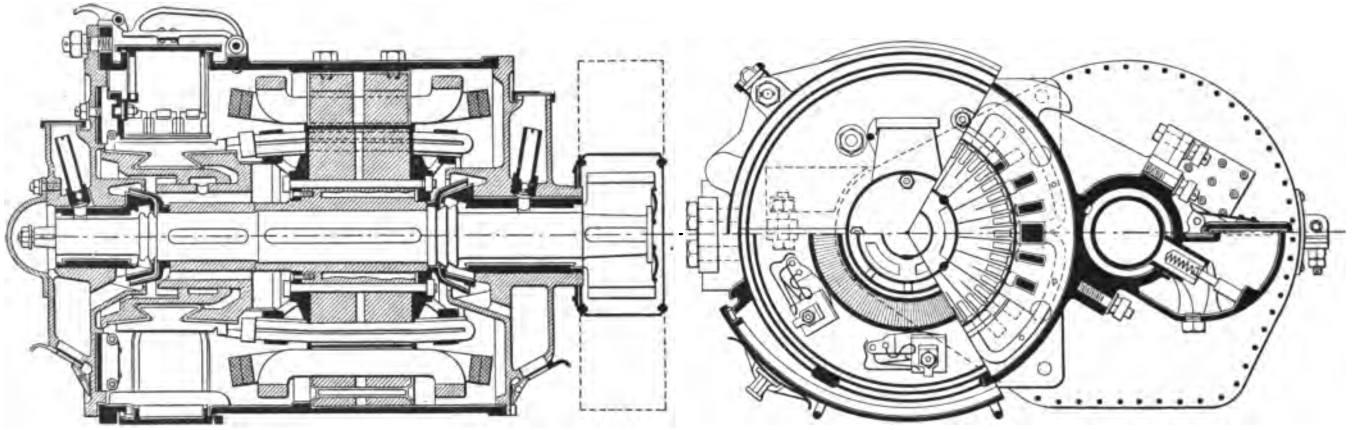

FIG. 94. Siemens-Schuckert 40 H.P. single-phase motor for the Vienna-Baden Railway.

are controlled on the series parallel system, each unit consisting of two motors in series.

The performance curves of the Siemens-Schuckert 175 H.P. single phase motor are shewn in figure 97, and give the results of the most careful tests. This motor is designed for operation on a 320 volt 25 frequency single phase supply and weighs 2·75 tons inclusive of gear and gear case. It is provided with compensation and commutation windings and two distinct excitation windings, one for each direction of rotation. At first sight this seems wasteful in copper; but a consideration of the diagram in figure 98 will shew that this is not so. This diagram represents a two-pole compensated series motor, in which the compensation winding is shewn all round the poles next to the armature, and the excitation winding away from the armature as shewn at A. As will be seen, part of the compensation winding cancels part of the excitation winding, and consequently these parts may be omitted as shewn at B. This will economise copper, if the motor is intended for only one direction of rotation; if the motor is

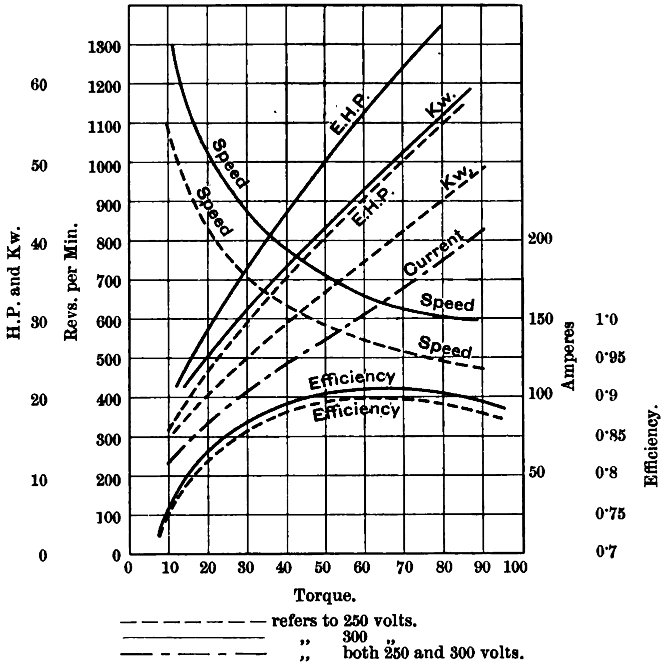

FIG. 95. Performance curves of Siemens-Schuckert 40 H.P. single phase motor with direct current.

required to be reversible it is necessary on this system to supply two sets of excitation windings, one for each direction as shewn at C. In that case the amount of copper is not altered; the arrangement, however, has certain advantages. These motors have been supplied to the Midland Railway, to the Rotterdam-Hague Railway, and to the Oranienburg Experimental Railway.

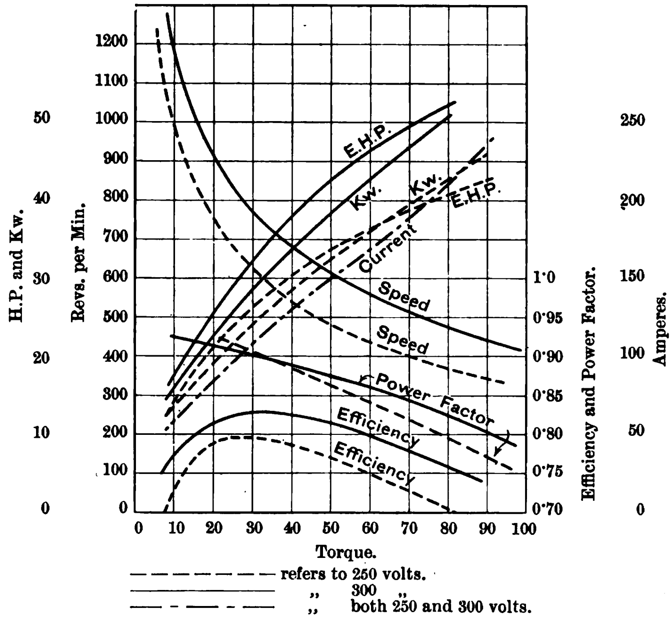

FIG. 96. Performance curves of Siemens-Schuckert 40 H.P. single phase motor with alternating current.

The 200 H.P. single phase series motor manufactured by the Oerlikon Company for the locomotive on the Seebach-Wettingen Railway is shewn in figure 99, and a general view of the same is given in figure 100. The motor is rated at 200 H.P., 650 r.p.m., 260 volts, 15 frequency, and weighs 3·4 tons exclusive of gearing. The performance curves are shewn in figures 101 and 102. As may be seen from the figures, the motor is wound for 8 poles, and is provided with distinct commutation poles between the main poles, each pole of both kinds having its own winding. The arrangement of the high resistance commutator connections is indicated in figure 99, these being situated between the armature and the commutator. The design of the gearing is peculiar; the motor is supported above the centre of the bogie and

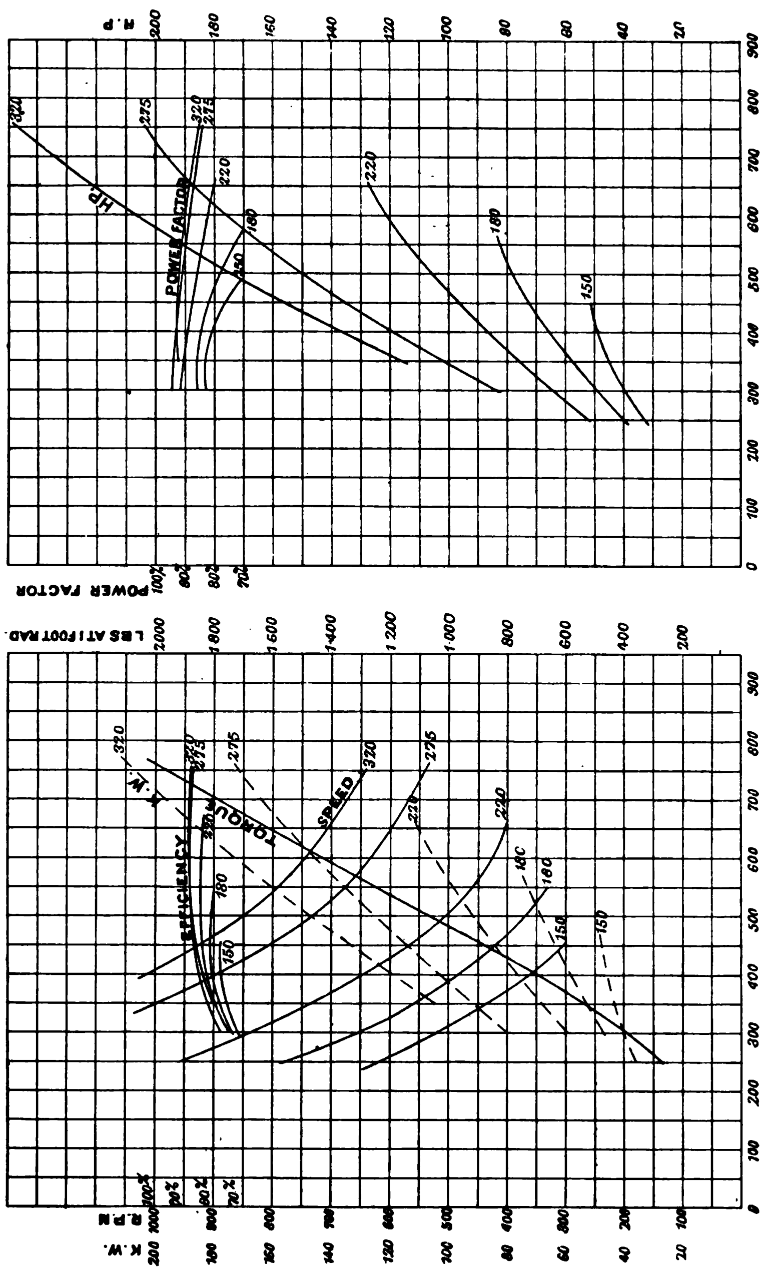

Fig. 97. Performance curves of Siemens-Schuckert 175 H.P. single phase motor, 320 volts, 25 frequency.

contains a countershaft which revolves in bearings fixed to the motor case vertically below the armature shaft. This countershaft carries the large gear wheel which gears with the motor pinion, and the same shaft is fitted at both ends with cranks connected by coupling rods with both the driving wheels. The locomotive has two bogies, each equipped with one motor.

Messrs Siemens-Schuckert also have built and equipped a locomotive for the Seebach-Wettingen line, which contains six 225 H.P. single phase motors.

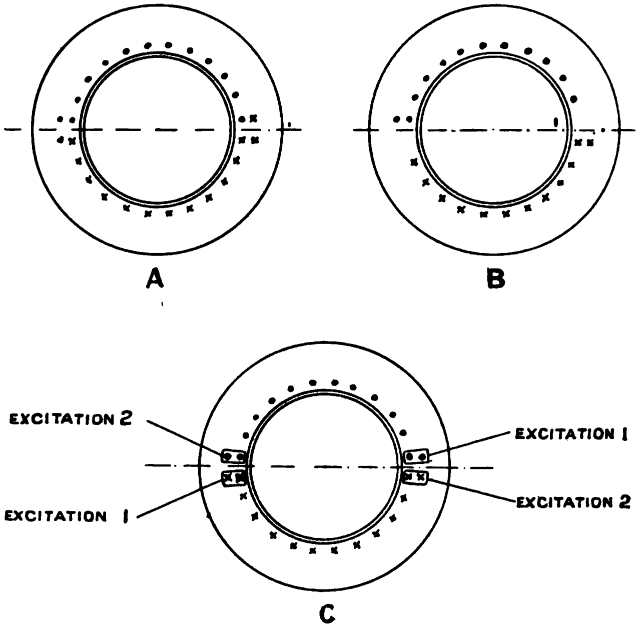

FIG. 98. Diagram illustrating the arrangement of the excitation windings in the Siemens-Schuckert single phase motor.

(*c*) Compensated repulsion motors.

Motors of this type are manufactured by the Allgemeine Elektricitäts Gesellschaft of Berlin. As examples may be quoted those for the Hamburg-Altona Railway, for the Borinage Railway and for the suburban system of the London, Brighton and South Coast Railway.

The Borinage motors are rated at 40 H.P. and are 6-pole motors for a voltage of 550 and a frequency of 40.

Those for the Hamburg-Altona Railway and for the London, Brighton and South Coast Railway are designed for 750 volts, 25 frequency and are rated at 115 H.P. for one hour at 600 r.p.m. They

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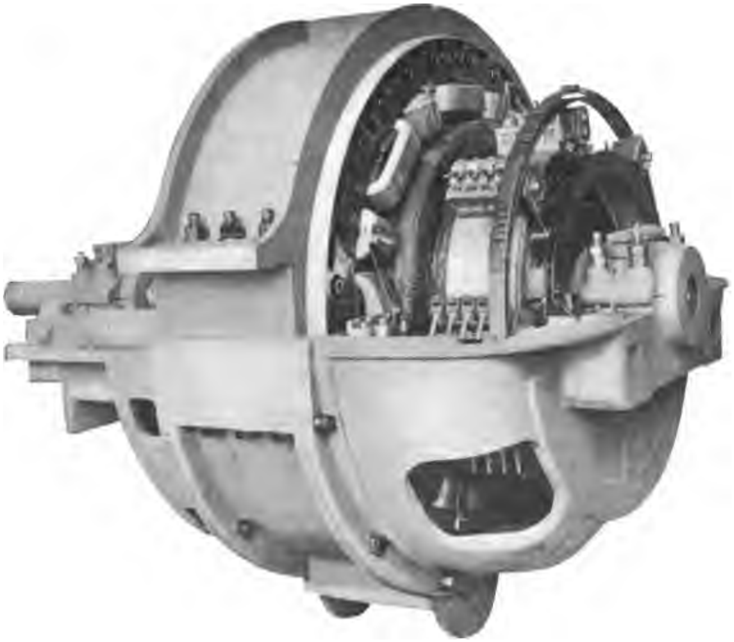

FIG. 100. View of Oerlikon 200 H.P. single phase motor.

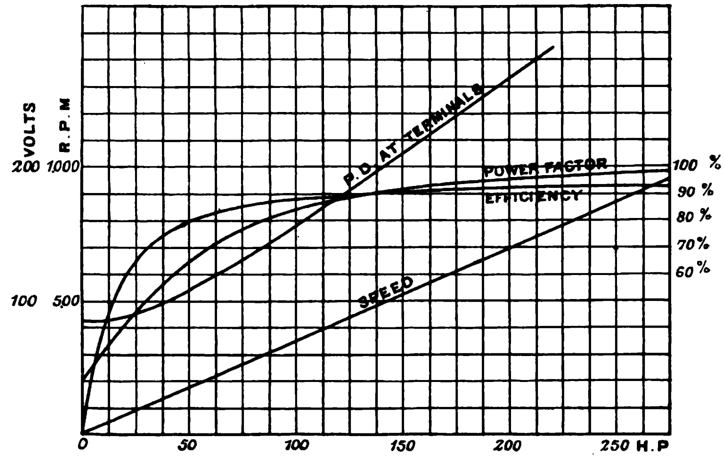

FIG. 101. Performance curves of Oerlikon 200 H.P. single phase motor with a constant current of 600 amperes.

are supplied with current from a step-down transformer which reduces the line voltage from 6000 volts to 450 or 750 volts. Performance curves are given in figure 103, and shew the power factor, efficiency,

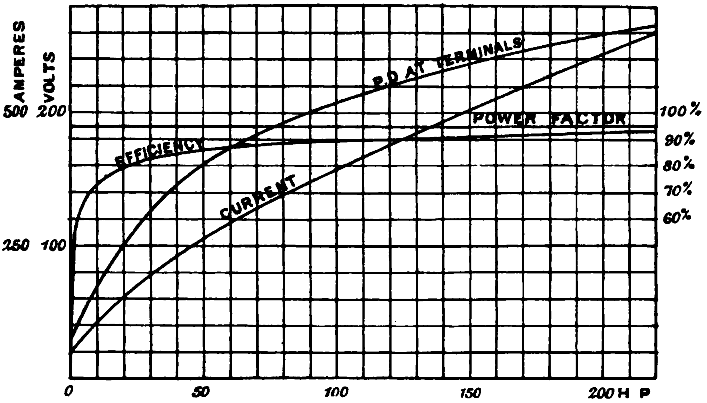

FIG. 102. Performance curves of Oerlikon 200 H.P. single phase motor at a constant speed of 650 revolutions per minute.

speed and pull for various currents and for four different positions of the controller. The efficiencies shewn make allowance for the losses in the excitation transformer and in the gearing.

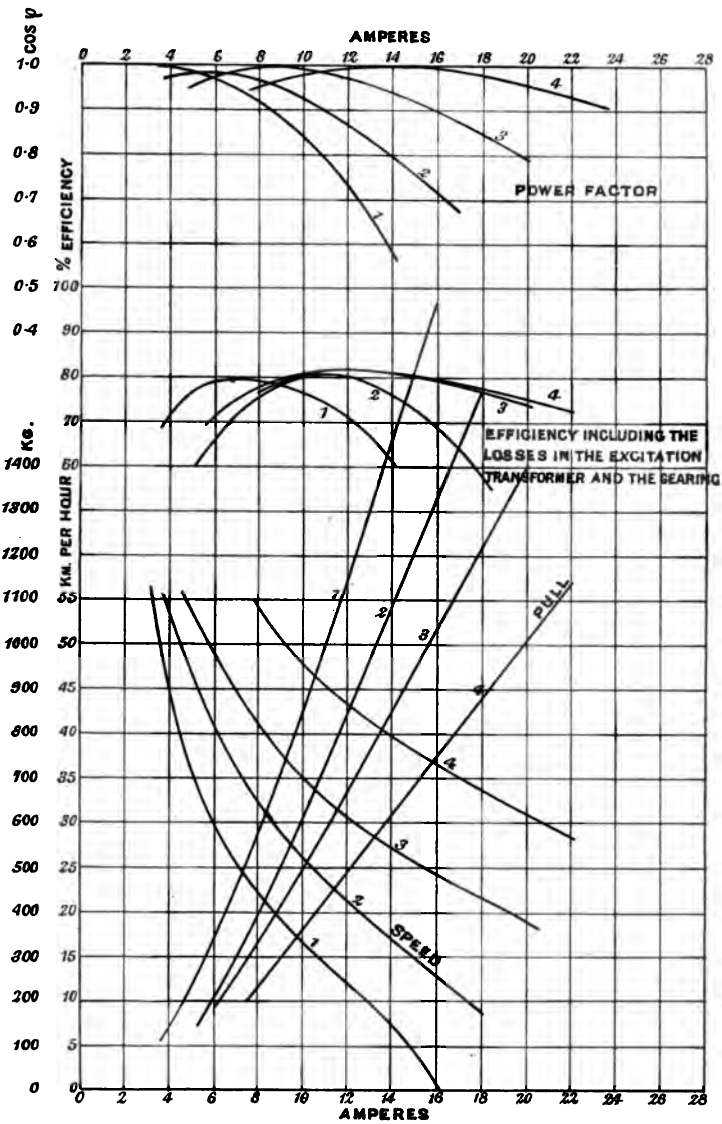

FIG. 103. Performance curves of 115 H.P. compensated repulsion motor, 6000 volts, 25 frequency, gear ratio 1 : 4·22, driving wheel 1000 mm. diameter. (Allgemeine Elektricitäts Gesellschaft.)

CHAPTER 6.

THE CONTROL AND EQUIPMENT OF ELECTRIC ROLLING STOCK ON THE SINGLE PHASE SYSTEM.

General. Single phase motors are controlled and regulated in the same way as direct current series motors, viz. by varying the terminal voltage. The great difference in the controlling apparatus is due to the fact that the chief means of obtaining this voltage variation with direct current is the use of series resistances, whereas in the case of alternate currents the variation can be obtained more economically by employing electromagnetic induction.

For example, if a railway car be equipped with single phase motors, and energy be supplied to the car from a high potential overhead wire, a step-down transformer is required between the trolley wire and the motors. In this case a simple method of providing the required voltage variation is to tap the secondary winding of the transformer at several points, which may be connected in turn to the motor terminals.

Requirements. The requirements of a control system for a railway car or a train of cars equipped with single phase motors are not very different from those in the case of a direct current equipment.

In the simplest form, for instance on a single car, it is necessary to regulate the motors by supplying a varying voltage to them, to reverse their rotation, to cut out any defective motors.

In a more complicated form, the same operations have to be performed simultaneously on a number of motor cars forming a train, that is to say, the control is a multiple unit system.

Examining the requirements more closely, single phase control systems are differentiated by the following points :

1. The voltage of the trolley line may be high, necessitating a transformer on the car. There must therefore be a high voltage switch between the collector and the primary winding of the transformer, and this should also be fitted with an automatic overload release to protect the transformer and incidentally all the other apparatus supplied thereby.

2. The methods of obtaining variable voltage for the regulation of the motors are peculiar to alternating currents. In the majority of cases there is no need to adopt the series parallel system of control, as a rheostat is not used, and the losses in the external circuit cannot be reduced thereby.

3. In some cases, the conditions of service require two or more different voltages on the trolley wire, such as, for instance, 500 volts in urban districts and 6000 volts in country districts. Switch gear must, therefore, be provided to enable the proper voltage to be applied to the motors whatever the potential of the trolley wire may be.

4. In other cases, the motors may be required to work on direct as well as on alternating current circuits. This may require that the control shall be on the series parallel system with rheostats in both cases, or that each case shall have the system of control most suited to it.

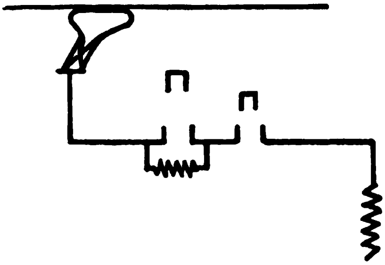

FIG. 104. Diagram of Siemens high voltage switch shewing connection of preliminary resistance to prevent a rush of current.

High voltage switch. A point of considerable interest in regard to the switch between the trolley wire and the primary of the transformer, is the device due to Messrs Siemens-Schuckert for overcoming the current rush which frequently occurs in switching on. The high tension switch is in two parts, forming practically two switches in series, one of which is shunted by a resistance. The switch is so arranged that the circuit is closed first through the resistance, which is almost immediately short circuited (see figure 104).

Methods of obtaining voltage variation with alternate currents. Probably the most important part of single phase control systems is the apparatus for obtaining a variable voltage.

The most obvious method is that already mentioned, viz. by tapping the secondary winding of a transformer at a number of points to which connection may be made.

A second method consists in the employment of an induction regulator, that is a transformer in which the secondary winding can be displaced and completely reversed in relation to the primary. This regulator is always made in a form similar to that of an induction motor, one of the windings, say the primary, being on the rotor, and the other, *i.e.* the secondary, being on the stator. If then the rotor be moved through an angle corresponding to one pole, the E.M.F. induced in the secondary will be reversed.

A third method consists in the combination of the first two methods, that is a transformer with a multiple connected secondary combined with a small induction regulator.

Multiple connected transformer. This method is at first sight quite simple. It would be supposed that it only required for its operation a number of switches in the form of a controller barrel or otherwise, by means of which the motors could be connected to the various taps in succession. A difficulty, however, arises in passing from one tap to the next; if one switch is opened before the next is closed, the whole current is broken at the working potential; on the other hand, if the switch in use is not opened before the next one closes, that portion of the transformer secondary between the taps is short circuited through the switches and a very heavy current has to be broken when the first switch is opened. There are various methods of overcoming this trouble, but all consist in the employment in one form or another of preventive resistances or inductances which are inserted between successive taps during the passage from one to the next. As an example take the method indicated in figure 105. In this figure S represents the secondary of a transformer tapped at points 1 and 2; X, Y represent the leads conveying current to the motors; and R the preventive resistance or inductance. Switch 1 is shewn closed and completes the motor circuit. To change the voltage it is necessary to close the switch 2, open 1, and then short circuit R. If R be suitably designed for the particular conditions to be met, there will be no voltage across switch 1 when 2 is closed, and switch 1 can be opened without any arc being formed.

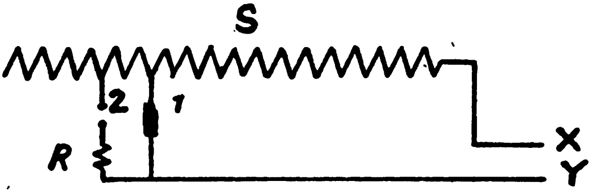

FIG. 105. Diagram of method for obtaining variable voltage from a static transformer.

Let V, figure 106, represent the voltage between the points 1 and 2, and let C be the motor current lagging behind the voltage by an angle ϕ. Then in order that no arc may be formed on opening switch 1, the voltage drop in R due to the passage of C must be equal and opposite to V, the voltage from 1 to 2. In other words the impedance of R must be $\frac{V}{C}$, the resistance $\frac{V}{C}\cos\phi$, and the reactance $\frac{V}{C}\sin\phi$.

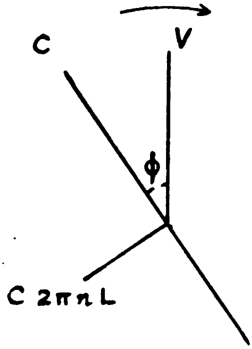

FIG. 106. Vector diagram of drop of volts in the preventive coil.

This method, therefore, would be quite satisfactory if the three quantities V, C and ϕ remained constant as the start progresses. The first can, of course, be made constant, but a reference to the performance curves calculated in the preceding chapter will shew that if a uniform starting current be maintained, the power factor or the cosine of the angle ϕ varies between wide limits, say from ·3 to ·9.

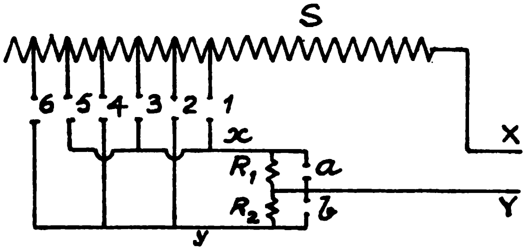

FIG. 107. Diagram of connections for obtaining variable voltage from a static transformer.

Consequently, one fixed value of the impedance R will not suffice to prevent arcs at all the switches in succession. There are two ways of meeting this difficulty, (1) by arranging the impedance to suit an average value of ϕ, (2) by providing a separate impedance for each step proportioned to suit the conditions at that voltage.

Figure 107 shews one way in which a constant impedance is used with a number of taps. Two impedances R_1 and R_2 are provided of the same value and are used with alternate steps, short circuited by switches *a* and *b*. The successive steps are as follows:

close switch 1—current flows through R_1;

close *a*—current flows direct to Y;

close switch 2—current may now flow through a parallel path switch 2, *y*, R_2, Y;

open switches 1 and *a*;

close *b*—current flows direct to Y, and so on.

Figure 108 shews a slightly different arrangement in which switches *a* and *b* are omitted. This is a little simpler but involves more switches for an equal number of steps and leaves one of the impedances constantly in circuit.

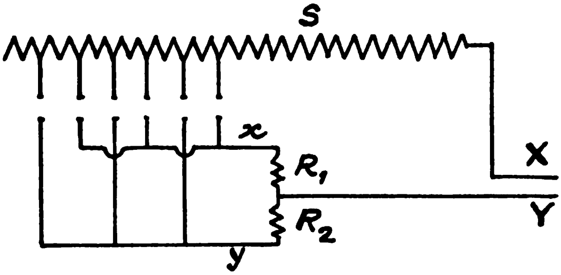

FIG. 108. A second method for obtaining variable voltage from a static transformer.

Figure 109 shews an alternative arrangement which is used in some cases, the impedance R_2 and the switch *b* in figure 107 being eliminated. In this arrangement the procedure is as follows: close switch 1, the current flows direct to the lead Y: next close switch 2, making sure previously that switch *a* is open; current may then flow through switch 2 and impedance R: open switch 1 and then close switch *a*, current flows through switches 2 and *a* to Y: to proceed, switch *a* is reopened, switch 3 closed and switch 2 opened*.

Figure 110 shews the arrangement in which each step has its own impedance.

Each of these arrangements is an example of the use of one or more preventive impedances, and they are all subject, in a greater or less degree, to the drawback that the voltage across any switch to be opened varies with the current. Although separate impedances can be adjusted for different power factors, they can only fulfil their purpose accurately with a definite current.

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* This method was devised by one of the authors in 1901.

Multiple switch methods. In all the methods mentioned above, only one switch at a time is used to carry the current except during the transition stages. These methods may therefore be called single switch systems in contradistinction to multiple switch systems in which two or more switches are used simultaneously.

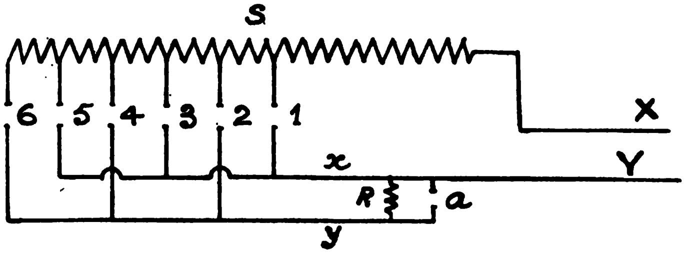

FIG. 109. A third method for obtaining variable voltage from a static transformer.

The simplest of these contains precisely the same apparatus as that shewn in figure 108, in which the transformer taps are connected alternately to the two ends of a single choking coil, the motor lead being connected to the middle point. If now switches 1 and 2 be closed simultaneously, the preventive coil becomes in reality an equal ratio transformer and the current from the transformer divides up into approximately equal portions, which flow through the two halves of the preventive coil, uniting at the centre to pass to the motors. The voltage applied to the motors is obviously that due to a point midway between the taps 1 and 2. To increase the working voltage it is only necessary to open switch 1 and then close switch 3; during the transition stage the full current flows through switch 2, and the voltage of the arc to be broken at switch 1 is only that due to the E.M.F. between taps 1 and 2.

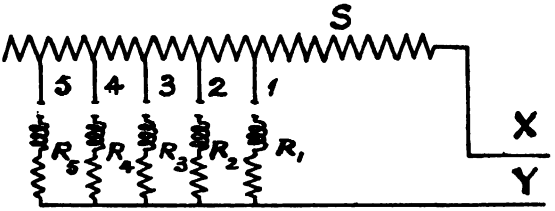

FIG. 110. A fourth method for obtaining variable voltage from a static transformer using a separate impedance for each step.

This system is used by the Westinghouse Company and has several advantages, viz. each switch need only be designed to carry half the main current, and the process of switching on is very simple; on the other hand, the preventive coil must be designed to carry the main current continuously and not only during the transition stages.

It is necessary to consider the effect of opening one of the two switches when passing to a higher or lower voltage. When two currents of equal magnitude flow in the two halves of the preventive coil they neutralise each other as far as any choking effect is concerned; but when one switch is opened the whole current passes through one side of the coil only, and may give rise to a very considerable choking effect, as the preventive coil is then acting as a choking coil pure and simple. This difficulty may be minimised by suitably designing the coil. For example, suppose the motor current which is kept approximately constant during the start has the value C; then this may be supposed to consist of two currents each equal to $\frac{C}{2}$ in the two halves of the coil. Superimposed on these two currents there is the magnetising current i necessary to produce a voltage equal to that between the transformer taps. Then the total current in one half is the resultant of $\frac{C}{2}$ and $+i$, and in the other half $\frac{C}{2}$ and $-i$ (see figure 111). In an ordinary transformer i would be very small compared to C, but in this case it is better to make it reasonably large. The resultant currents in the two halves will depend on the phase of C, and the choking will depend on this phase and on the magnitude of i.

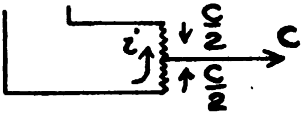

FIG. 111. Diagram shewing the currents in the halves of the preventive coil in Fig. 108.

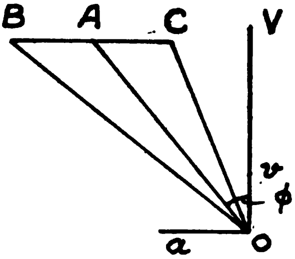

FIG. 112. Vector diagram for the currents in the halves of the preventive coil in Fig. 108.

Thus supposing Ov, figure 112, to represent the voltage between taps, OV the voltage on the motors, OA = C/2 and Oa = i the currents as above, then, neglecting the resistance of the preventive coil, the current OA will lag behind OV by the angle ϕ and Oa behind Ov by a right angle. The resultant current in one half of the coil will be OB and in the other half OC, AB and AC each being equal and parallel to Oa. If now the whole current pass through one half of the coil, the back E.M.F. of the coil will be due to 2OA in half the turns, *i.e.* will be $\frac{2OA \times \frac{1}{2}}{Oa} \times \frac{Ov}{2}$ or $\frac{OA}{Oa} \times \frac{Ov}{2}$, and this back E.M.F. will lag one right angle behind the current. This back E.M.F. will cause an alteration in phase and magnitude of the motor voltage, the amount of the alteration being easily determined with the aid of the diagram. From this figure it will be apparent that, provided $\frac{OA}{Oa}$ is not very great, this choking effect will be insignificant except for very low values of the power factor. As an example, the case is worked out for a start with two plain series motors, the performance curves for which are given in the previous chapter, pages 138, 139.

Assume that the current per motor varies between the limits 580 and 440, the mean being 510 amperes, and let the start be made in 5 steps. There will therefore be six transformer taps. In calculating the following set of figures it has been assumed that a current of 220 amperes flowing in all the turns of the preventive coil will produce an E.M.F. of 30 volts at its terminals.

This calculation is instructive in another way, as it shews how motor volts have to be varied in order to maintain the starting current within definite limits. The first voltage step is only 22 volts, whereas the last is 33 volts. The magnetising current of the preventive coil has been assumed more or less at random, but the calculation is sufficient to shew that the total current can be split up in fairly equal proportions between each pair of switches without causing any serious choking action due to the preventive coil.

This method may be extended so that in each working position of the controller four switches are closed, the total current being shared between them in fairly equal proportions. The diagram of connections is shewn in figure 113.

The working of this method is apparent. In any running positions four consecutive switches are closed; these switches are connected by leads x_1, y_1, x_2, y_2 to the four terminals of two inductances l_1 and l_2, the centre points of these two windings are connected to the terminals of a third inductance l_3, from the centre of which connection is made to the motor circuit. This arrangement ensures that the current in the leads x_1, y_1, x_2, y_2 shall be nearly equal as long as 4 consecutive switches are closed.

Suppose that 1, 2, 3 and 4 are closed, in order to step to a higher voltage 1 is opened and 5 is closed. The opening of 1 breaks ¼ the main current at a voltage due to the windings between 1 and 3.

Particulars of currents and voltages for a start with two single phase series motors.

Amperes per motor	Motor volts	Power factor	Current in one half of preventive coil	Current in other half of preventive coil	Motor volts when one switch is open	Position of controller
580	138	·4	710	420		1
440	138	·72	560	350		
					129	transition 1
580	160	·65	710	440		2
440	160	·83	560	370		
					157	transition 2
580	187	·79	710	465		3
440	187	·90	560	400		
					189	transition 3
580	217	·86	710	480		4
440	217	·93	560	420		
					222	transition 4
580	250	·92	690	515		5
440	250	·96	560	440		
					256	extra step

This method requires a large number of switches, but is very suitable for heavy work*. The number of switches, however, give an advantage in providing a large number of working voltages. Thus for instance on the Sarnia tunnel locomotive there are in all 20 con-

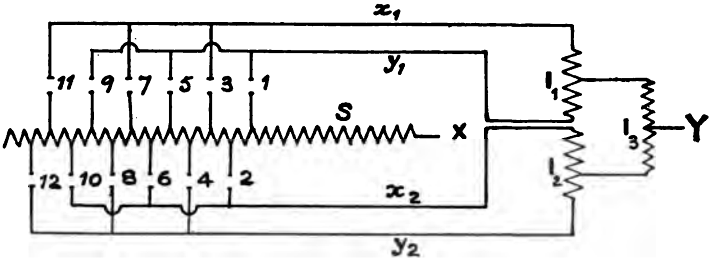

FIG. 113. A fifth method for obtaining variable voltage from a static transformer using four taps simultaneously.

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* This is presumably the method adopted by the Westinghouse Company in the equipment of the single phase locomotives for the Sarnia tunnel, see *Street Railway Journal*, Jan. 20, 1906.

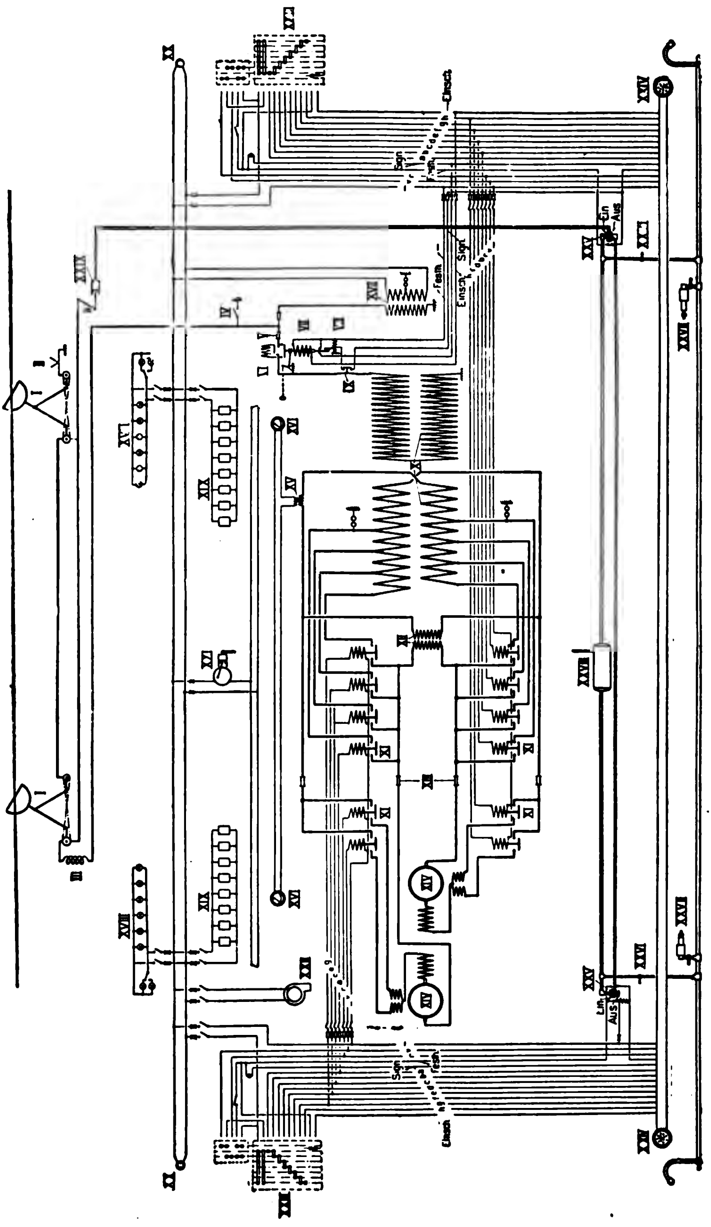

FIG. 114. Diagram of connections for Siemens-Schuckert single phase control system.

nections to the transformer and therefore 20 switches, and consequently 17 working voltages.

Siemens-Schuckert multiple switch control for single phase motors. Messrs Siemens-Schuckert of Berlin have devised a system of control which has various advantages, the diagram of connections for which is shewn in figure 114.

From this figure it will be seen that there are two transformers, the main (X) and the auxiliary (XVII); the former supplies power to the driving motors, and the latter to the control, heating and lighting circuits. The control is a duplex system, that is to say, for each working position of the master controller two contactors are closed. The principle of the control is shewn in the diagram, figure 115; the main transformer has two distinct low tension windings, S_1 and S_2, each of which is tapped in four places 1, 3, 5, 7 and 2, 4, 6, 8; in effect the two secondaries S_1 and S_2 and the two motors M_1 and M_2 are all in series and form a single closed circuit. Cross connections are provided in the shape of two choking coils C_1 and C_2 through which current is supplied to the motors from winding S_2 when the circuit of S_1 is opened between steps, or from S_1 when the circuit of S_2 is opened. The switching on is effected by closing switches 1 and 2, then opening 1 and closing 3, then opening 2 and closing 4, and so on.

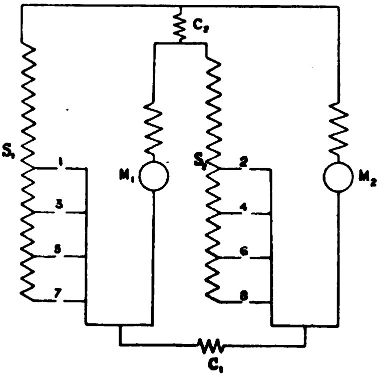

FIG. 115. Diagram illustrating the principle of the Siemens-Schuckert single phase control system.

The two choking coils are wound on the same core. The four contactors of each set are mechanically interlocked with each other, so that only one of each set can be closed at the same time.

The motors are provided with separate windings for each direction of rotation, and the reversers are simply two pairs of contactors, one pair for each motor (see page 193). The other details of the control are sufficiently clear from the diagram of connections, and from the following index.

Index of apparatus in the Siemens-Schuckert multiple switch control for single phase motors.

I. High voltage current collector.
II. Lightning arrester.
III. Lightning arrester choking coil.
IV. Earthing switch.
V. High tension fuse.
VI. High tension oil switch.
VII. Operating magnet for the high tension oil switch,
VIII. Hold-on magnet for the high tension oil switch.
IX. Hot wire overload relay.
X. Main transformer.
XI. Contactors.
XII. Choking coils.
XIII. Motor fuses.
XIV. Motors.
XV. Current transformer.
XVI. Ammeters.
XVII. Auxiliary transformer for lighting, heating and control.
XVIII. Lighting fittings.
XIX. Heaters.
XX. Couplers for lighting and heating circuits.
XXI. Electrically driven air compressor.
XXII. Ventilating fan.
XXIII. Master controllers.
XXIV. Control couplers.
XXV. Compressed air valve for current collectors.
XXVI. Driver's brake valve.
XXVII. Brake cylinder.
XXVIII. Compressed air reservoir.
XXIX. Compressed air cylinder for the current collectors.

Induction regulator method. As already explained, the induction regulator method consists in the use of a transformer with a reversible secondary, by which a certain definite voltage is first subtracted from that supplied by the step-down transformer and then by gradually rotating the regulator added to it. Thus for instance, supposing the range of variation to be from 200 to 400 volts, the step-down transformer would be wound for 300 volts and the induction regulator for 100.

The actual method of connection may be in one of two ways, either supplying the primary of the regulator with a constant voltage or with a voltage which depends on the position of the regulator. The two methods are shewn in figures 116 and 117, T being an autotransformer, IR the regulator with a moveable primary and a fixed secondary, and XY the motor circuit.

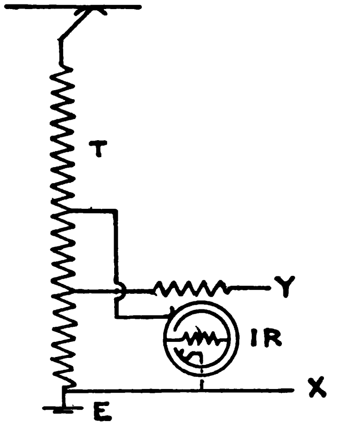

FIG. 116. Diagram of connections for induction regulator in combination with a static transformer.

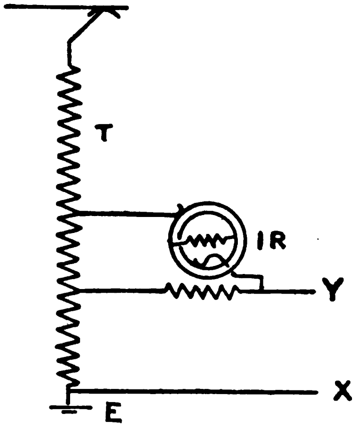

FIG. 117. Alternative method of connection of induction regulator in combination with a static transformer.

This method has obvious advantages, as it eliminates switches, and moreover provides a gradual change of voltage without jerks. It has objections, however ; the size and weight of the regulator are considerable ; it requires a certain amount of power, due as much to the gearing as to the actual electromagnetic force ; it lowers the power factor of the whole system due to the magnetising current required ; it is not very easy to move rapidly when occasion requires.

Induction regulator with multiple connected transformer. In this method, devised by one of the authors*, the induction regulator is used to facilitate the passage from point to point. Instead of an impedance being inserted between consecutive points, the current is passed through the secondary of a small induction regulator, by means of which an E.M.F. is produced in the local circuit equal and opposite to the E.M.F. in the section of the transformer. Figure 118 shews the arrangement and connection of the apparatus for carrying out this method. As before S is the secondary of the transformer, 1, 2, 3 etc.

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* (F. L.) British patent 1874 of 1905.

switch connections to the taps, and X and Y the leads to the motors; *p* and *s* are the primary and secondary of the small regulator. Alternate switches are connected by two leads *x* and *y* to the terminals of *s*, from the centre of which current is taken by Y. The regulator is wound so that the secondary E.M.F. is equal to the E.M.F. in the sections 1—2, 2—3 etc. of the transformer.

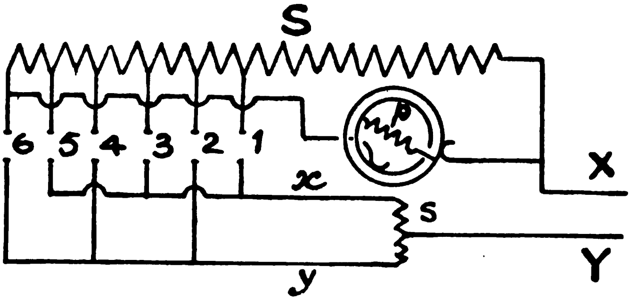

FIG. 118. Diagram of connections for induction regulator in combination with a multiple-connection transformer.

The operation is as follows: switch 1 is closed when the regulator is in such a position that its secondary E.M.F. opposes the transformer E.M.F. The regulator is then turned round through half a revolution (supposing it to be wound for 2 poles) when the secondary E.M.F. is assisting the transformer E.M.F. Switch 2 is then closed and switch 1 opened, and the process is repeated.

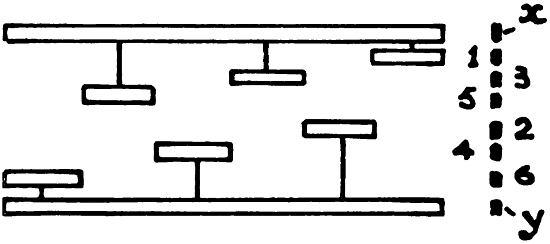

FIG. 119. Diagram of controller barrel for use with induction regulator and multiple-connection transformer.

This method is carried into practice by arranging the switches in the form of a controller barrel geared to the revolving part of the regulator, the development of the barrel being shewn in figure 119.

This system possesses the advantages of dividing the main current more or less equally between two switches at the moment of opening, so that the current to be broken is only half the main current, and of keeping down the voltage across the switch to very small values practically independent of the current and power factor of the motor circuit [experiments carried out by Messrs Siemens Bros. and Co. at Stafford have shewn that by suitably designing the transformer and the regulator the voltage across a switch can be kept down to about 4 volts with steps of 26 volts]. The voltage is raised gradually as with the previous method; but the regulator may be only $\frac{1}{6}$th of the size necessary in that system, thus eliminating many of the disadvantages. Figure 120 shews an improved arrangement* similar to that in figure 113, in which the two inductances l_1 and l_2 are replaced by two secondaries of a single induction regulator, the stator of which is wound like that of a 2-pole 2-phase induction motor. With this system the current to be broken is only $\frac{1}{4}$ of the total current.

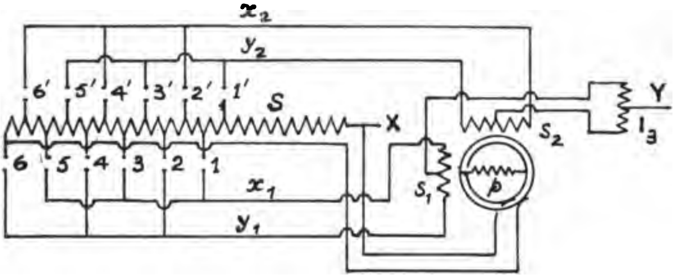

FIG. 120. Modified diagram of connections for induction regulator in combination with multiple-connection transformer.

Examples of single phase control systems for series motors. Murnau-Oberammergau Railway. On this railway, equipped by Messrs Siemens-Schuckert, there is no necessity for multiple unit system of control, as a train contains only one motor car. Figure 121 shews the general diagram of the control; M_1 represents one of the two motors, R the reverser, CB the main barrel controller, T_p the primary and T_s the secondary of the step-down transformer, A the main high tension switch operated by the driver by means of a small battery in combination with an overload relay, C a choking coil and L a lightning arrester, TT the bow collectors, S_w switches for connecting the car circuits to the low tension collector LT and to Earth.

The chief interest is centred in the main controller. The method in use for effecting the necessary voltage variation is practically that illustrated in figure 105, the lower contacts of the two switches shewn there being moved on gradually from step to step. The barrel consists of two parts, the shaded and the plain. The shaded part consists of two portions as shewn insulated from each other. The unshaded part serves for connecting the various taps to the motor circuit alternately direct and through the preventive resistance PR. The arrangement is such that arcs are broken only by the two small double contacts on the extreme left, which are fitted with magnetic blow-outs M.

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* British patent 21,068 of 1905.

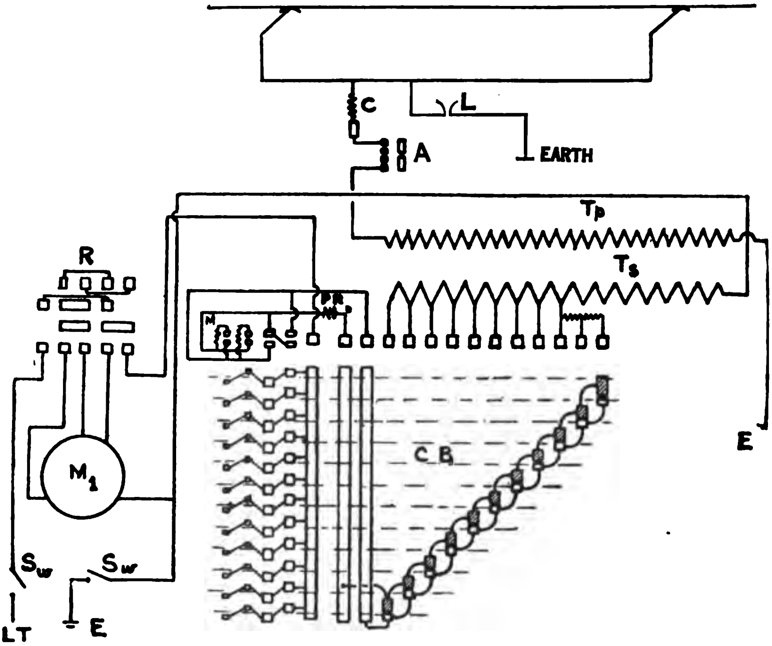


FIG. 121. Diagram of connections for the single phase controller on the Murnau-Oberammergau Railway. (Siemens-Schuckert.)

Long Island Railway. On this railway, equipped by the Westinghouse Company, the system of control is an example of the duplex method referred to on page 205. The equipment consists of two 25 kw. autotransformers connected in parallel, two 50 H.P. 300 volt 25 frequency series motors, two hand controllers of the barrel type, with a double preventive impedance consisting of a resistance and an inductance in parallel joined at their middle points to the motor circuit. The diagram of connections shewn in figure 122 makes plain the method of operation.

The same system arranged for multiple unit control is shewn in figure 123. The apparatus for a motor coach consists of the following parts: one or more current collectors, by means of which current is supplied to the high tension end of the autotransformer; one main autotransformer; one lightning arrester; one main fuse as near as possible to the collector; six unit switches A, B, C, D, E, F electro-pneumatically operated; one preventive coil; one series transformer

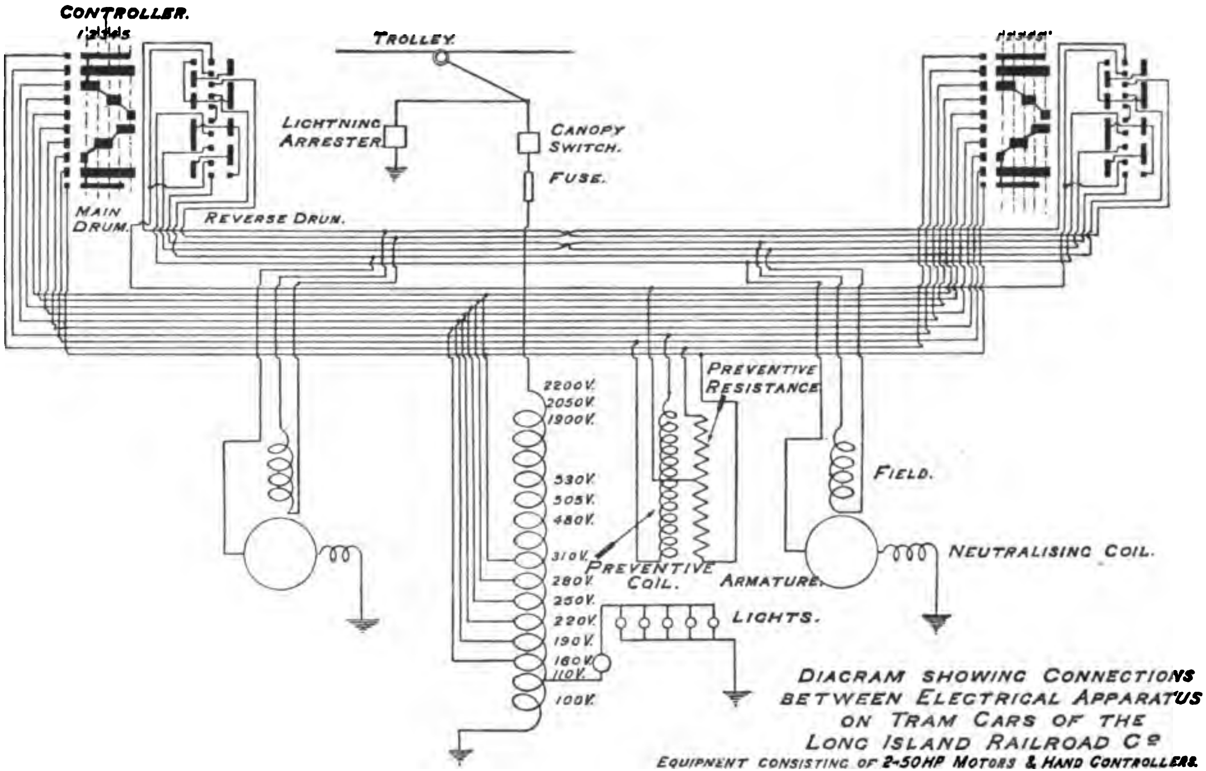
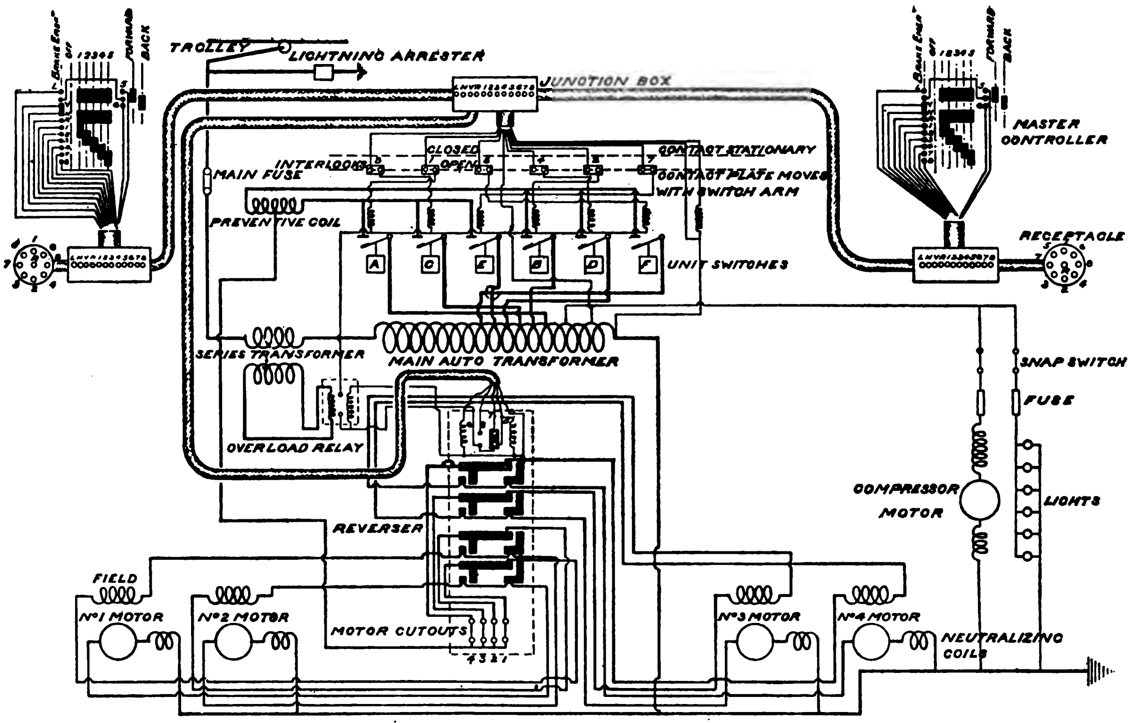


FIG. 122. Diagram of connections of the single phase controller on the Long Island Railway. (Westinghouse.)



WIRING DIAGRAM FOR A MOTOR COACH EQUIPPED WITH SINGLE PHASE SERIES MOTORS AND ELECTRO PNEUMATIC MULTIPLE UNIT CONTROL SYSTEM

FIG. 123.

connected in the high tension circuit, the secondary supplying current to an overload relay; one overload relay with resetting coil; one reverser with auxiliary contacts for interlocking; four main motors; four motor cutting-out switches; two master controllers; three junction boxes; two receptacles; one multiple core auxiliary train cable; one motor compressor; and the necessary lighting circuits.

The control current as well as the motor current is taken from the main transformer; a tap giving 50 volts to earth is connected to the wire L.

Schenectady-Balston Railway*. In this case cars have to run on a mixed system of alternating current at 2000 volts, 25 frequency, and on direct current at 500 volts. The line was equipped by the General Electric Company, and the control on the alternating system is effected by means of a barrel arranged on the method illustrated in figure 110, page 205, the impedances being formed in each case by the blow-out coil in the spark shield with a resistance. On direct currents the motors are connected two in series, the four motors being controlled on the series parallel method.

Indianapolis-Cincinnati Railway. On this line, equipped by the Westinghouse Company, each motor car is supplied with four 75 H.P. motors controlled on the induction regulator principle. The wiring diagram for the car is given in figure 124. The induction regulator is driven through worm gearing by means of a pneumatic motor, the valves of which are governed electrically from the master controller and a small storage battery. The operation of the system can be easily followed in the diagram. The master controller has 3 positions for forward and 3 for backward running. In the first position current from the battery pulls over the reverser and sets the overload relay. As soon as the reverser has responded, which can only take place when the induction regulator is at off, and the main switch has been closed by the operating coil through the overload relay, the motor circuit is made in the position of minimum voltage. If, now, the master controller be advanced to the third position the operating coil of the air motor receives current through the limit switch and the interlocking contacts on the main switch, and the induction regulator is rotated. The speed of rotation is set by the limit switch, which is governed by the high tension current through a series transformer. If at any time the master controller be put to the second position, the motion of the regulator is arrested. If to the first position, the regulator is turned back to the position of minimum voltage. If the master controller be switched off or if the main current becomes excessive, the main switch with its

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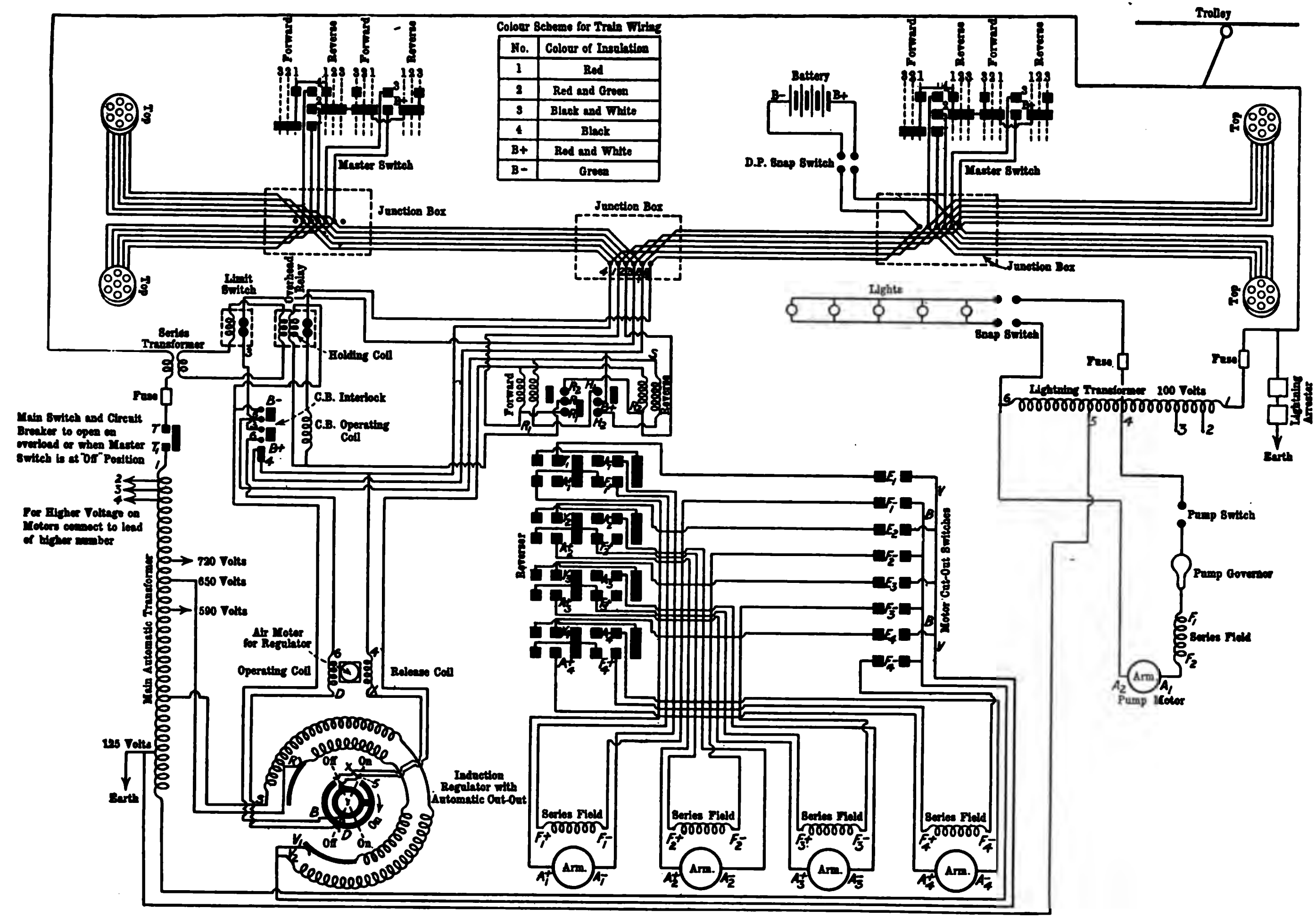
* See *Electrician*, Nov. 3, 1905, p. 98; and *Electrician*, Sept. 9, 1904, p. 826.



Colour Scheme for Train Wiring

No.	Colour of Insulation
1	Red
2	Red and Green
3	Black and White
4	Black
B+	Red and White
B-	Green

FIG. 124. Diagram of connections of Westinghouse single phase control using induction regulator.

interlocking contacts opens and current is supplied through these contacts to the release coil of the air motor whereby the regulator is turned back to the off position.

The disadvantages of this system have already been pointed out, and in recent work by the Westinghouse Co. this method of voltage variation has been abandoned in favour of the multiple connected transformer methods, an example of which has been described above.

1350 h.p. Westinghouse locomotive. This is equipped with six 225 H.P. single phase motors controlled on the induction regulator system. Sketches of this locomotive are given in the *Street Railway Journal* for June 3rd, 1905, from which a general idea can be obtained of the sizes of the various parts.

The Sprague Thomson-Houston multiple unit control for alternating current series motors. The diagram for this control system is shewn in figure 125. The control is on the multiple connected transformer system and is effected by means of twelve single phase contactors, of which four are utilised for reversing the motors. These contactors are controlled from a master controller, two of which are shewn in the figure. Current for control purposes is taken from a small auxiliary transformer and for power purposes from the main autotransformer or "compensator." The latter has six taps for regulating the voltage applied to the motors, these taps being marked T_1, T_2, ..., T_6; the other parts of the control may be seen from the figure. The operation of starting the motors is as follows: Assuming the main switch and the control switch in the circuit of the high tension switch closed and the main fuses (high and low tension) intact, and also that the motor cut-out switch is set for the operation of both motors, the master controller switch C at one end is closed and current is supplied from the small transformer to the top finger but one of the master controller. The reversing barrel is turned to "forward" say, and the main barrel to the first position. Current, therefore, flows to train wire 8 and through the interlock on switch 10 and closes the two forward switches 9 and 11. Current also flows to train wire 11, through the bottom contacts on the overload relay, through the control switch for the high tension main switch to the operating coil of the latter. The compensator therefore receives the high voltage. Current also flows through train wires 1 and 10 to the operating coils of switches 6 and 5, from the latter to the interlocks on switches 1, 2 and 3, and so to Ground G. These switches 6 and 5 therefore close and connect tap T_1 to the motors through switches 6 and 5.

As soon as switch 5 closes, its operating current passes through its own interlocks to earth, so that the switch is held closed independently

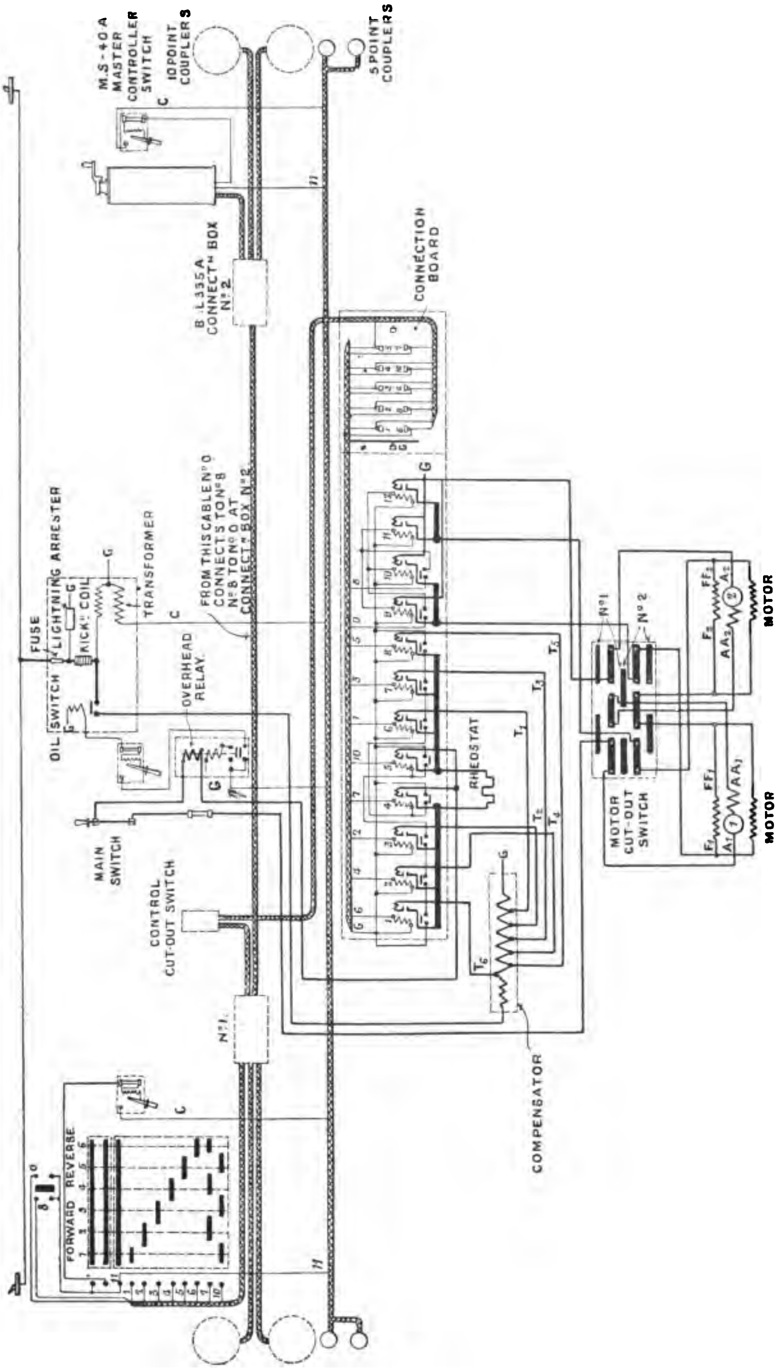


FIG. 125. Diagram of connections of Sprague Thomson-Houston multiple unit control for alternate current series motors.

of the interlocks on switches 1, 2 and 3. Proceeding to the second position of the master controller, train wire 2 is first energised, then train wire 1 is cut off, then train wire 7 is made alive, and finally train wire 10 cut off. These changes give rise to the following movements of the switches: Switch 3 closes, connecting tap T_2 to the motors through the preventive rheostat and switch 5; then switch 6 opens, cutting off tap T_1; the interlocks on switch 6 being thereby closed, the way is prepared for the closing of switch 4, which short circuits the preventive rheostat; finally switch 5 opens. This process is repeated step by step as the master controller is turned.

The overload relay is arranged with an overload coil and a holding coil. In case of an overload in the motor circuit, the armature of the relay lifts, and in so doing opens the circuit through the lower pair of contacts and closes that through the upper. This cuts off current from the operating coil of the main high tension switch, and at the same time supplies current to the holding coil of the relay. This coil keeps the relay up so long as it is supplied with current, so that there is no possibility of closing the high tension switch again, until after the master controller has been put back to the off position.

Examples of single phase control systems for compensated repulsion motors. The general question of the control of compensated repulsion motors has already been alluded to in the previous chapter (see pp. 157, 167). On the first systems installed the motors were wound for high voltage, so that there was no necessity for any step-down transformer. The control of the motors was effected entirely by altering the ratio of the series transformer which supplied the excitation. An example of this method is the installation on the railway* from Spindlersfeld to Nieder-Schönwerde, near Berlin.

In more recent installations, however, this method has been abandoned, and the motors are wound for a reasonably low voltage obtained from the trolley line by means of a step-down transformer. The regulating transformer for the excitation is retained, and the control is divided between this and the main transformer.

Examples of this method are the **Hamburg-Altona Railway** as shewn in figure 126, and, with some slight modifications, the projected installation on the **London, Brighton and South Coast Railway.** In this figure it will be seen that the main transformer has two taps in its secondary winding and the series regulating transformer has three. It will also be noticed that no provision whatever is made for passing from one tap to the next without arcs. The change of voltage is effected simply by opening one connection before closing the next. This method certainly possesses the merit of simplicity, but, naturally,

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* See *Electrical Review*, Dec. 23, 1904, p. 1012.

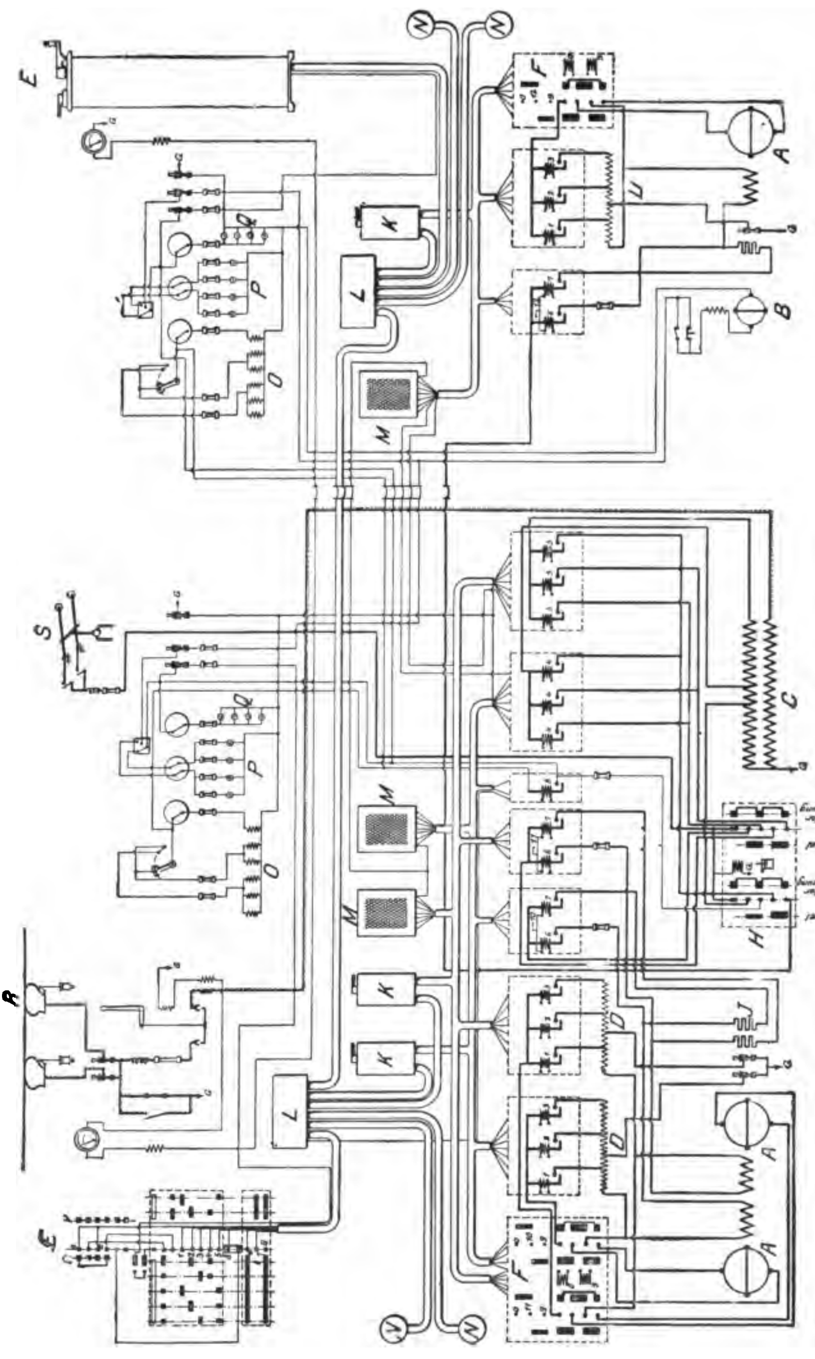


FIG. 126. Diagram of connections of single phase controlf or compensated repulsion motors. (Allgemeine Elektricitäts-Gesellschaft.)

the arcs formed at the switches are more severe than with the other methods. The dotted lines shew the connections for electric braking. The figure gives the diagram of connections for two motor coaches, one of which is equipped with two motors and the other with one. The method of operation is as follows :—Current at the line voltage passes from the collectors R through a choking coil, a high tension fuse, and an oil switch fitted with overload relay direct to the high tension end of the main transformer C, the other end of which is put to earth G. The secondary of this transformer is tapped at two points so as to supply three voltages. The lowest voltage is connected to the change-over switch H, and when this switch is in the proper position for high voltage operation (fingers along the line marked Bügel) connection is made from this tap on the main transformer through a fuse to the auxiliary circuits of the cars, which include the heaters O, the car lighting Q and the signal lights P. A second switch and fuse is provided through which current from this connection is taken to the master controller E, and a third switch and fuse for the pump motor B.

The control current passes through the deadman's switch to the main barrel of the master controller. The path of the current from this can easily be traced in the figure, through the reversing barrel to the main reversers F, and through the main barrel to the contactors 1, 2, 3, 4 and 5, through the connection boxes L, disconnecting switches K, and control resistances M, the numbering of the wires at the master controller corresponding to the numbering of the contactors. The order of switching is as follows :—the middle tap of the main transformer is connected to the motors A through contactors 4, and the motors are regulated by means of the exciting transformers D through the contactors 2 and 3; next the full voltage of the secondary is connected to the motors through contactors 5 and the speed regulated as before by the contactors 1, 2 and 3. There are thus 5 steps of control. Contactors 6 and 7 govern the connections for the ordinary operation of the motors and for their operation as generators for braking. In the latter case the motors are reversed by the backward movement of the master controller past the off position, and the braking resistances J are inserted in the motor circuits. In the off position of the master controller contactor 8 is closed; this supplies current to the heaters O. When the motors are taking current the heaters are cut off, and in this way the load on the power station is to a certain extent equalised. Provision is made by means of the throw-over switch H for working the motors directly from a low voltage circuit through the collector S.

Interchangeable control. An interesting problem arose in connection with the addition of new motor cars on the Hamburg-

Altona Railway. The first equipments consisted of compensated repulsion motors, whereas the new cars were to be fitted with series motors. As the service required a multiple unit system of control, it was obvious that the control of the new cars must be so designed, that

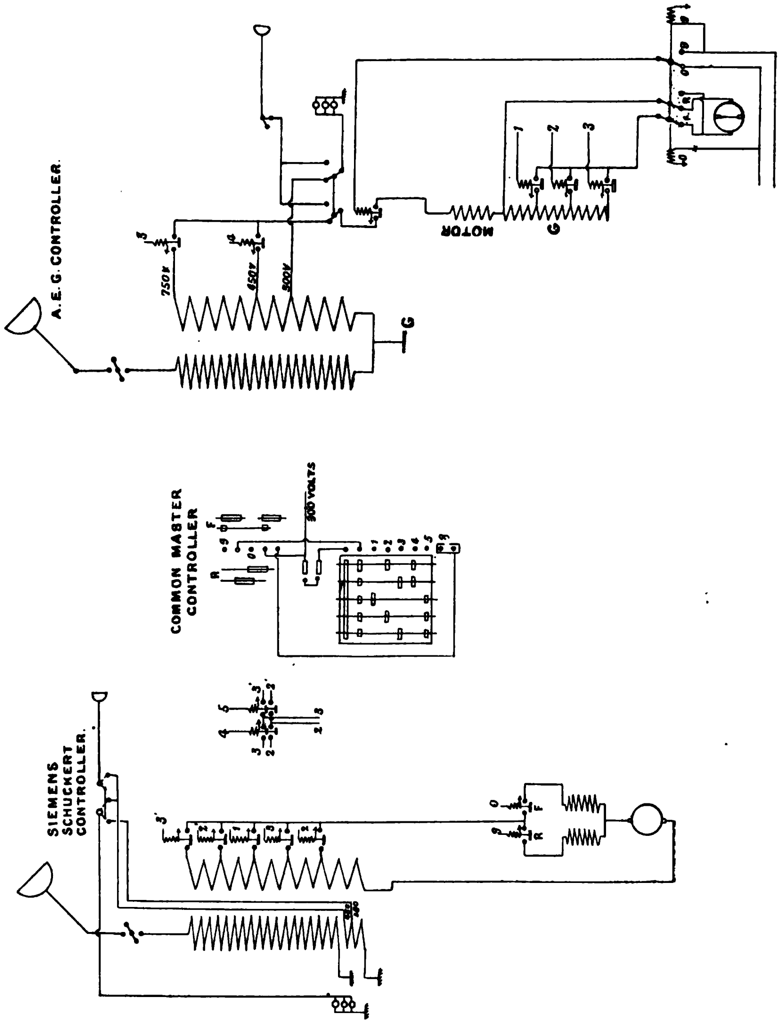


Fig. 127. Diagram of control of single phase series and compensated repulsion motors from the same master controller.

it could be operated by the master controllers fitted to the older cars. Figure 127 shews how the desired object was effected. On the right of the figure the control system of the compensated repulsion motor is represented in accordance with the system which has already been described. In the centre is shewn diagrammatically the master controller with the reversing barrel above the main barrel. To the left is the switching apparatus designed by Messrs Siemens-Schuckert for their series motors.

The essential feature of the problem lies in the fact that the compensated repulsion motor control requires the combination of three switches (connected to taps of the series transformer) with two switches (connected to the main transformer), whereas the series motor requires simply five switches connected to the main transformer. The proper working of these five switches is effected by means of the two-way switch operated by the control wires 4 and 5. These two wires govern the main transformer switches on the repulsion motor system, whereas the wires 1, 2 and 3 govern the regulating transformer switches. In the Siemens control, the wires 2 and 3 govern the first two switches when wire 4 is alive, and 1, 2 and 3 the other three switches when wire 5 is alive. With these few words of explanation there will be no difficulty in tracing out the various connections.

Interlocking. The subject of interlocking is one of considerable importance, and it may safely be said that any system of control is incomplete which does not provide a certain amount of interlocking devices. As to how far this should be carried is largely a matter of individual opinion; it is certainly possible to overdo it, an excessive number of devices generally being likely to add to the possibility of faulty working.

In single phase control systems, however, several details are more or less recognised as being advisable. In the first place, it is strongly to be recommended that all high tension apparatus apart from transformers and collectors should be enclosed in a high tension chamber which can only be opened when the collectors are not touching the overhead wires. Other interlocks are introduced into single phase systems from the corresponding apparatus in direct current control systems.

Thus, for instance, it is not unusual to interlock the main high tension switch with the reverser and the master controller, so that the switch cannot be closed unless the reverser is in its proper position as indicated by the master controller. An example of this has already been shewn in the Westinghouse single phase unit switch system. Similarly the contactors may be interlocked with the main switch and the reverser, so that both these must be in their proper positions before the contactors can operate.

Another instance of interlocking is strongly advisable and is in general use in one form or another, to the effect that it shall be impossible, by reason of the sticking of the contactors, to cause a local short circuit between two taps of the main transformer. In most of the control diagrams given above alternate taps of the transformer are connected through switches or contactors to one of two common leads; thus, for example, in figure 107 taps 1, 3, 5 are connected to lead x and taps 2, 4 and 6 to lead y. These two leads x and y are generally only connected together through a part or the whole of the preventive coil, and it is usually considered a sufficient precaution to interlock together the alternate contactors; thus, for example, 1, 3 and 5 would be interlocked so that only one can be closed at any time, and similarly 2, 4 and 6. This will prevent any direct short circuit, and the worst that can happen is that taps 1 and 6 may be short circuited through the preventive coil.

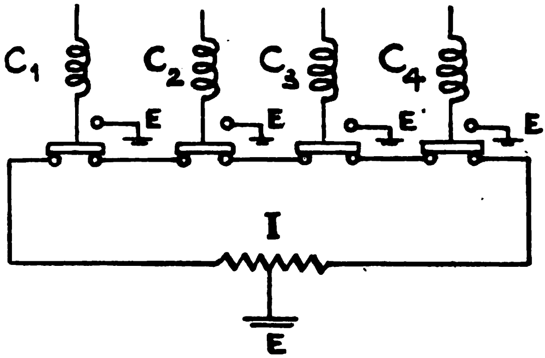


FIG. 128. Diagram of electrical interlocking with chain connection of auxiliary contacts.

The methods of carrying out such interlocking may be electrical or mechanical. The electrical method consists of various arrangements of auxiliary contacts on the contactors, whereby the operating coil of any one contactor cannot be energised unless the auxiliary contacts on the other switches are in their proper positions. An example of this method has been shewn in the Westinghouse control system with unit switches (see figure 123). Another method is shewn in figure 128; each operating coil C_1, C_2, C_3, C_4 is connected to a contact plate on its own switch, moving up when the switch is closed. In the open position this plate bridges a pair of fixed contacts, adjacent contacts being connected up as shewn so as to form a closed circuit through an inductance I, the middle point of which is connected to earth. When any plate is raised by the switch being closed it makes connection with a fixed contact permanently earthed. In this position the switch is held closed so long as current is supplied to its operating coil. Suppose now that switch C_1 is closed and sticks; when the train wire for C_2 is made alive, current flows through C_2 and through the auxiliary contacts of C_3 and C_4 to one side of the coil I. This coil has a very high reactance, sufficient to prevent enough current passing through to close switch C_2; if, however, switch 1 has opened, current from coil C_2 can pass to both ends of the coil I, which thus becomes non-inductive and allows the full current to flow to earth.

Mechanical interlocking is of the slack chain type. Figure 129 shews an arrangement used by Messrs Siemens-Schuckert which possesses the advantages that not only are all auxiliary contacts avoided, but also that any switch which sticks receives a blow tending to open it every time another interlocked contactor tries to close. The apparatus consists of a number of three-armed crank levers between the switches, the vertical arms of all the levers being linked together short links. Sufficient lost movement is allowed in the connections of the levers and links, and in the connections of the levers and the switches to allow of any one switch being closed if all the others are open. The actual apparatus for interlocking may be seen in figure 133, which shows a back view of a group of contactors.

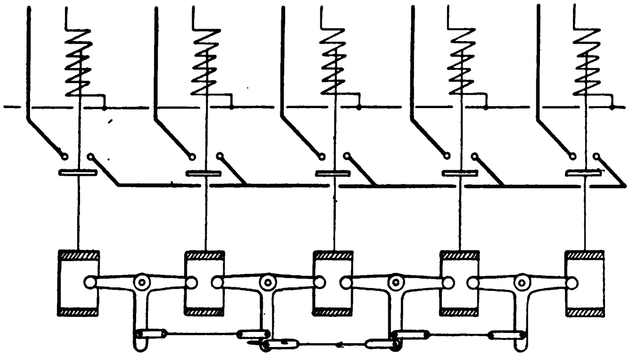


FIG. 129. Diagram of mechanical system of interlocking contactors. (Siemens-Schuckert.)

Apparatus required for carrying out the various control systems. In many ways the actual apparatus involved in the control of single phase motors does not require special mention.

High tension switches are almost invariably of the oil-immersed type, and as such are well known; in like manner overload and distant control relays are familiar from central station switch-board practice. Slight modifications only, chiefly of a mechanical nature, are necessary to fit them for railway cars.

For hand controllers the ordinary barrel form is applicable, and for multiple unit systems the contactors already developed and described in connection with direct current railway motors are available. In most cases the chief difference lies in the fact that the currents to be dealt with are perhaps double the currents in the corresponding case with direct currents. This applies, of course, to series motors connected in parallel, and not to compensated repulsion motors. Another slight difference is due to the fact that the blow-out field is alternating; it is sometimes considered advisable to provide an additional contact in the contactor for short circuiting the blow-out coil when the switch is closed, so as to avoid eddy currents in the contacts.

Single phase magnets and contactors. Probably the most important element of the single phase control as distinguished from direct current control is the electromagnet whereby pneumatic valves or contactors are operated. The magnetic field due to an alternating current is itself alternating, and consequently produces a magnetic pull which fluctuates periodically between zero and twice the average value. Unless steps are taken to prevent it, this produces violent chattering, and renders the magnet practically useless.

To avoid the difficulty direct current magnets may be used, supplied from a small storage battery. This, however, is not necessary, as magnets can be made that will give as great a steady pull as is required.

From another point of view single phase magnets are very satisfactory in that they are self regulating. In a direct current solenoid the current that flows is determined by resistance alone; in the case of alternating currents it is chiefly the reactance that settles the value of the current. With a properly constructed magnet the reactance depends almost entirely on the air gap; in other words, the current is such as to maintain a constant field strength through the magnetising coils whatever the gap may be. That is to say, the current is automatically regulated to maintain a constant pull over a wide range. In practice, leakage of magnetic lines of force which do not pass through the gap and, therefore, do not contribute to the pull, produces a falling off in the pull as the gap increases; even allowing for this, however, the regulation is very effective. As an example a set of curves is shewn in figure 130 for a single phase electromagnet manufactured by Messrs Siemens Brothers at Stafford. In this figure three curves are given shewing how the pull varies with the position of the plunger, with voltages of 109, 101 and 94 respectively applied to the magnet coil. It will be seen that the pull decreases rapidly at first as the gap is lengthened until it becomes practically constant in each case at about 1 inch. The current curves for these voltages shew that the magnets possess the quality of self regulation to a marked degree.

To avoid chattering and violent vibration when the magnet is closed, a slot is cut in the working face of the plunger and a ring of solid copper inserted so as to embrace a portion of the plunger area. This ring is equivalent to a short circuited secondary of a transformer, of which the primary is the magnet coil; the effect of the short circuit is to alter the phase of the magnetic field passing through it relatively to that of the field which passes through the other part of the plunger face. In this way a two-phase magnetic field is produced, and consequently the resulting pull never vanishes. The average pull is of course greater than the minimum or "hold on" pull (*i.e.* the greatest pull which can be applied to the plunger when closed without producing vibration); but it is the latter which should be taken into consideration when dealing with the construction of the contactor. The values for this magnet are 116, 114 and 109 lbs. respectively.

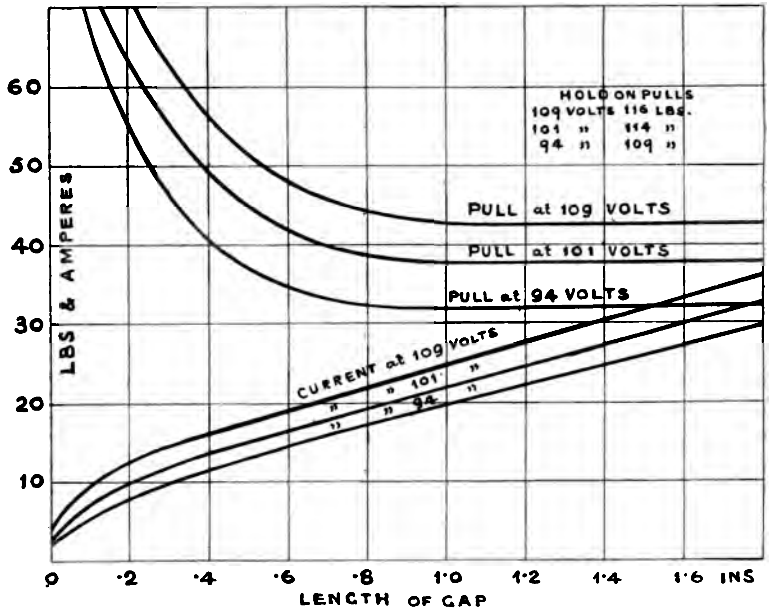
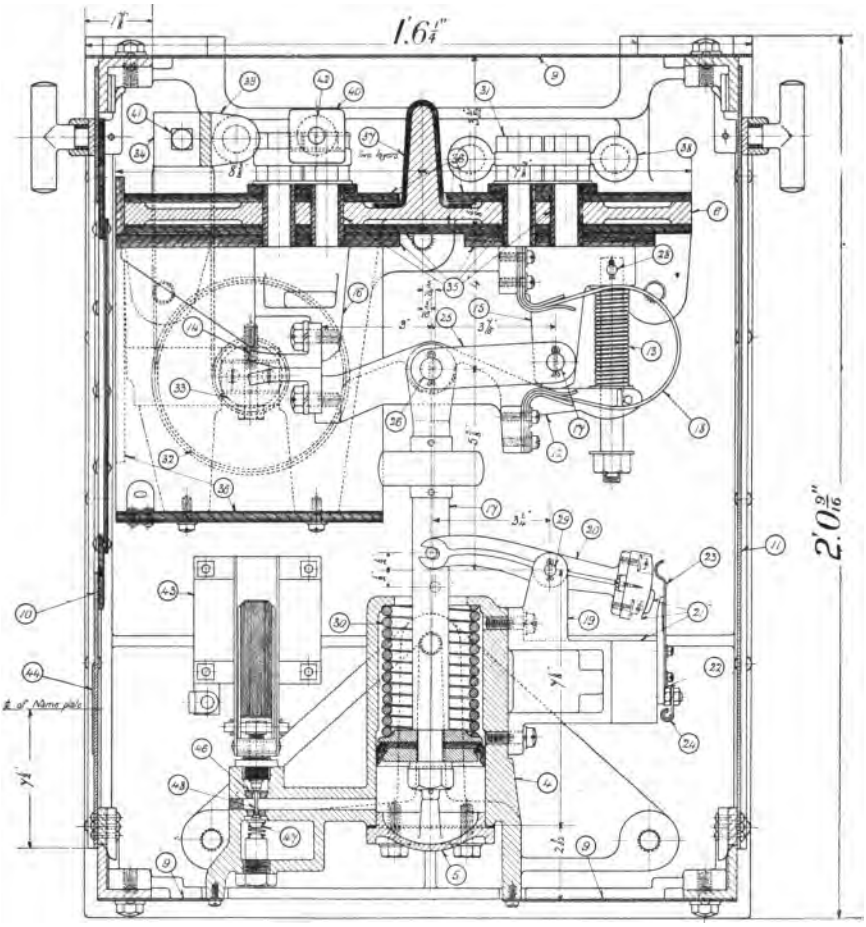


FIG. 130. Curves of pull and current for single phase magnet.

The overall dimensions for this magnet are $10'' \times 6\frac{1}{2}'' \times 5\frac{1}{2}''$. Smaller magnets would naturally be used for pneumatic valves and larger ones could be made for contactors to be used with very heavy currents.

The Westinghouse Company's pneumatic contactor or unit switch for single phase operation is shewn in figure 131. The arrangement is very similar to that of the unit switch for direct current systems, described and illustrated in Chapter 14, Vol. I. Messrs Siemens-Schuckert use a contactor which is operated directly by a single phase



4. Cylinder Casting.
5. Cylinder Cap.
6. Top Casting.
9. Sheet-steel Covering.
10. Door with Screens.
11. Door.
12. Switch-Arm.
13. Spring for Switch-Arm.
14. Contact Tip.
15. Switch-Arm Bracket.
16. Contact Bracket.
17. Piston Rod Assembly.
18. Shunt.
19. Interlock-Lever Bracket.
20. Interlock Lever.
21. Interlock Details.
22. Interlock-Finger Base.
23. Interlock Finger.
24. Interlock Clip.
25. Link for Switch-Arm.
26. $\frac{5}{8}$ in. Pin.
28. $\frac{5}{16}$ in. Pin.
29. $\frac{5}{16}$ in. Pin.
30. Piston Spring.
31. Contact Nut.
32. Blow-out Coil.
33. Blow-out Core.
34. Strap Connections.
35. Insulation for one Unit.
36. Insulation for six Units.
37. 3' 2" × 7½" × $\frac{1}{16}$" Sh. Asbestos.
38. Terminal.
39. Terminal.
40. Connector.
41. $\frac{3}{8}$" × 1⅛" Mach. B with nuts.
42. 1⅛" × $\frac{13}{32}$" × ⅛" Fin. B. Washer.
43. Valve Magnet.
44. Name Plate.
46. Valve Bushing.
47. Valve Spring.
48. Small Valve-Stem.

FIG. 131. Pneumatic contactor for alternate currents. (Westinghouse.)

magnet; it is fitted with a magnetic blow-out which is short circuited by special contacts when the switch is closed. The standard size made by this firm is capable of dealing with a current of 1000 amperes and an E.M.F. of 300 volts; the weight of a single contactor complete is 120 lbs. This contactor is shewn in figure 132, and a group of contactors shewing the interlocking gear in figure 133.

Transformers. The step-down transformer on a single phase motor coach is an important item in the equipment. The design of transformers is a distinct branch of electrical engineering and cannot be entered upon here. It may be of interest to remark here that for intermittent running advantage may be taken of the intermittency to work the iron at a higher density than would be economical for continuous running. This, of course, is only practicable if the high tension circuit of the transformer is opened whenever the motors are not receiving current. If this is not done the transformer design must follow the usual lines, and will, in general, be heavier as a consequence. It should be remarked that unless the high tension circuit is kept constantly closed an auxiliary transformer is necessary for the supply of power for lighting, heating and control. There are advantages and disadvantages in both alternatives, and engineers hold different opinions on the point.

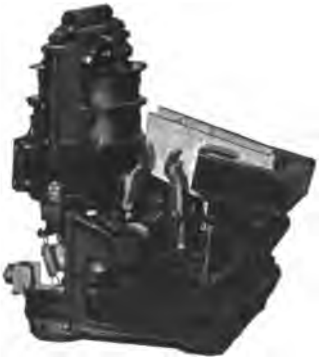


FIG. 132. Siemens-Schuckert single phase magnetic contactor.

Apart from this question, a traction engineer is chiefly concerned with the type of transformer, its weight and dimensions. As far as the type is concerned, it will be enclosed either partially or completely for the protection of the windings and to prevent any accidental contact with the high tension circuit. It may be air cooled by means of motion of the train or it may be completely enclosed in a case filled with oil. The former type has been used with voltages up to 2000,

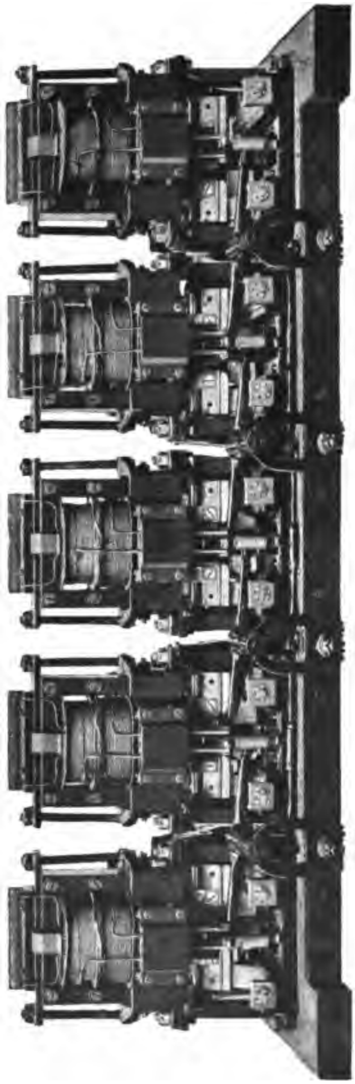


FIG. 133. Back view of a group of Siemens-Schuckert magnetic contactors shewing interlocking gear.

but above that oil insulation has been adopted. As to weights, the following information may be of use, in connection with oil-insulated transformers made by Messrs Siemens-Schuckert.

Weights of single phase transformers for railway work. 25 *frequency.*

Type	Volts		One hour load, K.V.A.	Nett weight without oil, lbs.	Weight of oil, lbs.
	Primary	Secondary			
W.B.T. 100	6000	250	115	2750	260
W.B.T. 200	6000	320	230	4630	440
W.B.T. 350	6000	360	400	6170	·770

These transformers can be wound for primary voltages up to 20000.

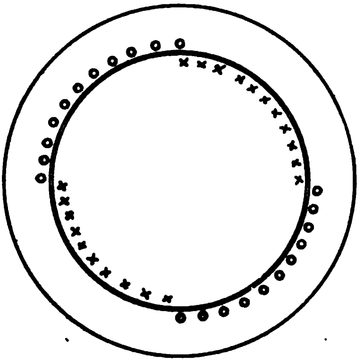


FIG. 134. Diagram of 2-pole induction regulator.

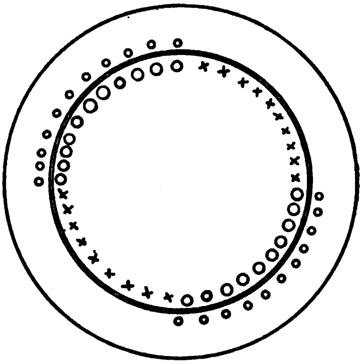


FIG. 135. Diagram of 2-pole induction regulator shewing short circuited compensation winding.

Induction regulator. The induction regulator is generally made in the form of an induction motor. It is obviously more convenient to arrange the primary winding on the rotor so as to avoid passing large currents through moving contacts.

In the position of maximum mutual inductance, the conductors of the primary and secondary windings are, of course, opposite. In mid position, the primary conductors are intermediate between the secondary conductors of adjacent poles as in figure 134 in which the primary winding is marked × and the secondary o.

In this position, there is no mutual induction, and consequently the secondary winding is highly inductive; and since the secondary current is practically independent of the position of the regulator (*i.e.* is not determined by the inductance of the winding as the primary current is), the field produced by the secondary winding alone would highly saturate the iron of the regulator, and give rise to a considerable back E.M.F.

The best way to avoid this is to utilise the vacant space on the rotor for a short circuited winding, marked O, figure 135. This effectually suppresses the cross field in mid position, and is quite inoperative in both extreme positions.

General equipment and suspension of the parts under a car. As an example of the arrangement of the various parts and the method of mounting them under a car, the design prepared for the Rotterdam to The Hague Railway may be cited.

On this line, which is being equipped on the single phase system by Messrs Siemens-Schuckert of Berlin, each motor coach contains two 175 H.P. series motors, and one 10000 volt transformer with a one hour rating of 230 K.V.A. The motors are mounted on the motor bogie and are artificially cooled by forced ventilation from an electrically driven fan in the driver's or guard's compartment. The arrangement is shewn in figure 136 from which it may be seen that the motor cases are connected by means of flexible tubes to the duct between two floors on the car body, into which air is driven by the fan. By this method a supply of air is secured which is practically dust free.

The motors are mounted on the bogies in the ordinary way by means of a nose suspension of the centre of gravity type. The main transformer is suspended under the car near the trailer bogie, the case being bolted up to the underframe. The arrangement is shewn in side elevation and plan in figure 137 and in end elevation in figure 138. The high tension chamber contains the high voltage oil switch with its overload relay, an earthing switch and a high voltage fuse. This chamber is mounted quite close to the transformer, the casing being hinged so that the contents can be inspected from the side of the car. The position is shewn in figure 137 and two sections and a side elevation in figures 138 and 139. In the latter is shewn the interlocking arrangement whereby the chamber is kept closed and cannot be opened unless the two bow collectors on the roof have been lowered so as to break the connection with the overhead high tension wire. The interlock consists of a rod passing up through the body of the car and terminating in a guide on the roof, through which passes also a rod in connection with the collectors. The latter has a recess into which the vertical rod can enter when the collectors are down, and by so doing it withdraws a bolt from the lock of the high tension chamber and therefore sets free the hinged case.

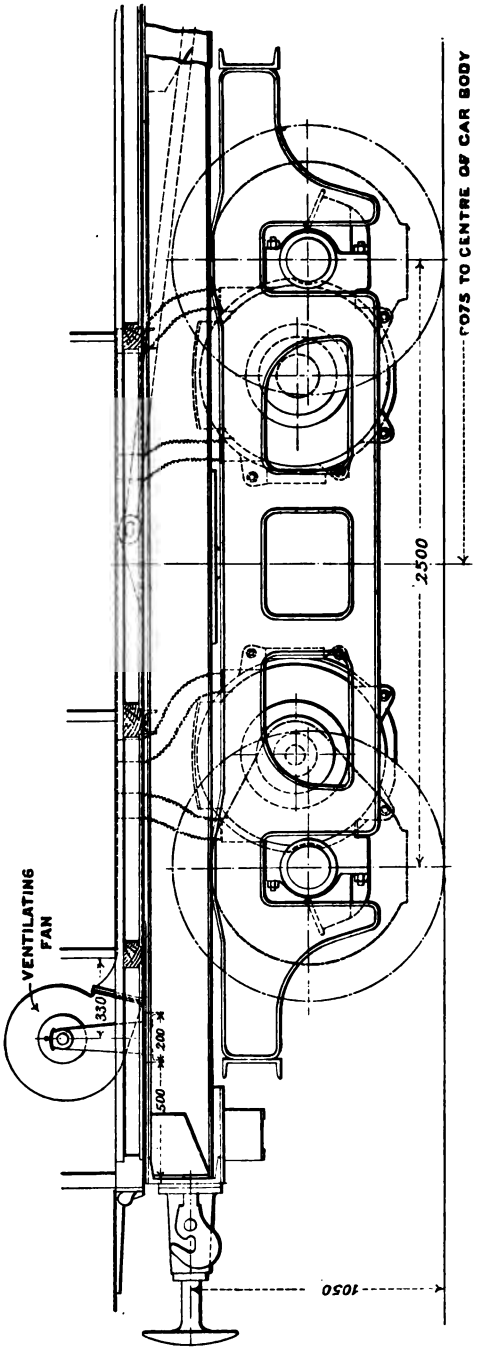
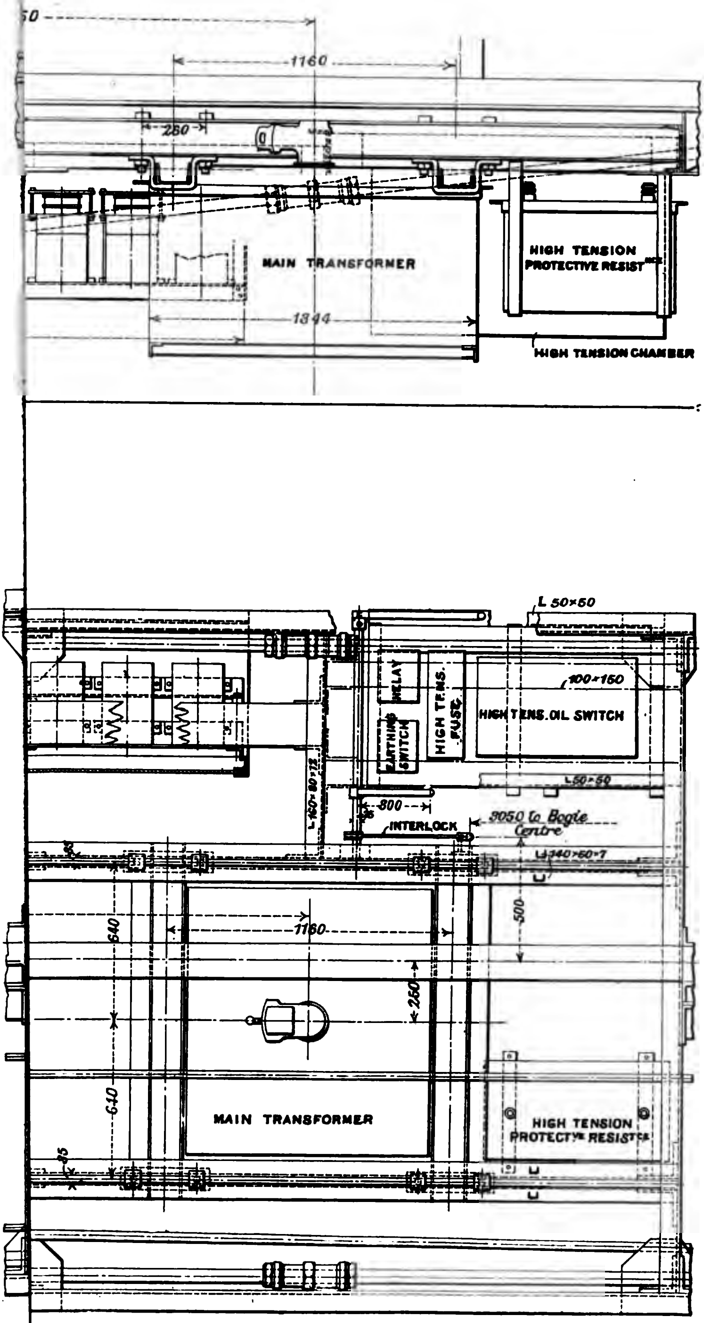


Fig. 136. Motor bogie for the Rotterdam-Hague single phase Railway, shewing the arrangement of forced ventilation.



ague single phase Railway.

The contactors, of which there are in all 20 including those utilised for reversing the motors, are arranged along the side of the car, being enclosed in a case opening downwards from the outside. These contactors are shewn in figure 137 and in more detail in figure 140.

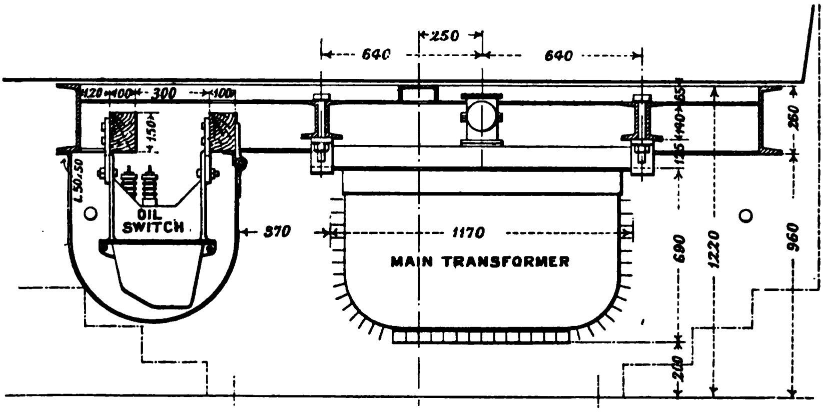


FIG. 138. End elevation of transformer and high-tension switch under the car body. Rotterdam-Hague single phase Railway.

Several other parts are shewn in the drawings including two preventive coils, one for the control of each motor, the resistance for the high voltage switch (see p. 201), and the motor fuses.

Table 2 gives the weights of the various parts.

TABLE 2. *Weights of the various parts of the motor coach for the Rotterdam to The Hague Railway.*

	Weight in metric tons
Transformer	2·80
High voltage switch with relay	·20
High voltage resistance	·20
High voltage fuse	·01
High voltage cables	·20
Bow collectors	1·00
Motors with gearing and suspension	6·00
Low voltage cables	·45
Contactors with case, etc.	1·60
Equipment in driver's compartments ...	·30
Master controllers	·20
Motor-driven fan	·25
Motor-driven compressor	1·00
Miscellaneous parts	·79
Total electrical equipment	15
Bogies	10
Car body	22
Total weight of car without passengers ...	47

Current collectors. The design and construction of current collectors for use with high voltage overhead lines have already been referred to in dealing with the equipment of locomotives and motor coaches on three-phase railways (Chapter 2 of this volume). Collectors for single phase railways differ in some respects from those already described, chiefly in that there is only one voltage to be dealt with, and that with the exception of the Berlin-Zossen line, much higher voltages are employed on single phase lines.

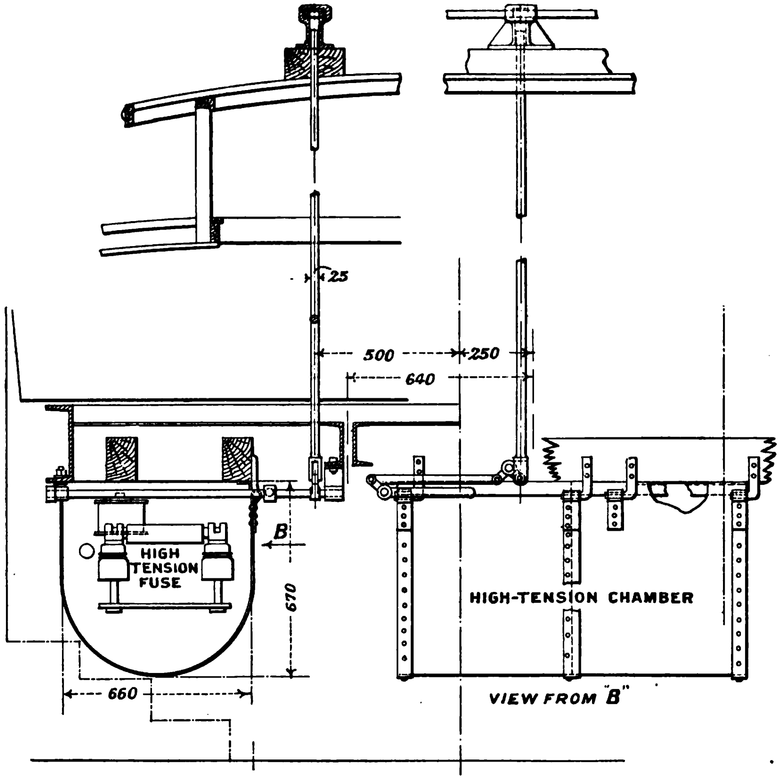


FIG. 139. Side elevation and section of high-tension chamber shewing interlocking device. Rotterdam-Hague single phase Railway.

Two examples of collectors may be given, as although they are somewhat similar, they differ in important details. The first, which is manufactured and used by the Westinghouse Company, is shewn in figures 141 and 142; in the former the collector being raised and in the latter lowered. This type is called the pantagraph type, and consists of a diamond-shaped frame at the top point of which is attached the actual collector, and the bottom of which is pivoted to four fixed insulated supports standing on the car roof. These four pivots are joined across in pairs by rods, these rods being fitted with cranks connected together by a cross arm so that the whole frame is compelled to rise equally and give a vertical movement to the collecting bar.

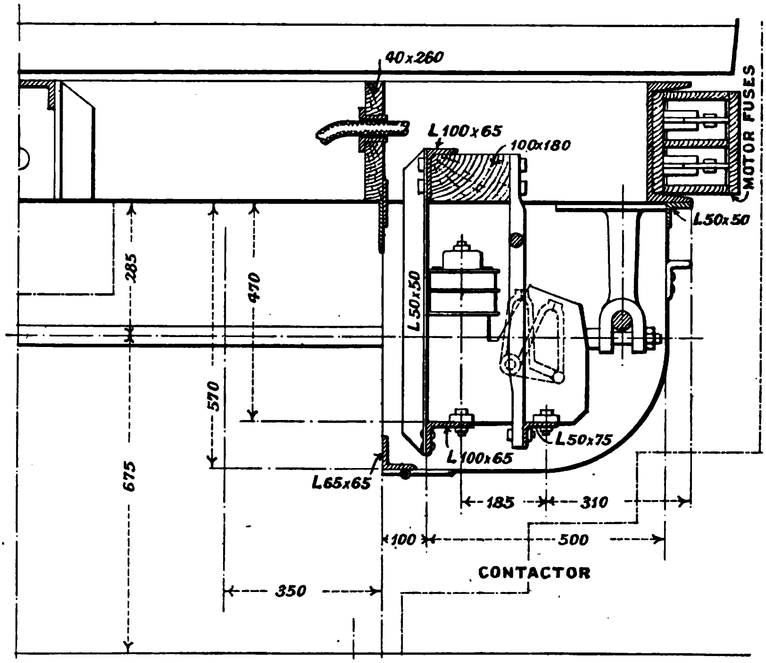


Fig. 140. End view of contactors under the car. Rotterdam-Hague single phase Railway.

The second example is that of the bow manufactured by Messrs Siemens-Schuckert for their motor coaches on the Hamburg-Altona Railway. This is shewn in figures 143 and 144 and consists of a main frame A of what may be called the "reversed pantagraph" type, with a small frame B pivoted to the top of the pantagraph carrying the collector bar. The four fixed pivots of the main frame are mounted upon short longitudinal girders which are attached to the coach roof by means of high voltage porcelain insulators C, the pivots being joined in pairs by cross rods which are connected together by means of a reversed sprocket chain D. A compressed air cylinder E mounted on the roof has its piston connected to a lever F to which are pivoted three rods; two of these marked H serve to apply pressure to raise the two collectors, while the third K is provided for the purpose of locking the high tension chamber (see p. 235). The rod is connected through a high voltage link porcelain insulator L, and a double crank G, and the helical spring M, to the crank lever N. The movement of the piston thus applies a pull to this crank and raises the pantagraph

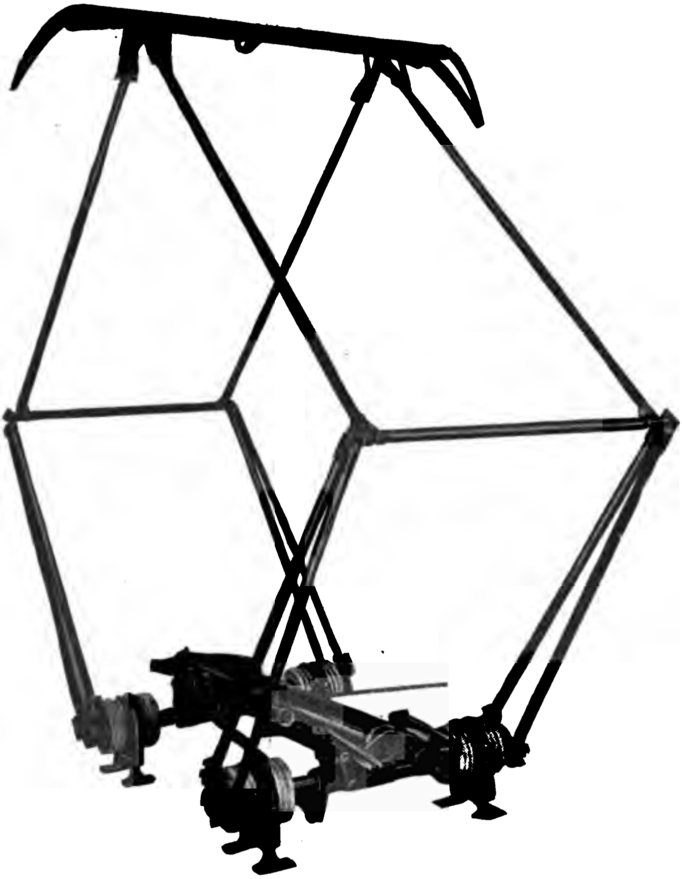


FIG. 141. Pantagraph collector for single phase Railways raised. (Westinghouse.)

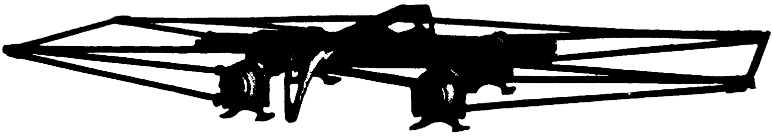


FIG. 142. Pantagraph collector for single phase Railways lowered. (Westinghouse.)

frame. The small frame B pivots about the upper angle of the frame A, being controlled by means of the two helical springs X which tend to keep it in a vertical position. The whole apparatus is made as light as possible, consistent with safety, and is constructed chiefly of bicycle tubing; the contact is made of aluminium and contains a groove for lubrication.

This type of collector can be made suitable for English railways on which the headroom is not so great as that shewn in the figures. Certain modifications will of course be necessary, but these do not present any difficulty that cannot be surmounted.

Alternative methods of utilising single phase current for electric traction. Before the advent of the single phase commutator, motor efforts had been made to devise some means whereby a single phase high voltage supply to the overhead wires could be utilised for traction purposes. The first important proposal was what is known as the Ward-Leonard arrangement which consisted of a motor generator mounted on a locomotive, transforming single phase high tension current into low tension direct current, the latter being supplied to standard direct current railway motors. Other methods which have been proposed include that of Arnold in the United States, which employed a single phase induction motor coupled direct to one of the members of an air compressor, the other member being coupled to the driving wheels; also systems in which some form of rectifier or permutator converts the single phase supply into a direct current supply.

The Ward-Leonard system. As already stated this consists of a motor generator driven from the overhead line, supplying current to direct current motors. This arrangement has obvious advantages and equally obvious disadvantages. The advantages are (1) it enables a high tension single phase distribution to be used, (2) it permits the use of well-known apparatus, such as direct current railway motors, single phase asynchronous motors, direct current generators, (3) it makes possible a very complete system of motor control without rheostatic losses, the voltage of supply to the traction motors being regulated by the excitation of the direct current generators, (4) it provides for regenerative braking with the same system of control, using shunt motors for traction. The disadvantages are (1) a motor generator is necessary which is kept constantly running, and probably a small storage battery to start up the motor generator, (2) unsuitability for a multiple unit system.

This system has not been entirely superseded by the single phase commutator motor, and is still being developed by the Oerlikon

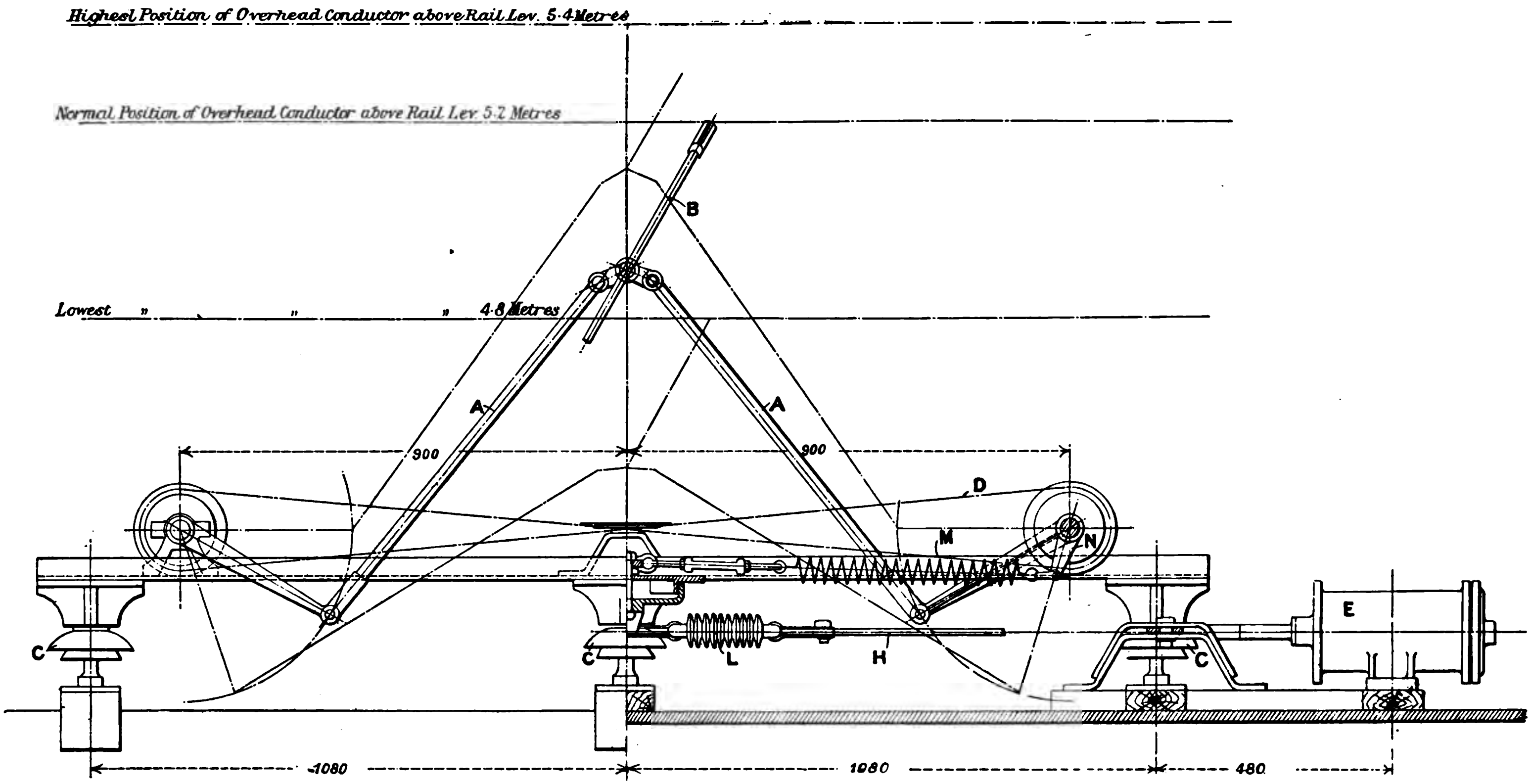

Highest Position of Overhead Conductor above Rail Lev. 5·4 Metres
Normal Position of Overhead Conductor above Rail Lev. 5·2 Metres
Lowest " " " 4·8 Metres
B
A
A
900
900
D
M
N
E
C
C
C
L
H
1080
1080
480

Fig. 143. Side elevation of current collector for the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

Company, who claim that in certain cases it has advantages over any other system. A locomotive designed on this system was built by this Company for the Seebach-Wettingen Railway in Switzerland and was put into regular service in January, 1905.

The equipment of this locomotive consisted of a single phase induction motor direct coupled to a direct current generator. The motor is supplied from a 14000 volt, 50 frequency overhead line, and drives a 400 kw. D.C. generator at about 980 r.p.m. The induction motor has a squirrel cage rotor, and a stator with two distinct windings, one for the high voltage, and the other for 700 volts, the voltage of supply in stations and yards. In the locomotive, there is also a small motor generator of about 6 kw. capacity which is used for excitation of the D.C. generator and for starting purposes. The excitation of the main generator can be regulated by means of a rheostat from zero to a maximum, and the voltage of the D.C. supply can in consequence be varied from 0 to 600 volts. The traction motors are two in number and are each of about 200 H.P. rating. The efficiency of the large induction motor is stated to be 94 per cent., and the power

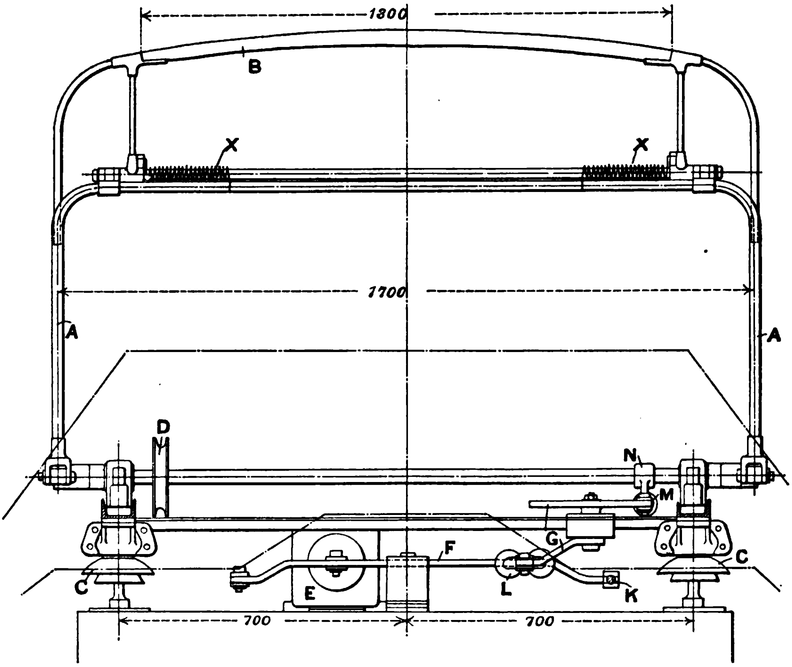


FIG. 144. End elevation of current collector for the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

factor ·89 at full load. The weight of the motor generator equipment is about 10 tons.

The following particulars have since then been made public by the Oerlikon Company in connection with a similar locomotive capable of developing a pull of about four tons, and running up to 44 miles per hour.

Voltage of supply	6000 volts		15000 volts	
	Weight lbs.	Cost £	Weight lbs.	Cost £
Motor generator, with exciter starter and regulating rheostats	33400	1620	33400	1620
Four 95 H. P. traction motors...	17600	1025	17600	1025
Collector, switch gear, etc. ...	1210	164	1210	164
10 kw. auxiliary transformer, fan, vacuum pump, motor and tools	2990	168		
500 kw. main transformer and auxiliaries			8750	428
Total for electric equipment...	55200	2977	60960	3237
Mechanical parts	41800	920	41800	920
Total	97000	3897	102760	4157

Arnold's single phase electro-pneumatic system. This system consists in the combination of a single phase induction motor with a rotary air compressor. The latter comprises two parts which can revolve independently, the pump action depending on the relative rotation of the two parts. The motor is coupled to one part, and the driving wheels to the other. When the car is at rest the induction motor is employed in pumping compressed air into a reservoir, the wheel part of the compressor being held still by the wheels or brakes. If the brakes are released, a torque is applied to the wheels by means of the back pressure of the reservoir, and the car starts running. As the speed rises, the relative motion of the two parts of the pump diminishes, and finally ceases; in this condition the induction motor is utilised solely for traction purposes. So far as the authors are aware this system has not been put into operation on any railway.

Rectifier or permutator systems. Several proposals have been made from time to time to use rectifiers for the conversion of single phase alternating current to direct current. The latest practical development is due to MM. Auvert and Ferrand, engineers of the

Paris-Lyons Mediterranée Railway. The principal novelty of this proposal lies in the use of a regulating rectifier by means of which not only is the current rectified but the voltage is regulated within sufficiently wide limits.

The rectifier itself consists chiefly of a special commutator by which the alternating current is rectified. In former types of this apparatus the commutator has always been a simple construction whose function was to reverse the current every half period. Assuming therefore that this commutator made one complete revolution per period, there would be only two segments suitably insulated from each other. In the new design of regulating rectifier, there are still two principal segments, but there have been introduced between them a number of small segments connected to intermediate points of a choking coil, the terminals of which are connected to the main segments. These small segments serve to effect a gradual reversal of the current, and reduce to a small value the voltage that has to be broken by the brushes. This enables the reversal to be carried out at any part of the period, with the consequence that the alternating voltage produced may oscillate equally above and below the zero line, or may be entirely above the zero line according to the position of the brushes. Figures 145

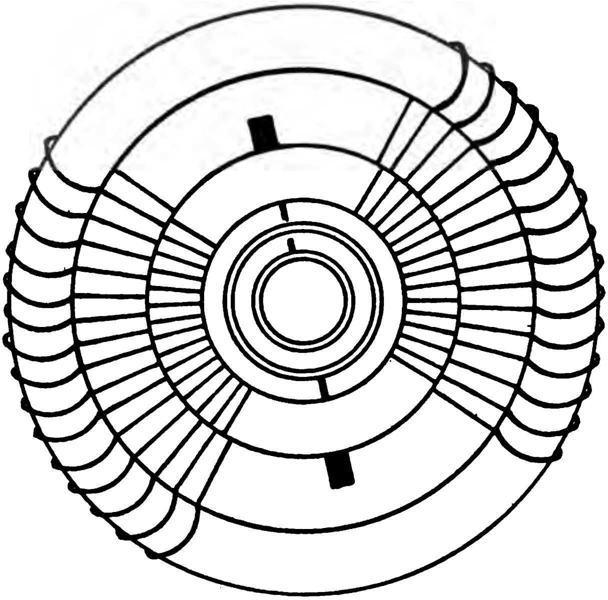


FIG. 145. Diagram of regulating rectifier for single phase Locomotive. (Auvert-Ferrand.)

and 146 illustrate this explanation, the former shewing diagrammatically the revolving commutator with the choking coils, the principal segments being connected by means of slip rings to the two poles of the single phase supply, and the latter the curves of the rectified voltage. The latter may be regarded as the combination of a direct and an alternating voltage; the alternating component may be suppressed by means of a low resistance choking coil in the motor circuits leaving a direct voltage which can be adjusted by the simple operation of moving the brushes.

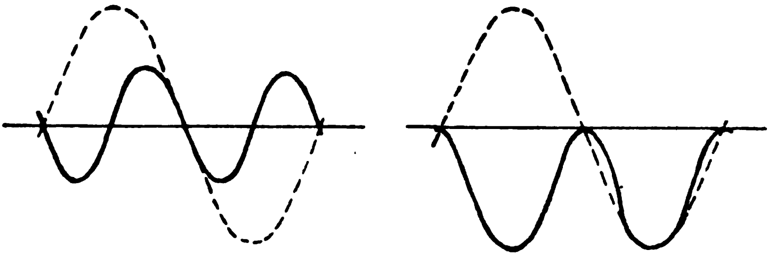


FIG. 146. Diagram shewing wave-forms of the voltage obtained with a regulating rectifier.

The Alioth Electric Company of Paris are constructing a locomotive for the P.L.M. Railway. This locomotive will be equipped with four direct current motors each with a capacity of 450 H.P., and with a permutator of 1650 kw. normal, and 2200 kw. maximum capacity. The weight of this regulating permutator will be about 18 tons. The locomotive will take current from a single phase line at 12000 volts, 25 frequency; the normal speed will be 60 kilometres per hour, and the maximum 80.

CHAPTER 7.

OVERHEAD CONSTRUCTION FOR SINGLE PHASE RAILWAYS.

General. Before proceeding to a study of overhead construction for single phase railways, the student is advised to make a careful examination of Chapter 3 of this volume, in which the various problems connected with overhead trolley wires for three-phase railways have been dealt with.

The chief difference in the requirements of these two systems of traction lies in the number of different voltages involved; in the three-phase system there must be at least two overhead wires to each track at different potentials, whereas with the single phase there is but one voltage to be dealt with, and, provided the return conductor is on the ground, one overhead wire per track will suffice, all overhead wires being at the same potential. This feature of the single phase system is one of its greatest assets, as it eliminates many of the difficulties inherent in the three-phase system.

Apart from this prevailing difference, the requirements of the two systems are very much the same. In both cases high voltages are used, and insulation must be designed accordingly; and precisely the same method of treating tunnels and bridges is applicable to both systems.

As far as actual constructions are concerned, however, there are greater differences than would seem to be indicated above, chiefly due to the introduction of what is known as the catenary suspension. There seems to be no very definite reason why this type of construction should be adopted for single phase rather than for three-phase railways; but the fact remains that this is so.

Catenary suspension. It would appear that the first public suggestion for a catenary suspension was made in a report by Mr Hughes*, on a projected high speed electric railway between Chicago and St Louis, in which it was proposed that the trolley wire should be hung from

* See *Street Railway Journal*, March, 1901, p. 203.

stretched cables, the object of this construction being to keep the trolley wire at a constant height above the ground.

The chief reason, then, for this departure is to be found in the requirements of high speed work. It has been stated* that with the arrangement adopted on the Valtellina Railway a current of 200 amperes per phase can be collected at 40 miles per hour, and that satisfactory trials have been made with 100 amperes at 60 miles per hour. In these trials the voltage was, of course, 3300 volts, and it is generally considered by engineers that at higher voltages, 6000, 10000 volts or higher, it would be better to provide a level wire, or at all events some arrangement whereby only very slight and gradual changes of elevation should be required on the part of the collector.

There are other advantages inherent in this method of support, chiefly the simplicity of the insulation.

As already indicated the peculiarity of the construction consists in the use of an extra wire generally called the "catenary" from which the trolley wire is hung. The catenary is stretched with only a moderate tension so that the sag is relatively considerable; the droppers by which the trolley wire is hung are, therefore, of different lengths according to the position in the span. The object of the large sag of the catenary is to reduce as far as possible the variation of level due to temperature. This matter has already been dealt with in Chapter 7, Vol. I., where the general principles were applied to the trolley wires in use on direct current tramway lines. The catenary wire is generally of stranded steel and the constants differ somewhat from those for hard drawn copper; but, as will be seen from the particulars given below, the tension is usually so moderate that the effect of elasticity is quite small. It is, however, not quite negligible, and the change of level due to temperature variations is kept within narrow limits by the combined effects of a large sag and elastic extension.

The catenary suspension in general. The curve assumed by a suspended wire, from which weights are hung at uniform intervals, is in reality composed of a series of small portions of catenary curves connected together at the points at which the weights are attached. As far as the trolley wire is concerned, the exact shape of these small catenaries is of very little interest; the important point to be attended to is the position of the joints, and the lengths of the various droppers so that the trolley wire may be level.

If the weight of the trolley wire is great in comparison to that of the catenary, it will be sufficiently accurate to treat the curve of the catenary as a series of straight lines. In the alternative case the curve differs very little from a simple catenary.

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* See *Street Railway Journal*, August 5, 1905, p. 192.

The calculation of the lengths of the droppers is most easily effected by using the principle of moments. An example will make the method clear. Take the case of the single catenary suspension shewn in figure 148, which has the following particulars:

Length of span 150 feet.

No. of droppers 10, spaced evenly and symmetrically.

Length between droppers 15 feet.

Weight of trolley wire ·639 lb. per foot—4/0 B. and S. copper wire.

Weight of catenary ·312 lb. per foot—strand of 7 steel wires each ·125″ diameter.

Consider a half span 75 feet long, and take moments in a vertical plane round the catenary support. Calling T the tension in the catenary in lbs., and S the sag in feet, the equation of moments is

$$\begin{aligned} T \times S &= 75 \times {\cdot}312 \times \tfrac{75}{2} + (15 \times {\cdot}639 + {\cdot}5)(7{\cdot}5 + 22{\cdot}5 + 37{\cdot}5 + 52{\cdot}5 + 67{\cdot}5) \\ &= 876 + 10{\cdot}1 \times 187{\cdot}5 = 2770 \end{aligned}$$

(the weight at each dropper being that of 15 feet of wire at ·639 lb. per foot plus ·5 lb. for each dropper and pair of clips).

In this case the construction is erected with a 34 inch sag so that

$$S = \tfrac{34}{12} = 2{\cdot}833 \text{ feet},$$

and therefore

$$T = \frac{2770}{2{\cdot}833} = 980 \text{ lbs.}$$

A table can now be prepared in which the length of each dropper is calculated by finding the sag below each point of attachment and adding 9 inches, the vertical distance between the lowest point of the catenary and the trolley wire.

Number of dropper	Distance from centre of span x feet	Moment of weight of catenary $\frac{x^2}{2} \times {\cdot}312$	Sum of moments due to droppers $10{\cdot}1\,(15+\ldots)$	Total moment	Sag $= \frac{\text{total moment}}{980}$	Length of dropper, inches
1	67·5	710	1515	2225	2·27	36·2
2	52·5	430	910	1340	1·37	25·4
3	37·5	220	460	680	·69	17·25
4	22·5	79	151	230	·234	11·83
5	7·5	9	0	9	·0092	9·11

It is interesting to compare the relation between sag and tension as found above and that which would be found if the weight were supposed to be uniformly distributed as in a simple catenary.

The average weight per foot of the total $= {\cdot}312 + {\cdot}639 + \frac{{\cdot}5}{15}$,

$= {\cdot}984$.

With a half span of 75 feet and a sag of 2·833 feet the constant of a catenary (see page 118, Vol. I.)

$$= \frac{75^2}{2 \times 2{\cdot}833} = 994,$$

and therefore the tension will be 994 × ·984 = 978; thus very little error is made in assuming a simple catenary as far as the calculation of the tension is concerned; but on the same assumption the lengths of the droppers would be 9·33, 12·07, 17·5, 25·6, and 36·5 inches respectively. These values are very close, but are not quite so accurate as the value obtained for the tension.

Variation of level with temperature. For the purpose of considering this question it will be sufficiently accurate to assume that the curve of the steel wire is a simple catenary.

The method of dealing with this problem has already been explained in Chapter 7, Vol. I., and it is only necessary here to apply it to a definite case such as that given in the previous paragraph.

The equation for the curve giving the relation between temperature (t) and the constant (c) of the catenary is

$$1 + \frac{l^2}{6c^2} = 1 - \frac{wc_0}{a\mathrm{E}} + \frac{l^2}{6c_0^2} + (t - t_0)\,\alpha + \frac{wc}{a\mathrm{E}},$$

in which l is the half span in feet,

w the weight per foot,

c_0 the constant of the catenary at a temperature t_0,

α the coefficient of linear expansion,

a the cross section of the catenary wire in sq. inches,

and E the modulus of elasticity in lbs. per sq. inch.

In this case $l = 75$, $w = {\cdot}984$, $c_0 = 994$ at say 60° F., $\alpha = {\cdot}0000064$, $a = {\cdot}086$ sq. inch and $\mathrm{E} = 22{,}000{,}000$.

[This value of E is considerably below the ordinary values for steel; but these values do not apply to stranded cables. The value given here was obtained experimentally by the engineers of the Westinghouse Company*.]

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* Acknowledgements are due to the British Westinghouse Company for this item of information.

The above equation may therefore be written

$$1+\frac{938}{c^2}=1-\frac{978}{1{\cdot}9\times10^6}+953.10^{-6}+(t-60)\,6{\cdot}4.10^{-6}+\frac{{\cdot}984c}{1{\cdot}9\times10^6},$$

$$\text{or } t=\frac{10^6}{6{\cdot}4}\left\{\frac{938}{c^2}+513\times10^{-6}-953\times10^{-6}+384\times10^{-6}-{\cdot}516\times10^{-6}+c\right\}.$$

Substituting different values of c, the corresponding values of t can be obtained, and hence the curve in figure 147 giving the sag in inches for various temperatures.

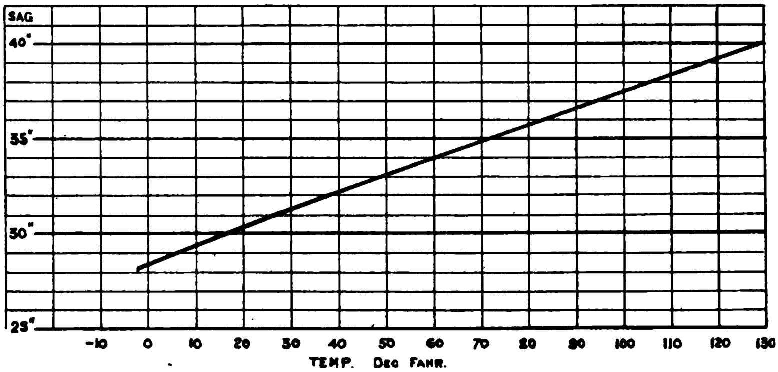


FIG. 147. Diagram of sag for catenary suspension at various temperatures.

A reference to this curve will shew that if the temperature fall to 10° F., that is drops 50° F., the sag of the trolley wire will be

$$29{\cdot}4-34=-4{\cdot}6 \text{ inches},$$

and at 110° F. which corresponds to a rise of 50° F. the sag will be positive and equal to 4·3 inches.

At 60° F. when the trolley wire is level the equation of a catenary will be approximately $y=\frac{x^2}{2000}$. At 110° F., the constant of the catenary is about 880 and the equation is $y=\frac{x^2}{1760}$. Therefore since the lengths of the droppers are the same the equation of the trolley wire will be

$$y=\frac{x^2}{2000}-\frac{x^2}{1760}=-{\cdot}00007x^2 \text{ nearly},$$

the radius of curvature at the centre of the span

$$=\frac{1}{\frac{d^2y}{dx^2}}=-\frac{1}{{\cdot}00014}\text{ feet}=7100\text{ feet}.$$

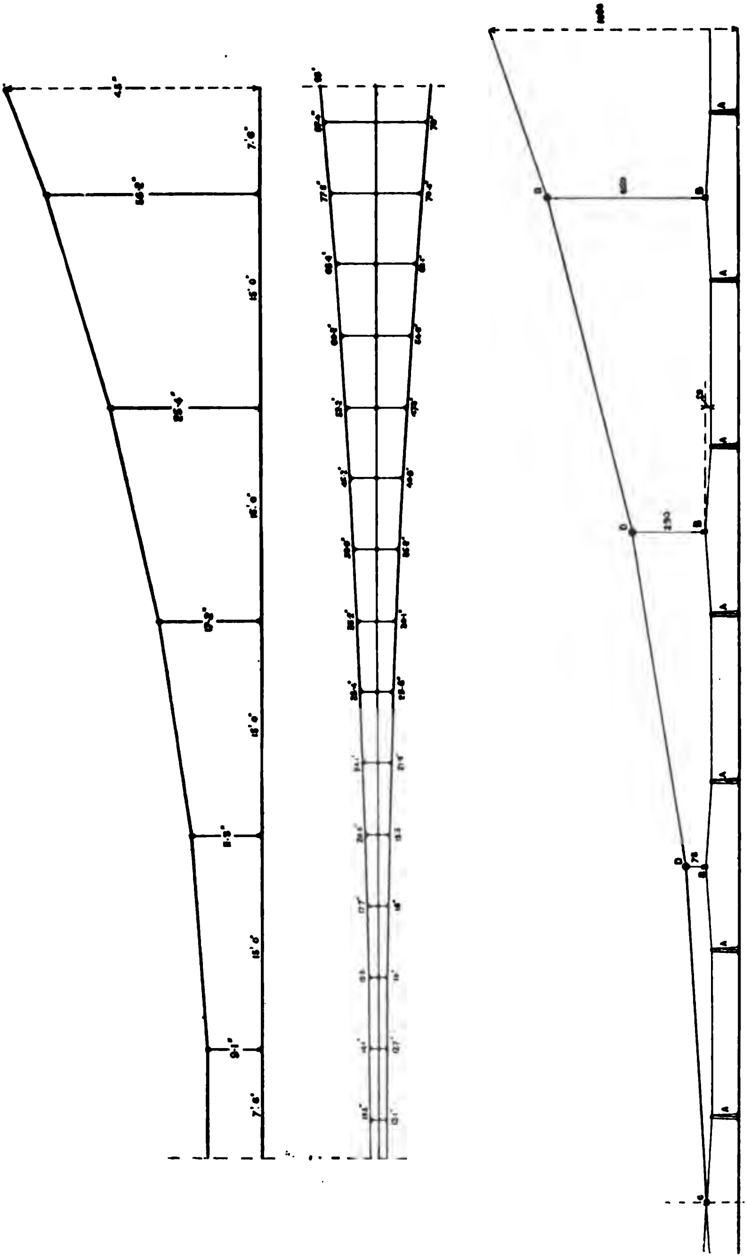


FIG. 148. Diagram of three different types of catenary suspension. (1) Single catenary. (2) Duplex catenary. (3) Compound catenary.

If a collector having an effective mass of 8 lbs. at the point of contact be moving past this point at the rate of 60 miles per hour, the pressure required to maintain contact will be

$$\frac{wv^2}{gr} = \frac{8 \times 88^2}{32{\cdot}2 \times 7100} \text{ lbs.} = {\cdot}\mathbf{27} \text{ lb.}$$

The normal pressure between the collector and the wire would be very much in excess of this; so that under these conditions there would be very little tendency to leave the wire.

Classification of catenary suspensions. Up to the present three types of catenary suspension have been employed which may be called respectively (1) the single catenary, (2) the double or duplex catenary, and (3) the compound catenary.

(1) The single catenary consists simply of two stretched wires, the lower being the trolley wire supported practically parallel to the track by means of droppers from the upper wire. This type is shewn diagrammatically in figure 148, which represents a half span. For the sake of clearness the lengths of the droppers are exaggerated, being drawn to five times the longitudinal scale. The actual lengths are marked in the figure, being the same as have been worked out in the foregoing example.

(1 *a*) With this type of suspension the trolley wire is sometimes duplicated, both being held by one clip to the centre of which the dropper is attached.

(2) The double or duplex catenary consists of three wires. Two of these wires are steel strands hung from points in the same horizontal plane and the third is the trolley wire supported by droppers from the two catenaries. This type is shewn diagrammatically in plan in figure 148; and a good idea of the construction can be obtained from the general view in figure 152. In the diagram, figure 148, a half span is shewn, the length of the span being 300 feet; two sets of figures are marked on the drawing, the upper ones being the distances between the points of attachment of the droppers to the two catenaries, and the lower ones the lengths of the droppers themselves. At each support the two droppers and the cross wire form a triangle, all such triangles being similar. The figures have been calculated on the following assumptions:

Length of span	300 feet
Number of points of support	30
Distance between points of support	10 feet
Horizontal distance between fixed points of catenary	93 inches
Sag of each catenary	72 ,,
Length of shortest dropper	12 ,,

Length of longest dropper	84 inches
Each catenary wire	$\frac{5}{8}$ inch diameter stranded steel
Trolley wire	4/0 B. and S. hard drawn grooved copper

These data correspond approximately to those on the New York, New Haven and Hartford Railroad on which the voltage is 11000 at 25 frequency.

(3) The compound catenary. This type also contains three wires, but in this case they are all stretched in the same vertical plane. The uppermost wire is the steel catenary; suspended from this by droppers is the intermediate wire, which in its turn supports the trolley wire. Figure 148 shews this type of construction, the vertical scale being made five times the horizontal. The lengths of the droppers are marked in the drawing and are calculated on the following assumptions:

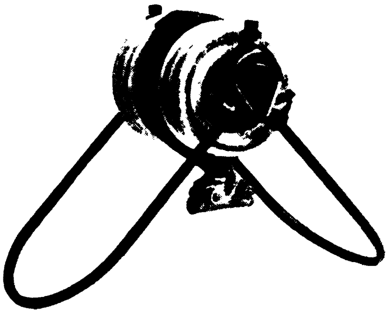


FIG. 149. Catenary Insulator. (Westinghouse.)

Length of span	50 metres
Number of droppers	10
Distance between droppers	5 metres
Distance between points of support of trolley wire	2·5 ,,
Sag of the catenary	·92 metre
Catenary wire	Strand of 7 steel wires each 2·5 mm. diameter
Intermediate wire	Single steel wire 6 mm. diameter
Trolley wire	95 sq. mm. hard drawn figure 8 copper

This type of suspension has been developed by Messrs Siemens-Schuckert in Berlin.

Constructional details. (1) The single catenary has been put up in several places by various manufacturing firms, each of which has worked out its own designs of clips, insulators, etc. The following illustrations shew the various parts of one of the forms designed by the Westinghouse Company. Figure 149 shews the catenary insulator consisting of a hollow porcelain cylinder surrounding an iron sleeve. This sleeve is slipped on to a T iron bracket, and clamped by means of the studs at the two ends of the sleeve. Round the centre of the porcelain cylinder is a metal band which supports the catenary clip.

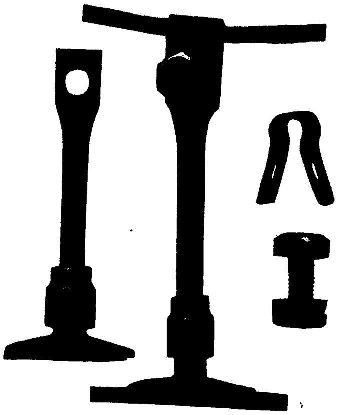


FIG. 150. Details of droppers for catenary suspension. (Westinghouse.)

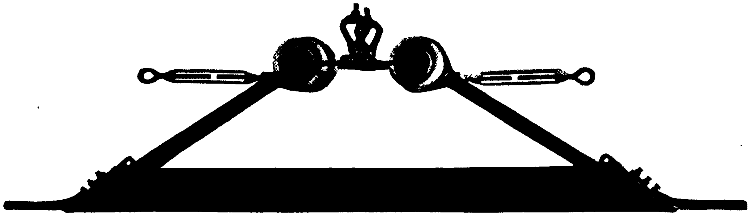


FIG. 151. Section insulator for catenary suspension. (Westinghouse.)

Earthed loops are provided, through which the catenary is threaded, so that in case of a breakage the live wire may come into contact with the loop and be put to earth. The details of the droppers are shewn in figure 150. A section insulator, shewn in figure 151, consists of a long piece of specially prepared hard wood bolted into metal clamps which take the ends of the trolley wire.

(2) The double or duplex catenary. A typical span for this type of construction is shewn in figure 152 illustrating the overhead trolley wire on a railway in the Westinghouse Works at East Pittsburgh, U.S.A. The length of the span is 300 feet, and the catenary supports are

porcelain insulators fixed on the top of a lattice-work arch spanning the tracks.

This type of construction has been adopted for the New York, New Haven and Hartford Railroad which is being equipped for single phase traction by the Westinghouse Company. On this railway there are four and sometimes six tracks running parallel, and steel lattice girder bridges are erected about every 300 feet spanning all the tracks. Every two miles or so, special anchor bridges are put up, which are of heavier section and are secured by anchor bolts to heavy concrete pedestals. At each of these anchor bridges each line is divided by a section insulator, and the ends are connected to oil break switches placed on the lattice bridge. The general arrangement is shewn in figure 153 which represents an anchor bridge spanning four tracks. Four oil switches are shewn, one for each line, by which all four trolley

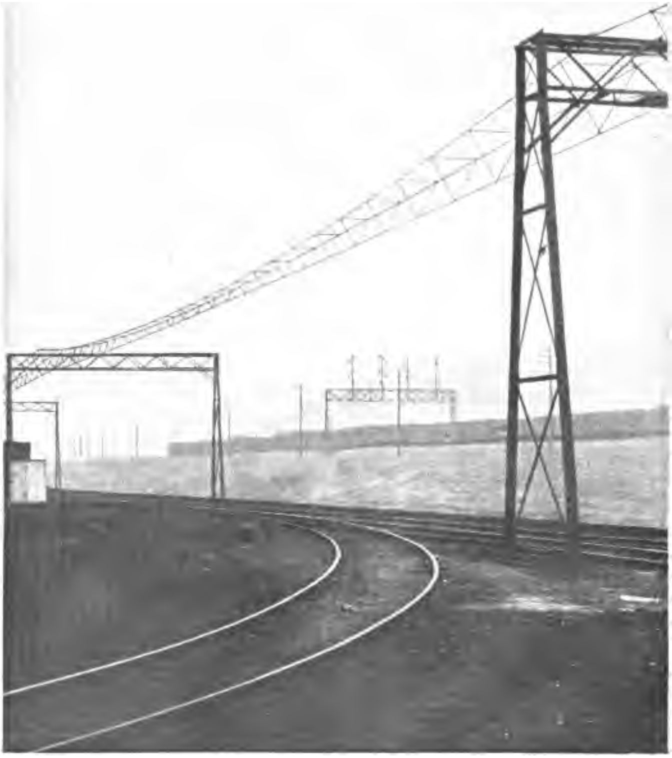


FIG. 152. Duplex catenary suspension in Westinghouse Works, East Pittsburg, U.S.A.

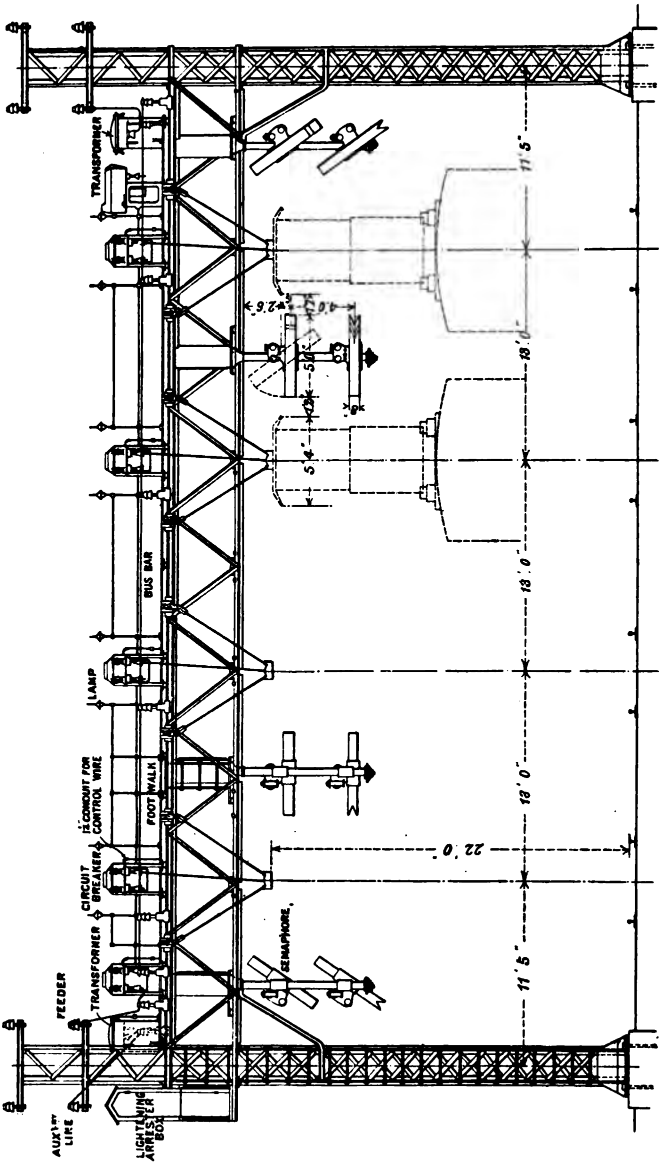


FIG. 153. Anchor bridge on the New York, New Haven and Hartford single phase Railway. (Westinghouse.)

wires are connected in parallel through a bus-bar to two feeders which run the whole length of the line. These switches can be operated by remote control apparatus from the nearest signal box, so that in case of necessity any section can be entirely cut off from the source of current.

(3) The compound catenary. This type of overhead construction has been adopted on the Hamburg-Altona single phase railway, and has been erected by Messrs Siemens-Schuckert of Berlin. The trolley wire voltage in this case is 6000 volts, at 25 frequency.

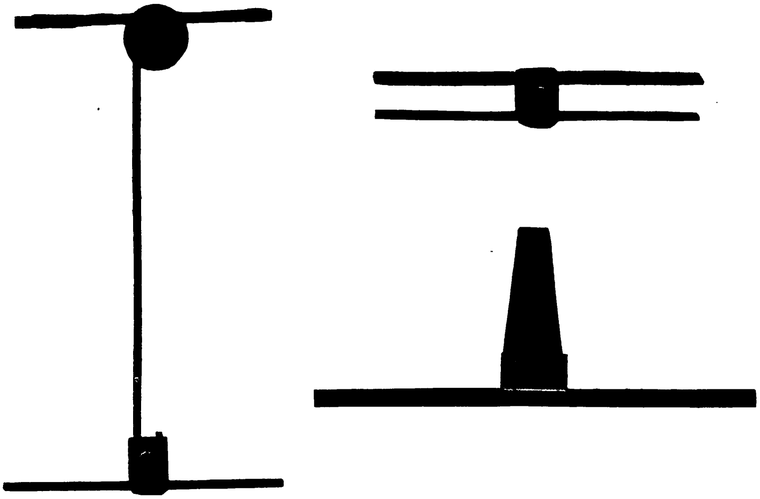


Fig. 154. Trolley wire suspension clips on the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

The catenary wire is clamped by means of a cap mounted on the head of a triple-petticoat porcelain insulator, which is screwed on to a stalk of wrought iron covered with hard rubber. This insulator is mounted on the top of a bracket supported by a lattice or wooden pole or when necessary on a lattice arch over the track.

Droppers are spaced at intervals of 5 metres, carrying the intermediate wire. Every 2½ metres, symmetrically arranged, are special trolley wire clips whereby the trolley wire is hung from the intermediate wire. These trolley wire clips are of special design to permit a slight amount of longitudinal motion of the trolley wire. A constant tension is kept on this wire by automatic tightening gear, and as the temperature varies and the wire expands and contracts the slack is taken up by endways motion. The clips are of four kinds and are marked distinctively A, B, C and D in the drawing in figure 148. These clips are shewn in figure 154; A represents the trolley wire clip, consisting of a clamp at the bottom for the trolley wire, and a loop at the top round the intermediate wire; B is the clip joining the intermediate wire and the dropper; C is a special form of B used at the centre of the span where the dropper is eliminated; and D is the clip clamping the dropper to the catenary wire.

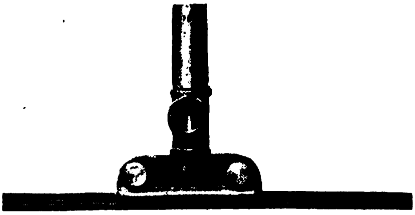


FIG. 155. Trolley wire pull-off for the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

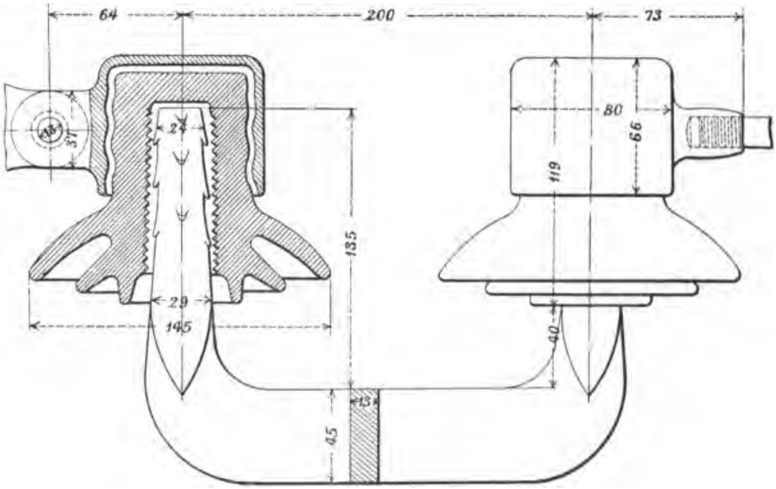


FIG. 156. Porcelain insulator for trolley wire pull-off. Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

It will be seen, on consideration, that this type of construction has important advantages; it reduces the number of droppers to less than half the number of trolley wire supports; it provides a highly flexible connection between the trolley wire and the porcelain insulator supporting the catenary so that any shock is minimised as far as possible; it permits automatic adjustment of the tension in the trolley wire so that it cannot be either too slack or overstrained. It should be noticed

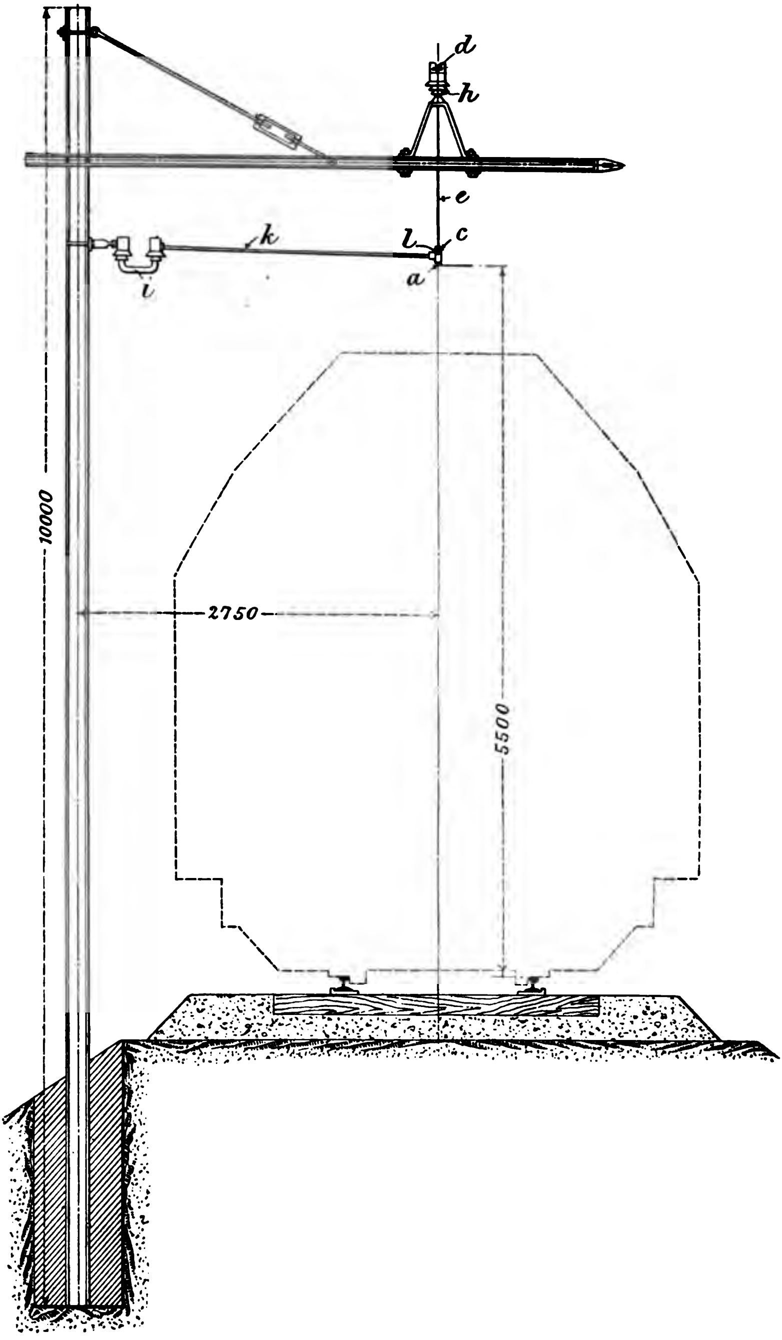


Fig. 157. Typical support for overhead construction on the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)

that expansion and contraction of the intermediate wire will not affect the straightness of the trolley wire.

The line is, of course, zigzag, to avoid undue wear at any point of the collector, and at each post both the intermediate wire and the trolley wire are anchored sideways. This anchorage is necessary also in view of possible swinging due to wind, more particularly in exposed situations; in the ordinary construction without a catenary suspension each support provides a sideways anchorage, and it is advisable to do the same for the more elaborate construction. The pull-off is shewn in figure 155 and consists of a mechanical clip for the trolley wire, combined with a slotted tube for the intermediate wire and a tapped socket for the end of a gas pipe. The other end of this pipe is screwed into the metal cap of a porcelain insulator such as is shewn in figure 156; dimensions being shewn in millimetres.

A typical construction made of simple H iron is shewn in figure 157, which shews the general arrangement of the catenary insulators and the pull-offs, and a general view of several spans is shewn in figure 158.

For the Hamburg-Altona railway a special design has been worked out for a combined section insulator and automatic tightener. The section insulator is not a single piece of apparatus but consists in anchoring off the two ends of the overhead line with a short length of overlap. The two ends are anchored by means of porcelain strain insulators to flexible connections which pass over suitable rollers and are attached to heavy weights which serve to supply the constant tension. The general arrangement of this special work can be seen in the drawing in figure 159, which shews the end elevation of an arch composed of H iron poles joined together by a pair of channel iron girders, and the side elevation and plan of the special span. Two such arches are erected at a distance of 10 metres, and the cross girders of the two arches are connected together by longitudinal girders, there being two such girders over the centre of each track. In this way a horizontal parallelogram is formed, and is supported by four poles. Further stiffening is provided by means of four anchor wires from the corners of the parallelogram to neighbouring posts. Over each corner is fixed a high tension insulator for the support of the catenary wires, which are brought together at the centre of the span, and connected together through porcelain strain insulators as shewn in the side elevation. To the same four corners are brought the intermediate wire and the trolley wire from opposite directions along the two tracks. Along each track the wires which approach each other are slightly displaced sideways so as to avoid any possibility of actual contact, and are then taken slightly up and anchored off at the far end of the quadrilateral through inverted porcelain strain insulators, the intermediate wires to fixed points and the trolley wires to chains which pass over suitable guiding pulleys and terminate in heavy weights arranged for sliding up and down the poles. These sliding weights serve to maintain a constant tension in the trolley wire, and the method of separate insulated anchoring provides a perfect section insulator, the collector bow sliding off from one wire to the other without any jar or shock. It will be obvious that this type of sectionalising can be made as suitable for 20,000 volts as for 2000. The whole arrange-

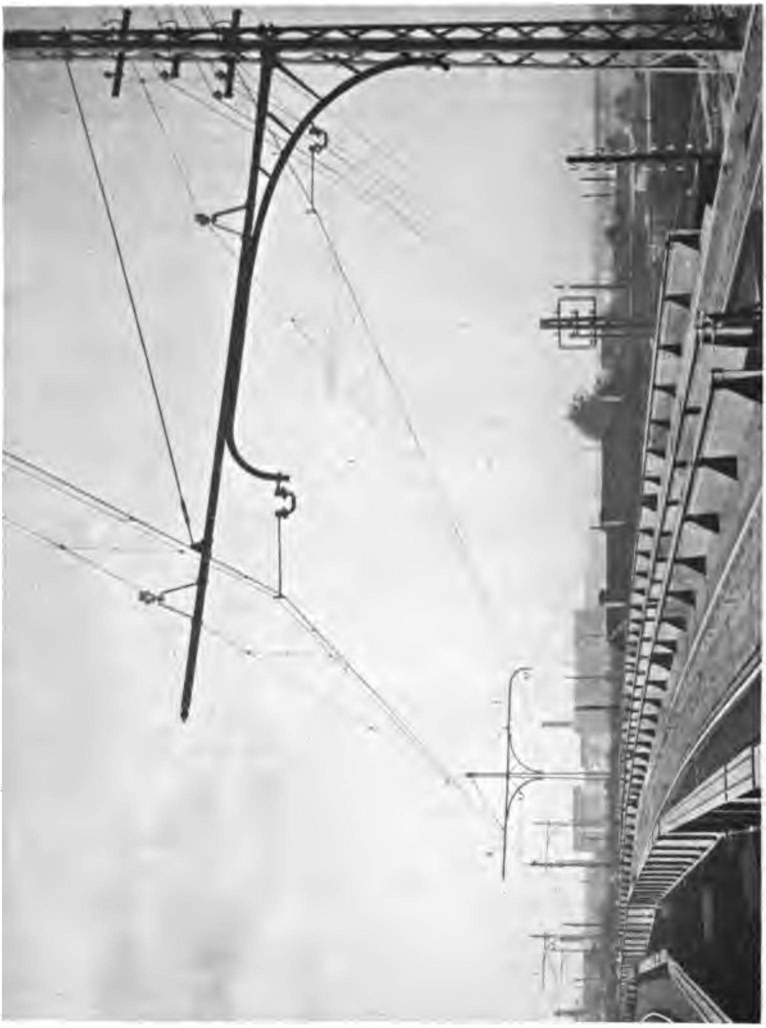
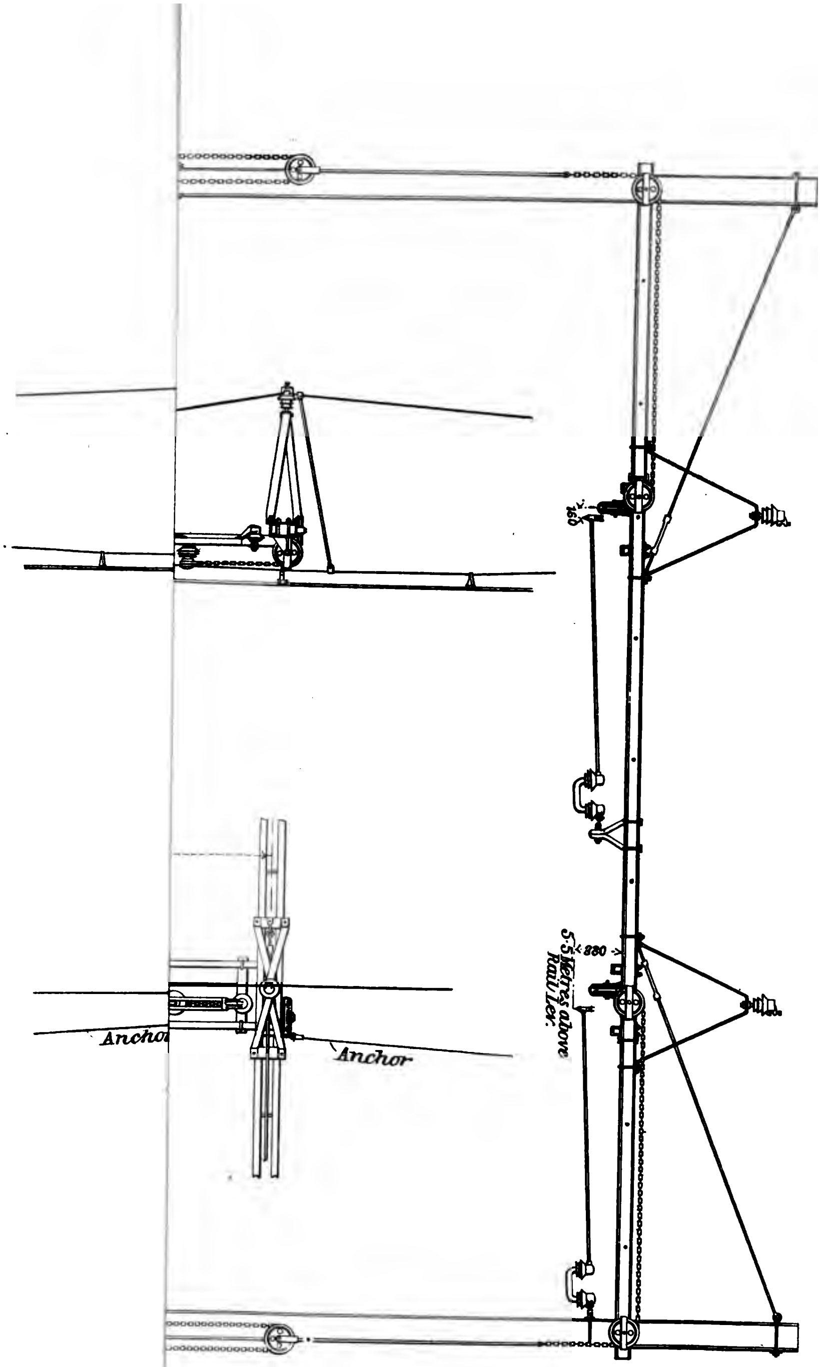


FIG. 158. View of overhead construction on the Hamburg-Altona single phase Railway. (Siemens-Schuckert.)



emens-Schuckert.)

ment can be clearly seen from the three views of the construction in figure 159.

Single phase overhead construction in tunnels and under bridges. The overhead construction in tunnels has been already dealt with in a general way in Chapter 3 of this Volume (to which reference should be made) and more particularly in relation to three-phase railways. On the single phase system, in which only one trolley wire is required for each track, the limitations of space are not, in most cases, so severe as where two wires per track are necessary. In extreme conditions neither system can be installed without modifications to existing structures; but these extreme cases are less likely to occur on the single phase system.

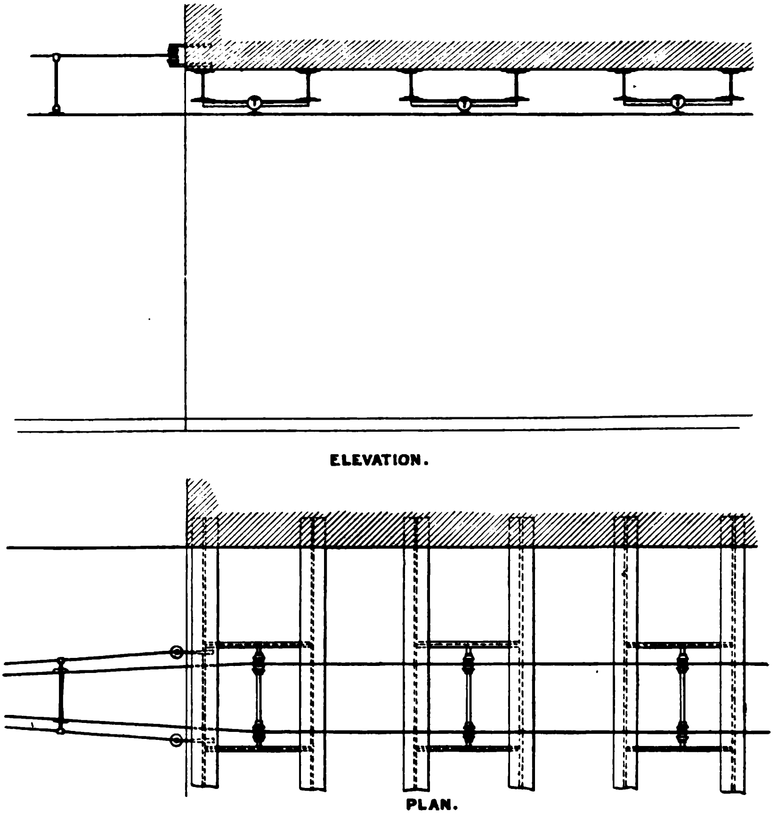


Fig. 160. Single phase overhead construction for a bridge or tunnel. (Westinghouse.)

As a rule, the level of the trolley wire under bridges and in tunnels is lower than in the open; and in designing the overhead construction care must be taken to make the change of level as gradual as possible.

Except in a few cases, where the tunnels are very roomy, it will not be possible to carry the catenary wire through the tunnel; when it is possible, the normal construction can be continued, the catenary insulators being supported by cross beams let into the tunnel walls.

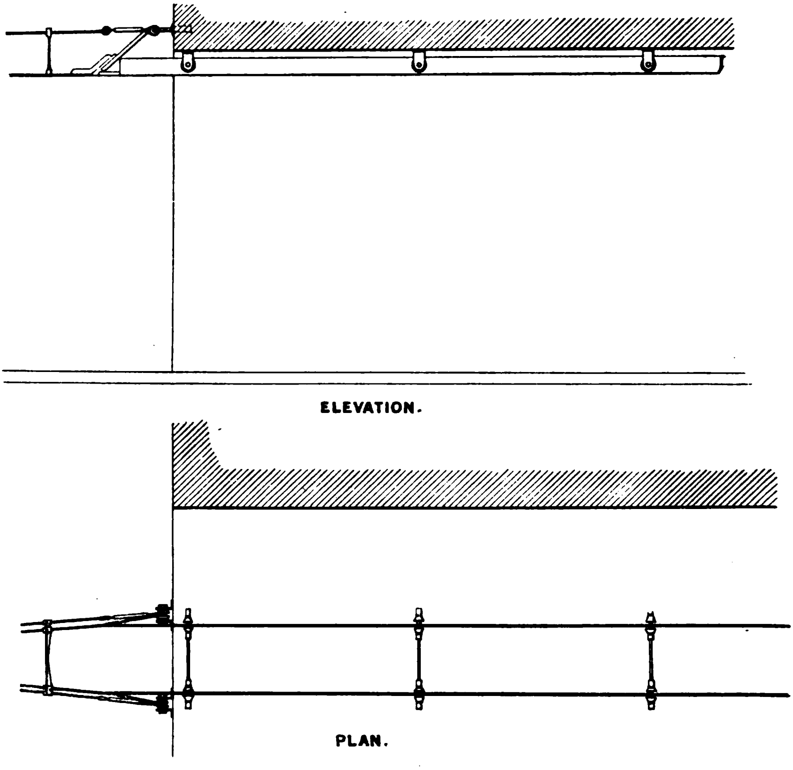


Fig. 161. Single phase overhead construction with dead section under a bridge. (Westinghouse.)

In other cases, the catenary must be anchored at each end of the tunnel or bridge, and the trolley wire carried through by means of some suitable construction, such as span wires insulated with porcelain strain insulators (as shewn, for example, in figure 156).

Figure 160 shews an arrangement for a single catenary construction for carrying wires under bridges, designed by the Westinghouse Company. The catenary and the trolley wire are both bifurcated a short distance outside the bridge or tunnel; the former is anchored

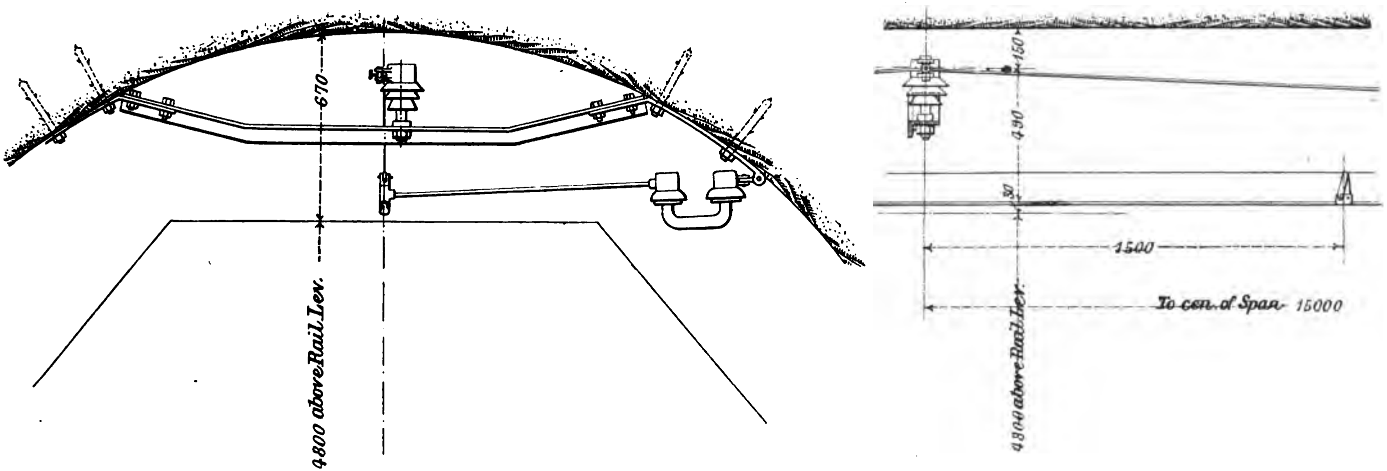


Fig. 162. Single phase overhead construction in single-track tunnel with sufficient head room for normal construction. (Siemens-Schuckert.)

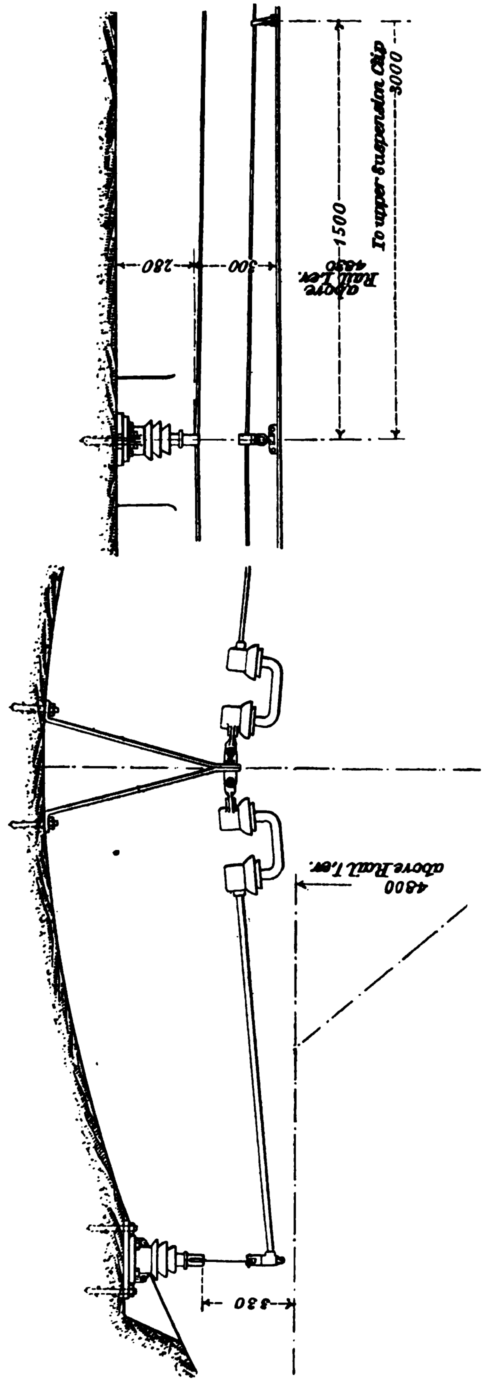


Fig. 163. Single phase overhead construction in double-track tunnel without sufficient head room for normal construction. (Siemens-Schuckert.)

by porcelain shackles attached to the structure of the bridge and the latter is carried through on porcelain insulators placed at frequent intervals as shewn. This method of bifurcation is specially adapted to low tunnels, so that the live wires may be brought close to the rolling stock gauge without being directly over the locomotive funnel.

A similar design is shewn in figure 161 in which the section under the bridge is earthed and consists of two hard wood boards. The trolley wires at both ends are bifurcated as before and finished off by section insulators attached to the bridge ends.

Figure 162 shews the method of construction of the overhead line in a single track tunnel in which there is sufficient head room to keep to the usual arrangement, and figure 163 gives similar details for a double track tunnel in which there is not sufficient clearance to permit the use of the ordinary support of the catenary wire; dimensions in both cases being given in millimetres. It will be seen that in the former case there is a clearance of 15 centimetres between the tunnel roof and the highest point of the catenary, and 64 centimetres between the tunnel roof and the trolley wire; in the latter case the distance between the trolley wire and the tunnel roof is only 49 centimetres. The position of the catenary wire relative to the insulator is reversed in the latter case, the insulator being supported from the roof by means of its cap and supporting the catenary wire by its stalk, which is held in the insulator by means of cement. Both these designs are due to Messrs Siemens-Schuckert.

The construction at iron overbridges is important, and should be studied with great care. These bridges are mostly for public use, and it is essential that precautions should be taken to prevent accidents from shock, due to passers-by reaching out to or interfering with the high tension wires. Moreover it is usually the case that such overbridges only clear the rolling stock by a small clearance, and there is a danger that sparks formed between the trolley wire and the collector of a passing car may set up a continuous arc between the wire and the bridge. To prevent this Messrs Siemens-Schuckert interpose between the trolley wires and the bridge an iron plate carried from the bridge by means of insulators. This plate projects beyond the limits of the bridge far enough to preclude any possibility of any arc being formed. Figure 164 shews the general arrangement of this protective shield, which consists of corrugated iron laid between two parallel angle irons resting on cross bars supported by porcelain insulators. This shield is absolutely isolated from the high voltage wires, and cannot therefore cause any shock. This figure shews also the anchoring of the catenary wire, and the protecting plates put round the anchorages to prevent any interference from the bridge. A similar arrangement is shewn in figure 165 for a simple footbridge; in this case however the catenary

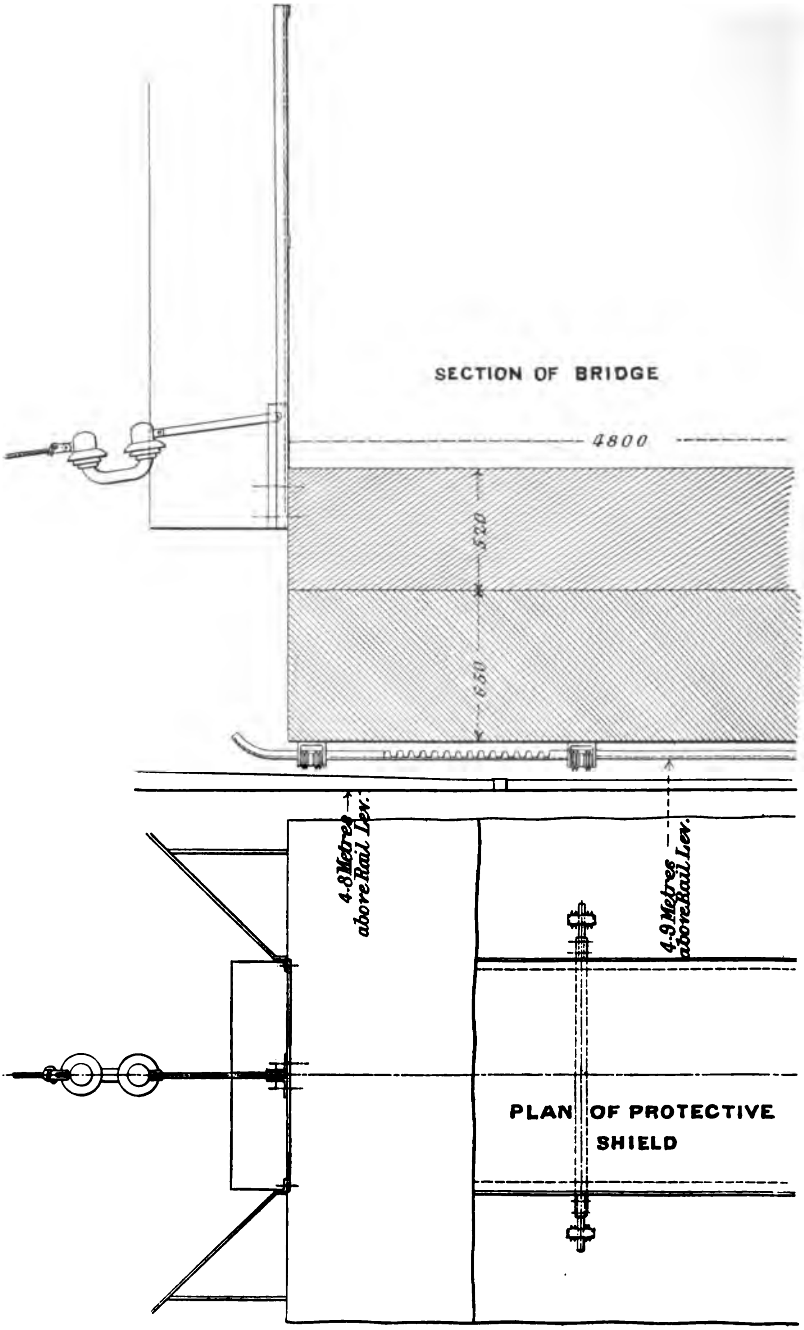

SECTION OF BRIDGE
4800
520
650
4·8 Metres above Rail Lev.
4·9 Metres above Rail Lev.
PLAN OF PROTECTIVE SHIELD

FIG. 164. Protective shield between high-tension overhead construction and over-bridge. (Siemens-Schuckert.)

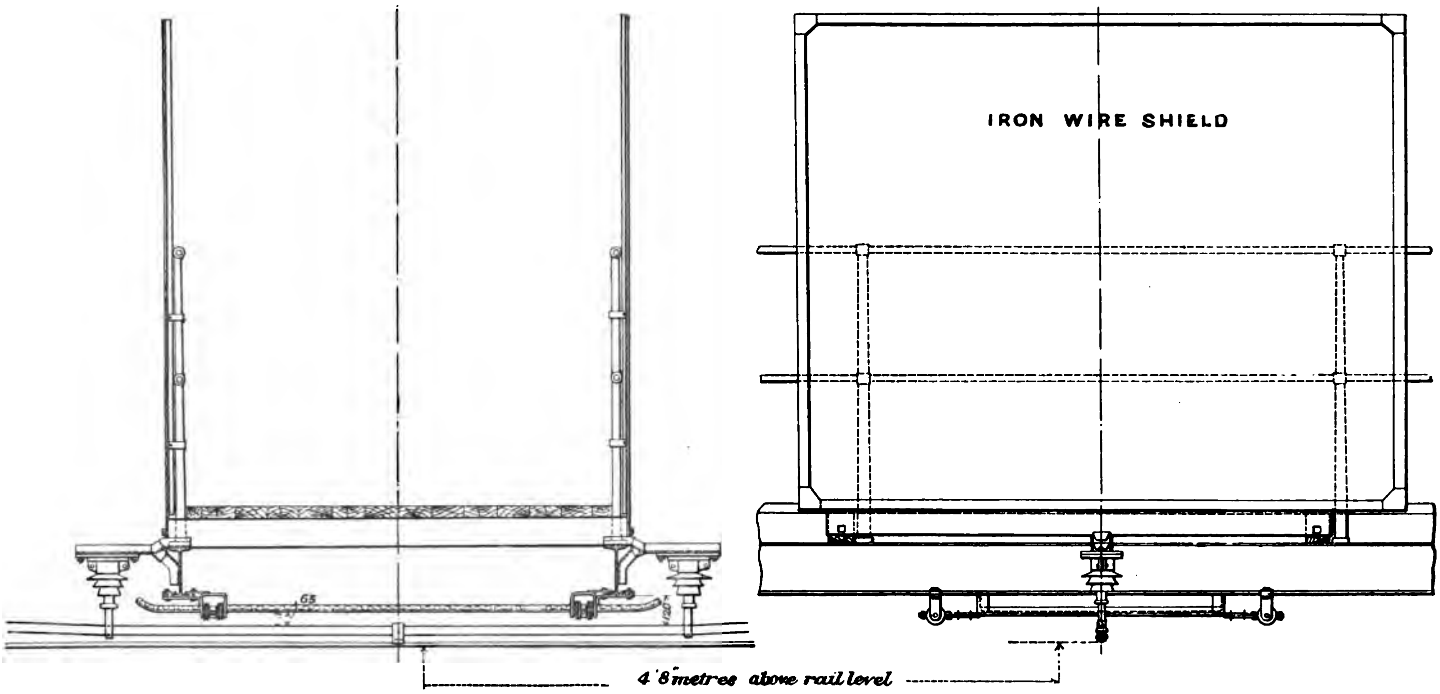


FIG. 165. Protective shield and catenary supports at a foot-bridge. (Siemens-Schuckert.)

wire is not anchored off but taken through under the bridge as shewn. A large wire screen is erected at each side of the bridge to prevent access to the wires.

Single phase overhead construction at junctions and cross-overs. The question of overhead construction at junctions has been discussed with reference to the three-phase system in Chapter 3 of this Volume; and it is only necessary here to point out the great simplification which can be made by the elimination of the second voltage overhead. A comparison may be made between figures 49 and 50 in Chapter 3, and figures 166 and 167 in this chapter. In the former case it was necessary to insert carefully designed lengths of insulated wires to avoid short circuiting the two voltages; in the latter case no dead wires are necessary and the construction can be simplified a good deal. These alternatives do not really provide a fair comparison of the two systems in this particular, as the designs are made to suit different conditions. In the three-phase case the apparatus is as simple as possible, chiefly in this respect, that the insulation between a live and a dead wire consists only of a piece of hard wood. If such an arrangement were adopted for the single phase system, the construction would be much simpler than that shewn in the figures. On the contrary, in these figures it has been assumed that the voltage is too high for such an arrangement, and section insulators are provided of the overlap type shewn in figure 159.

In figure 166, which shews one arrangement for a cross-over road, the two catenaries, one for each track, are continued straight on past the points, and a special catenary is put up, stretched between the two anchor poles shewn in the figure. The trolley wire over each track is brought up to the point of divergence, and is there diverted from one direction along the cross-over road to a section insulator in the centre, and from the other direction out to the anchor pole on that side. In this way, the catenaries are maintained in the normal positions and directions, and a continuous support is provided for the collector at a uniform level.

This arrangement provides for isolation of the "up" and the "down" line; but in case this is not required the construction can be simplified.

Figure 167 shews a similar arrangement for a double track junction.

Switchgear for the overhead line. Connections between different sections of the overhead line may be made by remote-control oil switches as in the arrangement shewn in figure 153, or by air switches mounted on the tops of the poles and operated by rods from the nearest signal box. Figure 168 shews a type of high tension air switch with horns designed for this purpose by Messrs Siemens-Schuckert.

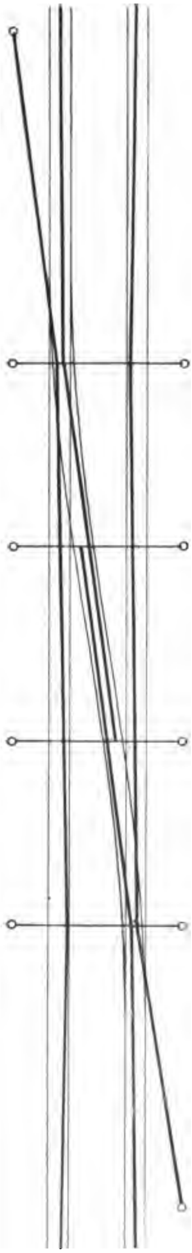


FIG. 166. Arrangement of single phase overhead construction at a cross-over road.

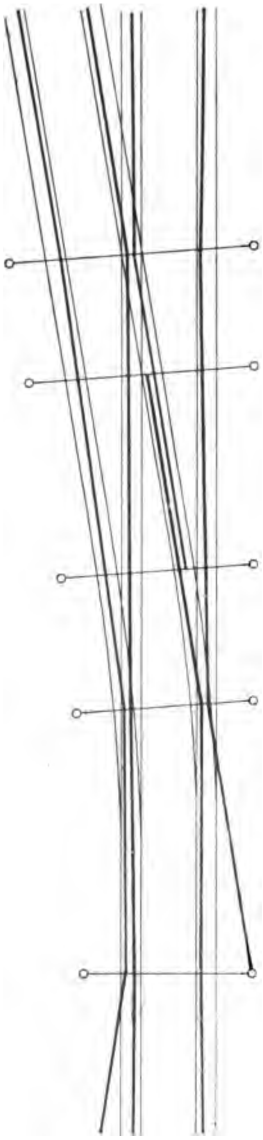


FIG. 167. Arrangement of single phase overhead construction at a double-track junction.

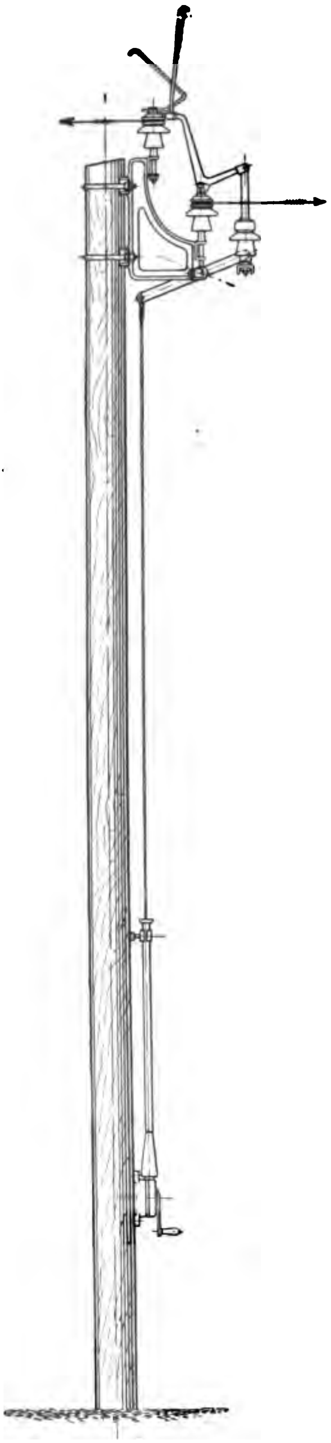


Fig. 168. High-tension switch for mounting on the top of a pole. (Siemens-Schuckert.)

CHAPTER 8.

FEEDER SYSTEMS FOR ALTERNATING CURRENT ON ELECTRIC RAILWAYS.

Drop of volts in alternating current transmission. It must be clearly understood in dealing with the transmission of power by means of alternating currents that the voltage dropped in the transmitting leads is by no means necessarily the same as the drop in voltage at the far end. The latter is simply the difference between the voltages at the transmitting and at the receiving end, entirely irrespective of phases. It is proposed here for the sake of clearness to call the potential difference between the two ends of any one of the transmission leads the "*impedance voltage*" in that lead as being the resultant of the resistance and the reactance volts, and to call the difference between the voltages at the receiving end and at the transmitting end the "*transmission drop.*" Both these terms may be expressed in volts direct or as percentages of the voltage at the generating or transmitting end.

The transmission drop. Taking this first as being the simplest, it is necessary, before calculating the drop, to know the two components of the "impedance volts," and also the power factor of the load.

Let r represent the resistance of the transmission lines in a single phase circuit,

x the reactance of the same lines,

z the impedance $(= \sqrt{r^2 + x^2})$,

and $\cos\phi$ the power factor of the load at the far end. Figure 169 shews the vector diagram for deducing the voltage at the transmitting end from that at the receiving end.

In this diagram OV represents the voltage at the receiving end, and OC the current, lagging behind OV by the angle ϕ. Va at right angles to OC represents the reactance volts and aV_1 parallel to OC represents the resistance volts in the transmission lines. Then OV_1 will be the voltage at the generating end, and if a circle be drawn through V with centre O, cutting OV_1 in b, V_1b will represent the "transmission drop," and VV_1 the "impedance voltage," in the go and return.

By reference to figure 169 it will be easily seen that

$$OV_1^2 = (OV + Va\sin\phi + aV_1\cos\phi)^2 + (Va\cos\phi - aV_1\sin\phi)^2.$$

Now in almost all cases in actual practice, no appreciable error is made by neglecting in this formula $(Va\cos\phi - aV_1\sin\phi)^2$; in which case

$$OV_1 = OV + Va\sin\phi + aV_1\cos\phi.$$

That is to say the "transmission drop" $OV_1 - OV$ is equal to

$$Va\sin\phi + aV_1\cos\phi,$$

or in other words

$$\mathbf{Cx\sin\phi + Cr\cos\phi}.$$

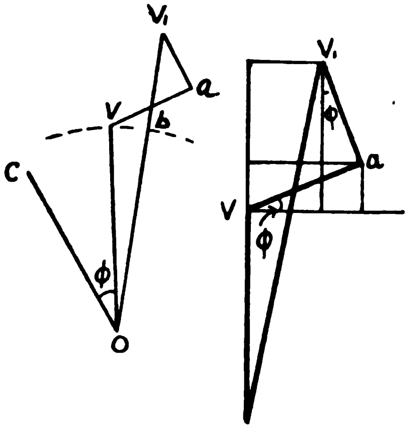


FIG. 169. Vector diagram of the voltages at the end of a transmission line.

For facility in calculating this expression, the relation between cos ϕ and sin ϕ is given graphically in figure 170, from which, knowing the power factor, *i.e.* cos ϕ, sin ϕ can be read off.

For example, if the load be as follows: 150 kw. at 6000 volts ·75 power factor, and the constants of the line be 10 ohms resistance and 15 ohms reactance, the transmission drop will be calculated thus:

Current transmitted $\frac{150}{\cdot 75} \times 1000 \div 6000$ = 33·33 amperes
Resistance volts = 333·3
Reactance volts = 500
Cos ϕ = ·75
Sin ϕ = ·66
Transmission drop = 333·3 × ·75 + 500 × ·66 = 580 volts
Transmission voltage = 6580 volts
Impedance volts in transmission line = $\sqrt{333{\cdot}3^2 + 500^2}$ = 602 volts

Impedance voltage. As already explained the impedance voltage is the resultant of the resistance volts and the reactance volts.

In the simplest case, viz. that in which the transmission line consists of two parallel copper wires, the resistance term needs no comment, being in practice identical with that which occurs with direct currents. The reactance volts may be calculated from the formula

$$\text{reactance} = 2\pi n\text{L},$$

where n is the frequency and L, the coefficient of self induction, is equal to the expression

$$4l\left(\log_e \frac{\text{D}}{\text{R}} + \frac{1}{4}\right) \text{ in absolute measure,}$$

where l is the length of each transmission wire,
D the distance between the two wires,
and R the radius of each wire.

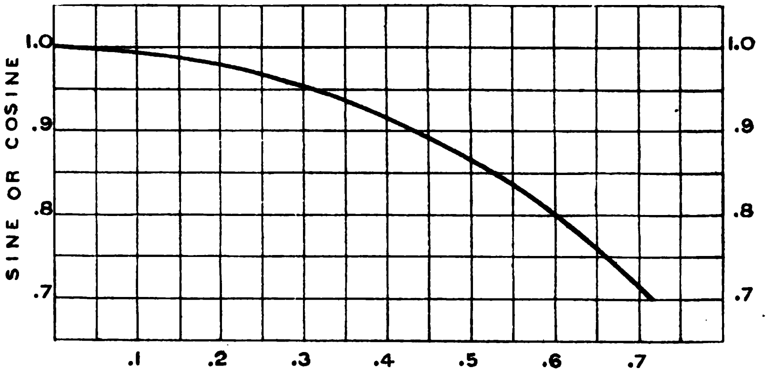


FIG. 170. Curve shewing the relation between sin θ and cos θ.

In using this formula it is important to notice that the various quantities must be expressed in centimetres; thus, supposing the two wires are ·4 inch in diameter and 18 inches apart, the coefficient of self induction per mile is

$$4 \times 5280 \times 30{\cdot}5\left(\log_e \frac{18 \times 2{\cdot}54}{{\cdot}2 \times 2{\cdot}54} + \frac{1}{4}\right) \text{ in absolute measure}$$

$$= 644{,}000 \times 4{\cdot}75 = 3{\cdot}06 \times 10^6.$$

And if the frequency be 25 periods per second the reactance will be

$$2\pi \times 25 \times 3{\cdot}06 \times 10^6 \times 10^{-9} \text{ ohms per mile} = {\cdot}\mathbf{48} \textbf{ ohms}.$$

Having thus calculated the resistance and the reactance the impedance can be obtained at once from the formula $z = \sqrt{r^2 + x^2}$.

In several respects, however, it is convenient to be able to calculate the impedance volts of the various parts of the circuit; for instance, it is important when the power is transmitted by one wire above a railway track, and the rails themselves are used for the return, to know the impedance voltage in the rails alone. For this purpose, it is best to split up the formula given above for the coefficient of self induction of a circuit into four parts. Thus the self induction for a circuit composed of two wires is the sum of the self inductions of each wire by itself and the mutual inductions of each wire on the other. Thus calling L_s and L_m the self induction and the mutual induction coefficients, the total induction coefficient for each wire $= L_s - L_m$*. Using these terms

$$L_s = 2l\left(\log_e \frac{2l}{R} - 1 + \frac{\mu}{4}\right)$$

and

$$L_m = 2l\left(\log_e \frac{2l}{D} - 1\right),$$

where μ is the magnetic permeability of the conductor, being practically unity for all ordinary materials except iron.

This method is very useful in making calculations on composite circuits; for instance, in cases where part of the circuit consists of one wire, and the return of several wires in parallel. In such a case, to calculate the impedance voltage in one of the return wires, it is only necessary to combine with the single term L_s as many terms L_m as there are other wires taking part in the transmission.

To facilitate such calculations curves, figure 171, have been prepared for L_s and L_m for varying values of D and R assuming $\mu = 1$, the actual values being in terms of self inductive and mutual inductive ohms per mile for a standard frequency of 25 periods per second †.

Example. The outgoing current of 50 amperes is carried by a wire ·4 inch in diameter, and the return current by two wires in

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* Following the method suggested by Behn-Eschenburg, *E. T. Z.* April 21, 1904.

† Strictly speaking the values of L_s and L_m cannot be expressed simply in terms of length, *i.e.* as so much per mile, since the length l enters into the logarithmic expression. In practice, however, this point does not arise since the values of L_s and L_m are never required apart from the value of the total induction. That is to say, since in every circuit the current is equal in the outgoing and the return portions the term $\log_e 2l$ never enters into the result. For example, L_s may be written

$$2l\left(\log 2l - \log R - 1 + \frac{\mu}{4}\right)$$

and L_m may be written $2l(\log 2l - \log D - 1)$,

and obviously the difference, viz.

$$2l\left(\log D - \log R + \frac{\mu}{4}\right)$$

is independent of the value of the term $\log 2l$. The curves have, therefore, been calculated on the most convenient assumption, viz. that $\log 2l = \log 2$.

parallel, each ·3 inch in diameter, 5 feet apart, and 15 feet from the outgoing wire. Find the reactance voltage in each of the return wires per mile.

$$\begin{aligned}\text{Reactance volts} &= 25 \times {\cdot}046 \text{ (S. I. volts)}\\ &\quad + 25 \times (-{\cdot}27) \text{ (M. I. volts from parallel return wire)}\\ &\quad - 50 \times (-{\cdot}325) \text{ (M. I. volts from outgoing wire)}\\ &= 25\,[{\cdot}046 - {\cdot}27 + {\cdot}65] = 25 \times {\cdot}426 = \mathbf{10{\cdot}65\ volts.}\end{aligned}$$

The following table will be found convenient, giving constants for some of the standard sizes of wire.

TABLE 3. *Constants of resistance and reactance for hard drawn copper wire.*

L. S. G.	Diameter		Area		Ohms per mile	
	Inches	mm.	sq. in.	sq. mm.	Resistance at 60° F.	S. I. reactance
00000	·432	10·97	·147	94·6	·298	·027
0000	·400	10·16	·126	81·1	·347	·031
000	·372	9·45	·109	70·1	·400	·035
00	·348	8·84	·095	61·4	·460	·039
0	·324	8·23	·082	53·2	·53	·042
1	·300	7·62	·071	45·6	·618	·046
2	·276	7·01	·060	38·6	·73	·051
3	·252	6·40	·050	32·2	·875	·055

Rails as conductors for alternating currents. The conduction of alternating currents by steel rails is by no means a simple problem. Such rails have a much higher specific resistance than copper; but, on the other hand, they have a very high magnetic permeability, which complicates the problem very greatly.

In all cases of electrical conduction, the energy lost in the conductor is conveyed to it from the surrounding ether; in the case of alternating currents the energy is conveyed by means of waves. These waves, naturally, reach first the external layers of the conductor and then penetrate to the interior. The rate of penetration depends partly on the frequency of the alternations, and partly on the material of the conductor.

This problem has been dealt with by many investigators both theoretically and practically. Dealing first, very briefly, with the results of the theoretical study, it has been shewn[*] that in cases in which the penetration is small, the resistance is the same as if the current were uniformly distributed over a surface layer the thickness of which is given by the expression

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[*] Lord Kelvin, Lord Rayleigh, J. J. Thomson.

$$\sqrt{\frac{\sigma}{2\pi\mu p}},$$

in which σ represents the specific resistance of the material,
μ the magnetic permeability,
and p the frequency multiplied by 2π.

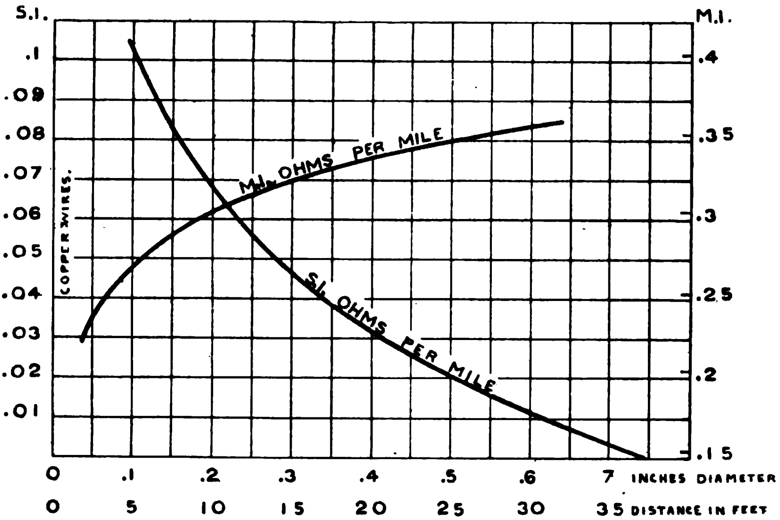

FIG. 171. Self and mutual reactance at frequency 25.

For copper μ is, of course, equal to unity, but for steel rails the value of the permeability is much greater. The chief difficulty of the problem, however, arises from the fact that the value of μ in this case is not a constant quantity. It is, therefore, necessary to rely upon experiment for actual results, basing them on the ideas propounded by the theory.

Experiments have been made in several quarters. Messrs Mordey and Jenkin quote some tests by Herr Blathy, from which it appears that the apparent resistance of steel rails when traversed by alternating currents is much higher than with direct currents. Unfortunately the conditions of the test are not sufficiently clearly stated to enable any very precise conclusions to be drawn from them. Experiments* have also been made in the Siemens Laboratory in King's College, London, by Messrs Siemens-Schuckert Werke in Germany, by the Westinghouse Company on their Interworks Road in Pittsburg, and by Messrs Siemens Bros. and Co. at Stafford. All these experiments point in the same direction, viz. that the penetration that occurs with steel rails under conditions that are likely to arise in practice is small.

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* See note at the end of this chapter.

Equivalent resistance of rails. It is not considered worth while to attempt here any elaborate analysis of the results, for two reasons. In the first place, the precise value of the resistance is not of great importance in practice ; and in the second place any attempt to reconcile the results with theory should take account of hysteresis. For these reasons it is considered sufficient to proceed on the assumption that for steel rails under working conditions, the current is confined to a surface layer the thickness of which may be taken at

4 millimetres for 15 periods per second,
3 millimetres for 25 periods per second,
and 2·4 millimetres for 40 periods per second.

These values are intended to be used for rails in which the specific resistance is about 12 times that of copper, as is the case with ordinary track rails.

On this basis the table of values given below has been worked out for various types of rail of different weights, and in figure 172 the same results are shewn graphically in the form of curves.

It is proposed to call these values the equivalent resistances, to distinguish them from the true resistance with direct currents and from the apparent resistance (or impedance) with alternating currents ; and the equivalent resistance is defined as that factor which, multiplied by the square of the current passing in the rail, gives the actual loss in the rail, including all resistance losses and hysteresis.

Internal and external reactance of rails. As already stated on page 276 the self inductive reactance of a conductor is proportional to the expression

$$2l\left(\log_e \frac{2l}{R} - 1 + \frac{\mu}{4}\right).$$

In this expression the term μ refers to the magnetic permeability of the material of which the conductor is formed, and in practically all cases is equal to unity, except in the case of iron. The simplest way to treat this, is to regard the expression as made up of two parts, viz.

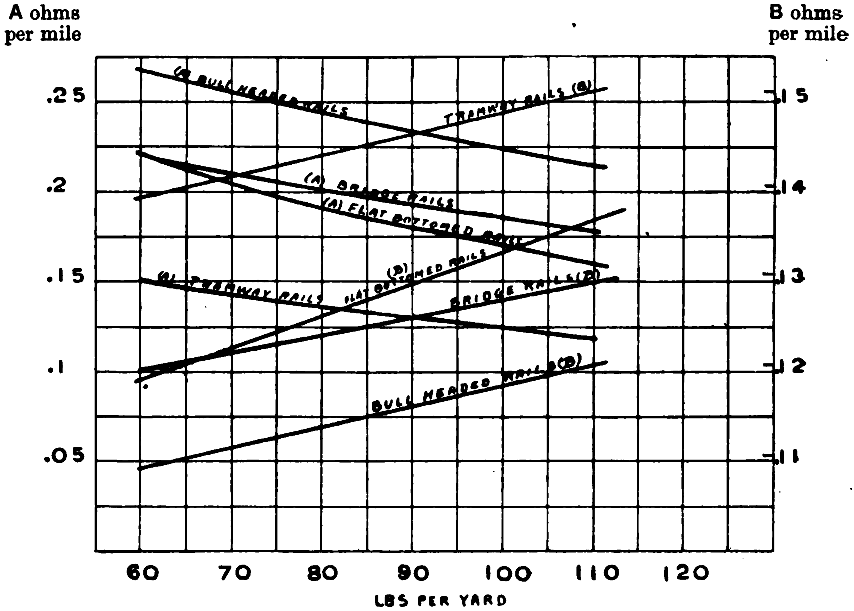
$$2l\left(\log_e \frac{2l}{R} - 1\right) \text{ and } 2l \times \frac{\mu}{4},$$

the former of which refers to the magnetic field surrounding the conductor and the latter to the field in the conductor itself.

Now, in the theory of the transmission by alternating currents in conductors already referred to, the coefficient of self induction due to the internal magnetic field is shewn to be

$$\sqrt{\frac{\mu\sigma}{2\pi p R^2}}$$

in which μ is the magnetic permeability of the material supposed constant.



A equivalent resistance and internal reactance.
B External s. i. reactance.

FIG. 172. Curves of equivalent resistance and reactance of steel rails at frequency 25.

From this expression the value of the internal reactance is obtained by multiplying by p, viz.

$$\sqrt{\frac{\mu p\sigma}{2\pi R^2}},$$

and it is interesting to note that the value of the equivalent resistance, viz.

$$\frac{\sigma}{2\pi R\sqrt{\frac{\sigma}{2\pi\mu p}}} = \sqrt{\frac{\mu p\sigma}{2\pi R^2}},$$

is precisely the same.

In practice, of course, the value of μ is by no means constant; but from experiment it appears that for reasonable currents (say up to 90 amperes in a 100 lb. rail) the internal reactance (at all events for 25 frequency) is approximately equal to the equivalent resistance worked out as stated above.

The same table of values will, therefore, be sufficiently accurate for both.

The external reactance, as mentioned above, is given by the expression

$$2l\left(\log_e \frac{2l}{R} - 1\right),$$

and a curve of values for various rails of different weights is given in figure 172. In this case R, the radius of the conductor, has no very obvious meaning; but it has been assumed to be equal to the perimeter of the rail divided by 2π.

TABLE 4. *Equivalent resistance and reactances of rails at frequency* 25.

Bull headed rails.

Weight, lbs. per yd.	60	70	80	90	100	110
Perimeter in cms.	40	42	44	46	48	50
Equivalent resistance and internal reactance, ohms per mile	·268	·256	·244	·234	·224	·214
External S. I. reactance, ohms per mile	−·1092	−·1115	−·114	−·116	−·1184	−·1205

Flat bottomed railway rails.

Weight, lbs. per yard	60	70	80	90	100	110
Perimeter in cms.	48·5	52·5	56	60	63·5	67
Equivalent resistance and internal reactance, ohms per mile	·222	·204	·192	·18	·17	·16
External S. I. reactance, ohms per mile	−·119	−·123	−·126	−·13	−·133	−·136

Bridge rails.

Weight, lbs. per yard	60	70	80	90	100	110
Perimeter in cms.	49	51	53	55	57·5	59·5
Equivalent resistance and internal reactance, ohms per mile	·22	·21	·202	·196	·186	·18
External S. I. reactance, ohms per mile	−·12	−·122	−·124	−·126	−·128	−·129

Tramway girder rails.

Weight, lbs. per yard	60	70	80	90	100	110
Perimeter in cms.	72	76	79·5	83	87	91
Equivalent resistance and internal reactance, ohms per mile	·15	·142	·136	·13	·124	·118
External s.i. reactance, ohms per mile	−·139	−·142	−·144	−·147	−·149	−·151

Example. A current of 120 amperes is transmitted to an electric train at a distance of 2½ miles from the generating station, by means of a single overhead wire of size L.S.G. 0000, returning by two 85 lb. bull headed rails at the standard gauge of 4 ft. 8½ inches, the overhead wire being 16 feet above the centre of the track. Find the impedance voltage in the rails, and the transmission drop, supposing the voltage at the generating station to be 6600 volts, at 25 frequency, and the power factor of the load to be ·85.

Current in each rail 60 amperes
Resistance of each rail per mile ·239 ohms
Internal reactance of each rail per mile ·239 ohms
Distance between rails 5 feet
Distance between rail and wire 16·2 feet
External reactance of rail per mile − ·115 − ·27 + 2 × ·328 = ·271
Total reactance of rail per mile = ·239 + ·271 = ·51
Impedance per mile $= \sqrt{\cdot 239^2 + \cdot 51^2} = \cdot 563$
Impedance volts in 2½ miles = 2½ × 60 × ·563 = **84 volts**
Resistance per mile of the overhead wire ·347 ohms
Reactance per mile ·031 + ·328 = ·359 ohms
Total resistance volts in transmission
120 × ·347 × 2·5 + 60 × ·239 × 2·5 = 140 volts
Total reactance volts in transmission
120 × ·359 × 2·5 + 60 × ·51 × 2·5 = 185 volts
Power factor $= \cos\phi = \cdot 85$
$\sin\phi = \cdot 53$
Transmission drop = 140 × ·85 + 185 × ·53 = **217 volts.**

Application to polyphase traction. The same method may be applied to transmission by polyphase currents. For electric traction pure and simple, that is excluding the general case of transmission of energy, only three-phase currents need be considered.

It is well known that the transmission drop in a three-phase system, in which the wires are similar and symmetrically placed, is the same as in a single phase system carrying half the load at the same voltage, the distance between the wires being the same in both cases.

This condition, however, is not fulfilled in cases of polyphase electric traction. Either three wires are placed parallel to one another in the same plane, as in the high speed experiments between Berlin and Zossen; or two wires are situated a considerable distance above the track, the rails themselves forming the third conductor.

In the latter case, which is the most usual, an exact calculation would be somewhat difficult, unless the resistance and reactance of all three conductors were identical, which is unlikely. Approximate figures, however, can be obtained as follows: Assume that the three currents are equal in quantity, and that their phase angles are equal, being each 120°. Calculate the resistance volts as before; and in calculating the reactance, take the mean of the mutual inductance from the other two conductors. Thus, if OA, figure 173, represent the self inductive reactance of one conductor, and OB, OC the mutual inductive reactances from the other two conductors, the total reactance will be OA minus the resultant of OB and OC. Now if OB and OC are

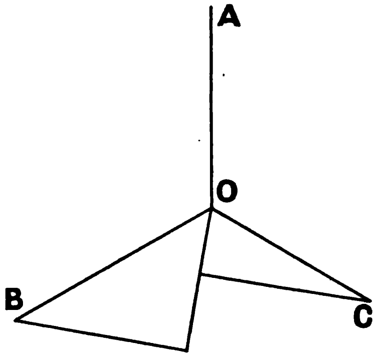


FIG. 173. Vector diagram of self and mutual reactance for three-phase system using the rails as a return.

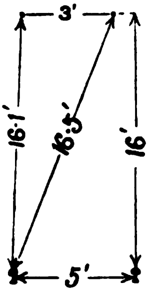


FIG. 174. Diagram shewing positions of two overhead conductors and the rails for a three-phase system.

not very unequal, their resultant will be nearly $\tfrac{1}{2}\mathrm{OB} + \tfrac{1}{2}\mathrm{OC}$, *i.e.* the mean of **OB** and **OC**. This method is, of course, somewhat rough.

Example. As a useful illustration, take the following data: the transmission conductors consist of 2½ miles of two overhead wires of size L.S.G. 0000, placed symmetrically 16 feet above the track and three feet apart in a horizontal direction, and for the third conductor the two track rails in parallel (bull headed rails 85 lbs. per yard at the standard gauge of 4 ft. 8½ inch). If the current per conductor be 120 amperes at 25 frequency, and the voltage and power factor at the receiving end be 3000 and ·88 respectively, find the transmission drop.

The positions of the conductors are shewn in figure 174.

From the foregoing tables and curves the various resistances and reactances can be written down:

Resistance of each wire	·87	ohms
Self inductive reactance of each wire ...	·077	„
Mutual inductive reactance from other wire	– ·61	„
„ „ „ „ one rail	– ·82	„
„ „ „ „ other rail	– ·825	„
Resistance of two rails in parallel	·3	„
Internal reactance of each rail	·6	„
External self-inductive reactance of each rail	– ·288	„
Mutual inductive reactance from other rail	– ·675	„
„ „ „ „ one wire	– ·82	„
„ „ „ „ other wire	– ·825	„

Hence, allowing 120 amperes in each wire, and 60 amperes in each rail,

Total reactance volts in each wire

$$= 120 \times {\cdot}077 - \tfrac{1}{2}\left[120 \times (-{\cdot}61) + 60 \times (-{\cdot}82) + 60\,(-{\cdot}825)\right]$$

$$= 120\left[{\cdot}077 + \tfrac{1}{2}\,({\cdot}61 + {\cdot}82)\right] = 120 \times {\cdot}79 = \mathbf{95\ volts.}$$

Resistance drop in each wire $120 \times {\cdot}87 = \mathbf{104\ volts.}$

Total reactance drop in two rails in parallel

$$60 \times [-{\cdot}288 + {\cdot}6] + 60\,[-{\cdot}675]$$

$$-\tfrac{1}{2}\left[120 \times (-{\cdot}82) + 120 \times (-{\cdot}825)\right]$$

$$= -22 + 99 = \mathbf{77\ volts.}$$

Resistance drop in two rails in parallel $120 \times {\cdot}3 = \mathbf{36\ volts.}$

From these figures it is obvious that the drops in the three phases are unequal. It can easily be shewn that in order to maintain three equal voltages of 3000 volts at the receiving end, the voltages at the generating end must be respectively 3240, 3180, and 3180.

If, however, the problem is stated in terms of three equal voltages at the generating station, the calculation of the currents and the transmission drops is very long and difficult.

As a practical method the following is proposed. In most cases the greatest drop will occur in the voltage between the two overhead wires; in the above example the drop was 3240 − 3000 or 240 volts. Now, if the voltages applied to a three-phase induction motor be unequal, say A, $A+\delta$, and $A+\delta$, it can be shewn that the value of the main rotating field in the motor is proportional to $A+\frac{1}{3}\delta$. The term $\frac{1}{3}\delta$ will be quite small compared to A, and there will be very little error in neglecting it. By so doing, it becomes necessary only to calculate the transmission drop in the overhead wires, which is comparatively simple.

Thus, taking once more the same example of 120 amperes transmitted $2\frac{1}{2}$ miles, but assuming a transmitting voltage of 3300 volts, the calculation of transmission drop is as follows:

Resistance of wire ... ... ·87 ohms

Reactance of wire $·077-\frac{1}{2}[-·61-·82]$ ... = ·79 ,,

Resistance volts $120 \times ·87$ ... ... = 104 volts

Reactance volts $120 \times ·79$ ... ... = 95 ,,

Generator volts between overhead wire and neutral point $\frac{3300}{\sqrt{3}}$ ... ... = 1900 ,,

Motor volts between overhead wire and neutral point $= 1900 - 104\cos\phi - 95\sin\phi$ ($\cos\phi = ·88$, $\sin\phi = ·475$) ... ... = 1763 ,,

Motor volts $= 1763\sqrt{3}$ ... ... = **3060 volts**

or more simply

Transmission drop $= \sqrt{3}\,(104\cos\phi + 95\sin\phi)$ = **237** ,,

Design of feeder systems. The principles which underlie the design of feeder systems on alternating current railways are very much the same as obtain in direct current tramways and railways. These principles are based on

(1) The necessity for limiting the drop of voltage in transmission.

(2) The necessity for limiting the difference of volts between any two points of an uninsulated return.

(3) The advisability of dividing the contact lines into a number of sections isolated from one another, for the purpose of localising as far as possible any trouble caused by partial or complete breakdown.

Drop of voltage in transmission. Of the first of these three points, it is not necessary to add much to what has been already said in the first part of this chapter. It only remains to discuss the amount of drop allowable.

This drop will depend on the circumstances of the case and will be different according to various factors that have to be taken into account. Of these three may be mentioned: (*a*) lighting the carriages from the contact wires, (*b*) controlling the motors by means of electromagnetic switches, and (*c*) the necessity for maintaining the schedule with an economical consumption of energy, and the satisfactory operation of the motors.

(*a*) If the first of these has to be taken into account, there can be no doubt that if the generators are not unnecessarily large, some system of compounded or compensated alternators is essential in the generating station (unless the number of separate units on the line is large). Assuming, then, in this case that the pressure at the terminals of the alternators is kept substantially constant, it may be taken, as a maximum figure, that the transmission drop should not exceed 15 or 16 per cent., and if it is practicable, it is better to limit it to 10 per cent.

(*b*) The necessary limitations on account of the electromagnetic switches will naturally depend on the design of this apparatus; but there is no difficulty in making allowance for a variation of 25 per cent. This, however, is to be regarded, not as a variation due to the transmission drop alone, but as due to all causes together, including, if necessary, contact lines, transformers, transmission lines, and generating plant.

(*c*) The necessity for economy in energy consumption depends very much on circumstances; but, in general, it must be taken into account. In any particular example, it would be quite possible to work out how far the energy wasted in the brakes is affected by low voltage; but a single phase system may have a great advantage in this respect over other systems, in the possession of a variable voltage transformer. By this means extra low voltage may be compensated by providing an additional tapping in the transformer secondary, only to be used at such times as may be necessary. Scarcely enough work has yet been done in alternating current railways to enable any general rule to be laid down, as the type of service has varied so much.

In three-phase systems, where induction motors are used, attention must be paid to the liability of such motors to fall out when overloads occur with low voltage. This matter has, however, been referred to already in Chapter 1 of this Volume.

Impedance volts in an uninsulated return. This consideration has already given rise to a good deal of discussion*.

It is well known that electrolysis may take place, when an alternate current is passed between metal electrodes immersed in an electrolyte. The amount of electrolysis depends upon the nature of the chemical action which results. The frequency of the alternate currents is also one of the important variables which influences the result. Mr A. P. Trotter has found † that corrosion can take place on lead buried in moist earth when the current density is varied between the limits 0·0039 and 0·00006 amperes per sq. cm. at 83 frequency. The figures in Table 5 ‡ give further information on the subject. No definite conclusions can be drawn from Laboratory Experiments as to what is going to take place in actual practice; the data are given more with a view to warn engineers that substantial electrolysis with alternate currents is a possibility.

On the other hand, negative results have been obtained in practice by Mr S. M. Kintner§, and on the Murnau to Oberammergau line||. The former experiments consisted in burying a number of iron and lead pipes in the earth, connected to an alternating current supply at 25 frequency. After the lapse of one year, no definite result could be attributed to electrolysis, although marked effects were produced in two weeks with continuous current under the same conditions.

On the Murnau line, one of the earth plates was taken up and examined, and shewed no signs of electrolysis after the passage of 90000 ampere hours.

Apart from the question of electrolysis, it is unwise to allow excessive voltages in an uninsulated return owing to the possibility of shocks to platelayers and others working on the line. It is quite possible for a man to complete an electrical circuit through his body, such circuit being a very extensive one; for instance, the circuit may be formed by one line of rails carrying the return current and another line of rails without any current. If there were no cross bonding, the voltage might easily be that due to two or three miles of rail.

It is prudent and advisable to provide efficient and frequent earthing of the track when used as a return conductor; but it is almost impossible to estimate what proportion of the current will

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* See Appendix to this chapter.

† *Trans. Faraday Society*, Vol. 1, August, 1905.

‡ See Appendix to this chapter.

§ *Electric Journal*, Nov. 1905, quoted by Mr C. F. Jenkin.

|| See "Single Phase Electric Traction" by C. F. Jenkin, *Proc. I. C. E.*, 1906—1907.

leave the rails, or what reduction of possible voltage is secured thereby.

On the other hand it seems reasonable to take into account the length of the line in settling the impedance voltage permissible. It is obviously irrational to allow the same limit for two points one mile apart as for two points 10 miles apart. Considered from this standpoint it may be suggested that in many cases a limit of 10 volts per mile for considerable lengths should be imposed*, and in some cases 15 or even 20 volts; but in settling such limits, account should be taken of the presence or absence of frequent earth plates, of long lengths of accessible conductors running parallel to the track and any other conditions which may bear upon the problem.

In this connection it is of interest to examine a practical illustration. It might be urged that as any conductor running parallel to the track is subject to the inductive action of the overhead wire in the same way as the rails, it is only necessary to consider the resistance and the internal reactance of the rails.

Consider the example worked out on p. 282, and suppose a point rod or wire to be connected to the rails at a certain place and to run parallel to the track at a distance of three feet from the nearest rail for a length of one mile.

The impedance voltage per mile in the rails = 34 volts and the voltage induced in the point rod can be shewn to be – 40 due to the overhead wire and + 32·4 due to the two rails, *i.e.* a nett induced voltage of 7·6 volts. Consequently if the gap between the point rod and the rails be bridged by a man at a distance of a mile from connection, he will receive a shock due to an alternating voltage of

$$34 - 7{\cdot}6 = \mathbf{26{\cdot}4\ volts.}$$

Practical methods of limiting the impedance volts in the rails. The example worked out on p. 282 is sufficient to demonstrate that the limits already mentioned may easily be passed in practice. In that example the impedance voltage was calculated to be 84 volts in $2\frac{1}{2}$ miles, *i.e.* 34 volts per mile. It is, therefore, evident that in many cases some provision must be made for keeping down the rail drop, since, with high tension overhead distribution to the trains, the distances between sub-stations (when there are any) may be easily 10 miles or more.

Supposing then a case in which the current has to be transmitted five miles, and a limit of 20 volts per mile be chosen, it becomes necessary to make special provision to keep to 100 volts.

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* From the point of view of shocks, it is obviously unnecessary to make provision for a drop of only 10 volts for a length of 1 mile.

There have been proposed several methods which are applicable to different cases. In direct current tramways the negative booster is the chief instrument for securing the necessary limitations in the rail drop; and in alternating current traction the booster transformer plays very much the same part.

The booster transformer or "sucking transformer" consists of two equal windings on the same magnetic core; and, in general, these windings are used to carry the outgoing and the return current respectively. Now in an ordinary transformer, the ampere turns on the primary are practically equal to those on the secondary, and if the two sets of turns are equal, the primary and secondary currents are practically identical. In other words, the booster ensures that the whole of the return current shall pass through it, since its secondary current must be equal to the outgoing current.

This booster may be applied in several ways.

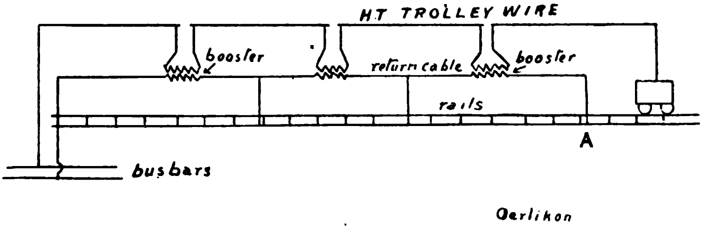

FIG. 175. Diagram shewing sucking transformers on the Oerlikon system.

The Oerlikon method. Probably the best method for a simple case is that shewn in figure 175. Here the trolley wire (or the two wires in case of a three-phase system) is divided into sections by insulators; at each of these points the current is diverted into the primary of a booster transformer; the booster secondaries are connected in series with a return cable which is itself connected to the rails at points midway between the section insulators. The overhead line and the return cable are then connected to the bus-bars at the generating station.

The action is as follows: Suppose a train at the far end of the line as shewn. The outgoing current passes along the trolley wire and through all the boosters in series; the return current flows along the track rails as far as the point A, at which point it leaves the rails and flows along the return cable through the boosters in series to the bus-bar.

Now it is not quite true to say that all the current leaves the rails at A, otherwise there would be two conductors in parallel connected at their ends, one of which (the rails) is taking no current, and the other (the return cable) is taking all the current without any E.M.F. being introduced into either. This is obviously impossible. As a matter of fact, a small current flows along the rails, and the greater part of the return current is sucked along the return cable by the E.M.F.'s in the boosters. In a properly designed transformer, the magnetising current is very small in comparison with the full load current (of the order of four or five per cent.). In the present case, this magnetising current is due to the difference between the outgoing current and the current in the return cable. This difference is, naturally, equal and opposite to the current in the rails between A and the bus-bar; and, therefore, the drop in the rails is very small, and in fact is generally negligible. In other words, the boosters adjust themselves so that practically all the current returns by the return cable, the impedance voltage in this cable being automatically counteracted by the boosters.

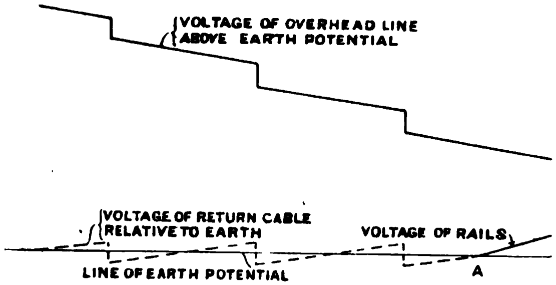
VOLTAGE OF OVERHEAD LINE ABOVE EARTH POTENTIAL
VOLTAGE OF RETURN CABLE RELATIVE TO EARTH
VOLTAGE OF RAILS
LINE OF EARTH POTENTIAL
A

FIG. 176. Diagram shewing distribution of voltage drop with sucking transformers on the Oerlikon system.

The impedance volts in the return are thus as shewn in the lower half of figure 176 (if the boosters E.M.F.'s are included in this term). The full line shews the rail drop between the train and the point A, and the dotted line the drop in the return cable.

It will be obvious, on consideration, that the negative drop is not eliminated by the action of the boosters; but is transferred to the positive side. The positive drop is, thus, as shewn in the upper half of figure 176, in which the short vertical lines represent those voltages added by the boosters.

The calculation of the transmission drop is, therefore, practically unaffected, since all the resistance volts are added together as before, and similarly all the reactance volts, whether in the trolley wire or the rails or the return cable.

The booster transformer must, of course, be constructed so that it will supply a voltage equal to the impedance voltage in the return cable without an excessive magnetising current under all conditions of practice. It is important, therefore, to keep this impedance voltage as low as possible. This can be done by supporting the return cable on the overhead construction as near as practicable to the trolley wire. This method has also the advantage of cheapness over an insulated cable laid in the ground.

Modified Oerlikon arrangement. A modification of the above method is shewn in figure 177, in which the return cable forms the outer of a concentric cable, the inner of which supplies the various sections of the trolley wire. By this means the impedance voltages in the outgoing conductor and also in the return cable are reduced to a minimum, as the reactance of a concentric cable is practically negligible.

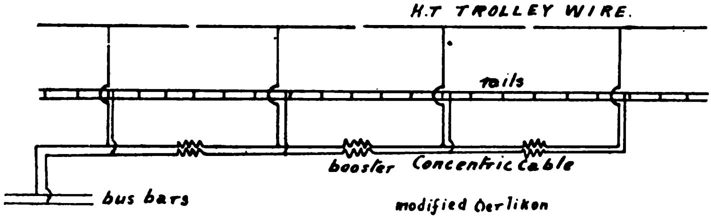

FIG. 177. Diagram shewing modification of Oerlikon arrangement of sucking transformers.

Kapp method. This method, illustrated in figure 178, is somewhat simpler than the above, in that there is no return cable. The rails are sectionalised at the same places as the overhead wire, and boosters inserted as shewn. This method is inferior to the Oerlikon plan, in that no definite path is provided for the current that flows in the rails in the latter system. Thus, for instance, the rails might be completely insulated from the earth, in which case the two currents in the booster windings would be exactly equal, and the booster would have no effect at all. Similarly, the arrangement would be inoperative if the rails are short circuited to earth at the points of section.

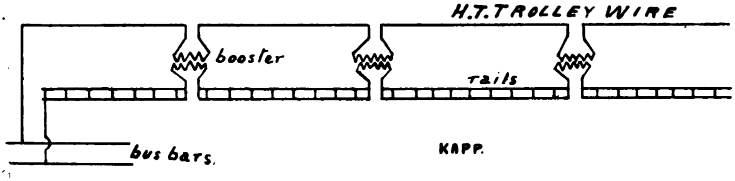

FIG. 178. Diagram shewing Kapp's arrangement of sucking transformers.

Combined feeder and booster. In the modified Oerlikon arrangement, the concentric cable may be regarded as a parallel distributor. Another arrangement is shewn in figure 179, in which the line is divided up into several divisions (each possibly containing a number of insulated sections). Each division is fed by means of a concentric cable, with which is combined in the generating or sub-station a booster transformer. In this system the control of all the divisions is kept in the generating stations, whereas in the systems described above such control is not provided for.

Very heavy traffic. In some cases it may easily happen that the rails alone are insufficient for the return current, even for moderately short distances. For example, suppose a train is equipped with single phase motors aggregating 1000 H.P.; then at 50 per cent. overload with 6000 volts, 25 periods per sec., and two track rails, the current would be about 250 amperes, and with an impedance per mile of ·28 ohm, the impedance volts per mile would amount to 70 volts. In such a case it might be advisable to supplement the rails with a parallel copper cable frequently bonded thereto. Probably two 000 L.S.G. wires in parallel supported overhead close to the trolley wire would reduce the drop as much as necessary.

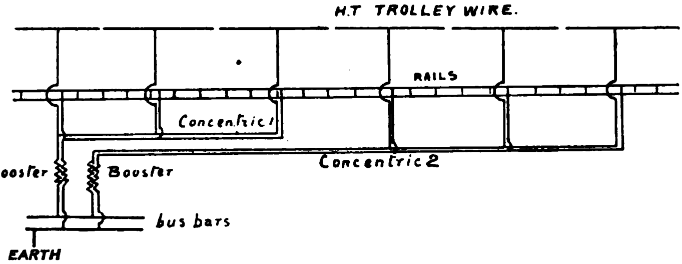

FIG. 179. Diagram of combined system with separate feeders and sucking transformers.

For very heavy traffic, however, there would probably be a double track cross-bonded at frequent intervals, and a higher voltage than 6000 would be almost necessary.

Control of the line. The system of controlling the various sections of the line in the case of direct current tramways has already been described in Chap. 9, Vol. I. This system consists in providing separate feeders from the generating station to the various sections, and is due to the fact that at no other place is there an attendant in constant readiness to exercise the necessary control.

On the other hand, in direct current electric railways, more particularly in those where there are sub-stations, control is generally exercised by the sub-station attendants.

In the case of alternating current railways, the range of line which needs no sub-station is much greater than with direct current systems, and when there are sub-stations they will probably be separated by very considerable distances. Moreover, the type and arrangement of the transforming apparatus are not such as to render necessary the constant presence of an attendant. It becomes necessary, therefore, either to control the line from the generating station, or to make some other arrangement.

In any particular case, the choice must be made by the officials of the electrified line, and no definite rules can be laid down. It may be possible to utilise the signalmen for the purpose; but in this case a telephone system should be provided, whereby each signalman can immediately communicate with the generating station or the electrical engineer's office.

Auxiliary feeder system. Not much need be said about the auxiliary feeder system, as in general it will be made to fit the circumstances of each particular case.

The telephone system, where necessary apart from the telephones already installed in connection with the signal arrangements, will naturally have to meet the requirements of the electrical engineer and the traffic manager.

The only point that need be touched upon is the position of the voltmeter leads in connection with the impedance voltage in the rails. It is obvious that in order to measure this voltage the measuring lead must be so situated that it is uninfluenced by induction from the outgoing or the return current.

Such a position can easily be found if the return current is confined to the rails [as is shewn in the example on page 288, in which the nett voltage induced in a conductor comparatively close to the rails is only 7·6 volts per mile]. If, however, a large percentage of the return current passes into the earth, there may be a considerable error introduced by placing the lead in such a position as to be uninfluenced when all the current is in the rails. The path of the current in the earth is, naturally, indefinite, and is scarcely suitable for calculation.

In any particular case, however, it should be sufficient to obtain an indication of the voltage to which a man might be subjected, and this can be done by running the voltmeter lead parallel to the track and at some convenient distance from it. So far this point has not received much attention in actual practice.

Interference effects. It is convenient to consider here the effect of the alternating current in the trolley wires upon neighbouring conductors, more particularly upon the telegraph and telephone wires which run parallel to the line.

The effects are of two kinds, and are due to (1) electromagnetic induction and (2) electrostatic induction. Telephone circuits are particularly sensitive, and even a very small induced voltage is enough to cause serious trouble. Of course a periodicity of 25 is not in itself very objectionable; but it is practically certain that harmonics will be present, and these make their influence felt by induction in proportion to the order of the harmonic. In general it may be stated that it is necessary to substitute a suitable cable for the overhead telephone wires in the neighbourhood of trolley wires*.

On the other hand telegraph circuits are not so sensitive, and in most cases interference with the working of block instruments may be avoided by the use of a metallic return supported close to the telegraph wires. The method of calculating the induced voltages in the two wires, given in this chapter, will enable one to examine any particular case.

Experiments have been made by Messrs Siemens Bros. and Co. at Stafford to ascertain to what extent railway signalling instruments are affected by alternating currents. The results of tests on three different instruments are given at the end of this chapter. These tests shew that such instruments are certainly affected by comparatively small alternating voltages; but there are obviously several ways in which they could be altered so that their sensitiveness to direct currents should be retained, while eliminating any tendency to respond to alternating currents. Thus the mechanical parts could be loaded, or the magnetic circuit could be sheathed in copper as proposed by Herr Dahlander†, or inductances with low resistances and high reactances could be connected in the electrical circuits; but how far such devices would enable the applied alternating voltages to be raised has not been ascertained.

Electrostatic induction may be dealt with in the following way.

Let A, figure 180, represent the trolley wire, and B a parallel telegraph wire; let

d_{12} be the distance between A and B,

s_{11} the distance between A and its image in the ground,

s_{22} the distance between B and its image,

s_{12} the distance between A and the image of B, or between B and the image of A,

r_{11} and r_{22} the radii of A and B respectively.

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* C. F. Jenkin, "Single Phase Electric Traction," *Proc. I. C. E.*, 1906—1907.

† Quoted by C. F. Jenkin, British Association, August 6, 1906.

Let the charge and the voltage of A be q_1 and V_1 and of B q_2 and V_2, and let

c_{11} be the capacity per unit of length of A in the presence of B and the ground,

c_{22} the capacity of B in the presence of A and the ground,

and c_{12} the mutual capacity of A and B in the presence of the ground.

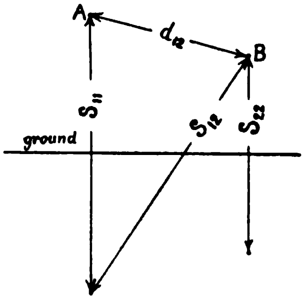

FIG. 180. Diagram shewing relative positions of trolley wire, telegraph wire, and ground.

Then the following equations (as given by O. Heaviside, *Electrical Papers*, Vol. I. page 44) express the relations between the various quantities.

$$V_1 = 2q_1 \log_e \frac{s_{11}}{r_{11}} + 2q_2 \log_e \frac{s_{12}}{d_{12}},$$

$$V_2 = 2q_1 \log_e \frac{s_{12}}{d_{12}} + 2q_2 \log_e \frac{s_{22}}{r_{22}},$$

whence

$$q_1 = c_{11}V_1 + c_{12}V_2,$$

$$q_2 = c_{12}V_1 + c_{22}V_2,$$

in which

$$c_{11} = \frac{2}{R} \log_e \frac{s_{22}}{r_{22}}; \quad c_{22} = \frac{2}{R} \log_e \frac{s_{11}}{r_{11}};$$

$$c_{12} = -\frac{2}{R} \log_e \frac{s_{12}}{d_{12}}, \text{ and } R = 2 \log \frac{s_{11}}{r_{11}} \times 2 \log \frac{s_{22}}{r_{22}} - \left(2 \log \frac{s_{12}}{d_{12}}\right)^2.$$

Now suppose the wire B connected to the ground through a resistance ρ. Then the current in this connection will be

$$\frac{V_2}{\rho} = -\frac{dq_2}{dt}. \quad \therefore -\frac{V_2}{\rho} = \frac{dq_2}{dt} = c_{21}\frac{dV_1}{dt} + c_{22}\frac{dV_2}{dt}.$$

If, now, an alternating voltage $V \sin pt$ be applied to A this equation becomes

$$-\frac{V_2}{\rho} = c_{21}pV \cos pt + c_{22}\frac{dV_2}{dt}$$

or

$$\frac{dV_2}{dt} + \frac{V_2}{c_{22}\rho} = -\frac{c_{21}p}{c_{22}} V \cos pt.$$

The solution of this differential equation is

$$\mathsf{V}_2 = \frac{c_{21}p\mathsf{V}}{c_{22}} \times \frac{\cos(pt-\epsilon)}{\sqrt{p^2+\left(\frac{1}{c_{22}\rho}\right)^2}} + \text{a term which vanishes as time goes on.}$$

Thus the maximum value of V_2 is

$$\frac{c_{21}p\mathsf{V}}{c_{22}\sqrt{p^2+\left(\frac{1}{c_{22}\rho}\right)^2}} = \frac{c_{21}\mathsf{V}}{\sqrt{c_{22}{}^2+\frac{1}{p^2\rho^2}}},$$

or the same expression gives the R.M.S. value of V_2 if V represent the R.M.S. voltage applied to A.

Now apply this result to a length of ten miles, the two wires having the following particulars:

$$d_{12} = 360 \text{ cms.},$$
$$r_{11} = r_{22} = \cdot 5 \text{ cm.},$$
$$s_{11} = 1000, \quad s_{22} = 600,$$
$$s_{12} = 855,$$

and $\qquad$ 10 miles $= 1\cdot 61 \times 10^6$ cms.

Then $\qquad \mathsf{R} = 212;\quad c_{12} = 13100;\quad c_{22} = 115000.$

Take $p = 2\pi \,.\, 25 = 157$, and let ρ be one megohm or 10^{15} absolute units.

Now all the quantities must be expressed in electromagnetic measure, and therefore c_{12} and c_{22} must each be divided by v^2 or $(3 \times 10^{10})^2$.

Then
$$\mathsf{V}_2 = \mathsf{V} \times \frac{\frac{13100}{9 \times 10^{20}}}{\sqrt{\left(\frac{115000}{9 \times 10^{20}}\right)^2 + \left(\frac{1}{157 \times 10^{15}}\right)^2}}$$

$$= \mathsf{V} \times \frac{13100}{\sqrt{(115000)^2 + \left(\frac{9 \times 10^5}{157}\right)^2}} = \mathsf{V} \times \mathbf{\cdot 114}.$$

Similar results can be obtained for other values of the resistance thus:

for 1 mile

ρ in ohms	∞	1000000	100000	10000	1000
$\frac{\mathsf{V}_2}{\mathsf{V}}$,,	·114	·114	·102	·022	·0022

Thus suppose $\mathsf{V} = 10000$ volts, then the insertion of an earth connection of 10000 ohms lowers the voltage induced in the parallel wire from 1140 volts to 220, and a connection of 1000 ohms from 1140 to 22.

APPENDIX.

TABLE 5. *Results of Electrolysis with alternate currents**.

Name of metal	Solution			Frequency	R. M. S. amperes per sq. cm.	Max. coulombs per half period	Average gms. per hour per sq. cm.		
	Name	Average density	Average temperature				A. C. under conditions of test	D. C. for same current density	Ratio $\frac{\text{D.C.}}{\text{A.C.}}$
Lead	H_2SO_4 dil.	1·050	17·0	21·5	0·0236	0·0394	−0·00629	0·091	14·47
,,	,,	,,	17·5	92·5	,,	0·0094	−0·00344	,,	26·45
,,	,,	1·043	26·5	92	0·0847	0·0335	−0·03230	0·3265	10·11
Iron	Ferrous sulphate	1·118	21·2	92	0·0273	0·00956	−0·00046	0·0285	62·0
,,	,,	1·180	23·0	92	0·0929	0·0335	−0·00445	0·0970	21·8
,,	,,	1·084	20·0	32	0·034	0·0334	−0·00079	0·0355	44·95
,,	Sodium chloride	1·088	20·2	32	0·031	0·0334	+0·00010		
Copper	Copper sulphate	1·105	16·7	25	0·0265	0·0345	−0·00005	0·03127	625·5
,,	,,	1·076	24·7	92	0·1000	0·0335	−0·00008	0·118	1475·0
,,	Sodium chloride	1·135	21·2	92	0·0241	0·00956	−0·00003	0·0283	943
,,	,,	1·082	19·4	32	0·0340	0·0334	−0·00028	0·0402	144

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* *Trans. Faraday Society*, Vol. I. p. 305.

TABLE 6. *Measurements of self induction and resistance of 70 lb. steel rails with alternate currents**.

R.M.S. amperes, x	Direct current Rx volts			Frequency 100						Frequency 50							Frequency 27							
					Rail portion per mile							Rail portion per mile							Rail portion per mile					
	For whole circuit experimented upon	Per mile of rail portion	L for whole circuit experimented upon in 10^{-6} henries	L′ in 10^{-6} henries per 60 ft.	Reactance volts	Resistance volts	Impedance volts	Ratio of alternate to direct current volts	Impedance volts for whole circuit experimented upon	L for whole circuit experimented upon in 10^{-6} henries	L′ in 10^{-6} henries per 60 ft.	Reactance volts	Resistance volts	Impedance volts	Ratio of alternate to direct current volts	Impedance volts for whole circuit experimented upon	L for whole circuit experimented upon in 10^{-6} henries	L′ in 10^{-6} henries per 60 ft.	Reactance volts	Resistance volts	Impedance volts	Ratio of alternate to direct current volts	Impedance volts for whole circuit experimented upon	
50	0·66	3·92	55·1	17·2	47·5	26·1	54·1	13·8	2·1	56·0	18·1	25·0	10·6	27·1	6·92	1·15	66·0	28·1	21·0	7·45	22·2	5·68	0·9	[1]
100	1·32	7·84	57·6	19·7	108·0	58·4	123·0	15·7	4·0	59·0	21·1	58·3	29·8	65·5	8·35	2·55	71·0	33·1	49·4	22·7	54·4	6·94	2·05	
150	1·98	12·5	60·1	22·2	184·0	96·0	208·0	16·6	6·1	62·0	24·1	100·0	58·8	116·0	9 26	3·9	74·0	36·1	80·8	44·7	92·5	7·39	3·1	
200	2·64	15·7	63·0	25·1	277·0	154·0	318·0	20·4	8·1	65·0	27·1	150·0	94·0	177·0	11·3	5·2	78·0	40·1	119·8	75·2	141·0	9·0	3·9	
							Av.	16·6						Av.	8·7						Av.	7·2		
50	0·57	3·92	46·0	16·9	46·6	26·1	53·4	13·6	1·75	50·0	20·9	28·9	10·6	30·8	7·85	1·05	59·0	29·9	22·3	7·45	23·5	6·0	0·75	[2]
100	1·14	7·84	48·0	18·9	104·0	58·4	119·4	14·2	3·5	52·0	22·9	63·3	29·8	70·0	8·93	2·3	61·0	31·9	47·6	22·7	52·7	6·73	1·9	
150	1·71	12·5	50·0	20·9	173·0	96·0	198·0	15·8	5·4	53·0	23·9	99·1	58·8	115·0	9·18	3·4	63·0	33·9	75·9	44·7	88·1	7 05	2·85	
200	2·28	15·7	52·0	22·2	245·0	154·0	290·0	18·5	6·8	55·0	25·9	143·0	94·0	171·0	10·9	4·7	65·0	35·9	107·0	75·2	131·0	8·35	3·75	
							Av.	15·5						Av.	9·2						Av.	7·0		
50	...	3·92	...	...	...	...	...	...	...	33·0	15·8	21·8	10·6	24·2	6·18	...	41·0	23·8	17·75	7·45	19·2	4·9	...	[3]
100	...	7·84	...	...	...	...	...	...	...	33·5	16·3	45·1	29·8	54·0	6·89	...	44·0	26·8	40·0	22·7	46·0	5·88	...	
150	...	12·5	...	...	...	...	...	...	...	36·0	18·8	78·0	58·8	97·7	7·80	...	47·0	29·8	66·7	44·7	80·4	6·41	...	
200	...	15·7	...	...	...	...	...	...	...	37·0	19·8	109·0	94·0	144·0	9·18	...	50·0	32·8	97·8	75·2	123·0	7·84	...	
														Av.	7·5						Av.	6·26		
50	0·61	1·96	50·0	12·1	33·4	12·7	35·6	18·2	1·75	50·0	12·1	16·7	4·5	17·3	8·82	1·04	57·0	19·1	14·2	2·74	14·4	7·37	0·65	[4]
100	1·23	3·92	50·3	12·4	68·5	26·2	73·4	18·7	3·4	50·3	12·4	34·2	11·0	35·9	9·16	2·1	57·5	19·6	29·2	7·45	30·1	7·68	1·75	
150	1·84	6·26	51·0	13·1	108·4	41·7	116·2	18·6	5·1	51·0	13·1	54·2	19·3	57·5	9·18	3·15	59·0	21·1	47·2	14·1	49·4	7·88	2·5	
200	2·46	7·84	51·5	13·6	149·8	59·6	161·2	20·6	6·8	51·5	13·6	74·9	29·8	80·6	10·3	4·25	60·0	22·1	65·9	22·8	69·7	8·9	2·9	
							Av.	19·0						Av.	9·36						Av.	7·96		
50	0·55	1·96	37·0	7·9	21·8	12·7	25·2	12·8	1·45	37·0	7·9	10·9	4·5	11·8	6·01	0·75	45·0	15·9	11·9	2·74	12·2	6·23	0·55	[5]
100	1·09	3·92	39·0	9·9	54·6	26·2	60·5	15·4	2·75	39·0	9·9	27·3	11·0	29·4	7·51	1·8	47·0	17·9	26·7	7·45	27·7	7·07	1·5	
150	1·64	6·26	40·0	10·2	84·6	41·7	94·3	15·1	4·3	40·0	10·2	42·3	19·3	46·5	7·43	2·7	49·0	19·9	44·5	14·1	46·7	7·45	2·15	
200	2·18	7·84	42·0	12·2	137·0	59·6	148·0	18·9	5·9	42·0	12·2	68·5	29·8	74·7	9·53	3·6	51·0	21·9	65·3	22·8	69·1	8·82	2·6	
							Av.	15·5						Av.	7·62						Av.	7·4		

REMARKS.—[1] One rail and conductor 12 ft. overhead. [2] One rail and conductor 6 ft. overhead. [3] One rail and conductor 1 ft. overhead. [4] Two rails 1 ft. 6 in. or 4 ft. 8½ in. apart, and conductor 12 ft. overhead. [5] Two rails 1 ft. 6 in. or 4 ft. 8½ in. apart, and conductor 6 ft. overhead.

* *Electrician*, February 23, 1906, and July 27, 1906.

From the observations on the resistance drop in the 70 lb. steel rails, referred to in Table 6, the following figures are obtained for the equivalent penetration at different frequencies and currents:

	Equivalent penetration		
R.M.S. amperes	Frequency 100	Frequency 50	Frequency 27
50	1·58 mm.	3·89 mm.	5·53 mm.
100	1·41 „	2·77 „	3·63 „
150	1·29 „	2·10 „	2·76 „
200	1·07 „	1·75 „	2·19 „

TABLE 7. *Test of Westinghouse Interworks Road.*

Volts	Amperes	Power factor	True watts	Copper watts	Rail watts	Effective resistance, ohms	Remarks
23·5	50	·646	759	555	204	·0816	Trolley wire and rails
46·2	100	·637	2940	2175	765	·0765	
68·5	150	·639	6570	4880	1690	·0751	
89·6	200	·63	11300	8680	2620	·0655	
138·4	300	·62	25700	19500	6200	·0689	
48·9	100	·552	2700	1941	759	·0759	Feeder and rails
71·4	150	·55	5890	4380	1510	·0671	
95	200	·548	10400	7780	2620	·0655	

Particulars:

Frequency 25.

Length of track 4000 feet, single track.

Rails—two, each 70 lbs. per yard, flat bottomed rails, both bonded.

Overhead wire—4/0 copper wire supported 23 feet above the ground.

Feeder—4/0 copper wire 26 ft. above the track and 6 ft. from the centre.

Taking an average ohmic resistance of ·07 for the two rails in parallel and a length of 4000 feet, the resistance per mile of single rail is ·185 ohm, which corresponds to an equivalent penetration of 3·3 mm.

TABLE 8. *Tests on a* 1·85 *kilometre length of the Marienfelde-Zossen line*. The circuit consists of a single* 83 *lb. flat bottomed track rail, and an overhead copper wire of* 100 *sq. mm. section* 7·3 *metres above the rail.*

Amperes	Volts	Watts	Frequency	Apparent resistance, ohms per kilometre	Internal reactance, ohms per kilometre	Equivalent shell, millimetres	Total impedance, ohms per kilometre	Drop in rails, volts per kilometre	Drop for A.C. / Drop for C.C.
43·5	49·75	1160	25·0	0·1104	0·108	3·0	0·282	12·3	8·2
51·5	58·9	1640	24·5	0·1152	0·112	2·86	0·288	14·8	8·4
68·5	78·3	3000	24·5	0·1220	0·101	2·7	0·285	19·5	8·3
88·5	101·2	5010	24·3	0·1242	0·1125	2·65	0·286	25·3	8·4
120·0	136·2	9200	23·4	0·1200	0·121	2·75	0·29	34·8	8·5
129·2	145·5	10600	23·3	0·123	0·117	2·7	0·286	37·0	8·4
143·0	164·5	13200	23·3	0·1258	0·13	2·6	0·3	43·0	8·8
164·8	186·6	17800	22·3	0·1325	0·13	2·5	0·298	49·0	8·7
35·0	65·2	880	47·6	0·1654	0·144	2·0	0·46	16·1	13·5
37·5	71·1	1100	48·8	0·1648	0·161	2·0	0·454	17·0	13·3
53·8	101·8	2200	48·5	0·187	0·134	1·77	0·47	25·3	13·8
70·0	131·3	3750	47·6	0·192	0·148	1·72	0·47	33·0	13·8
78·5	146·8	4760	47·6	0·1953	0·135	1·69	0·465	36·5	13·7
93·5	177·0	6900	47·3	0·205	0·149	1·61	0·477	44·6	14·0
122·0	223·6	11600	44·3	0·199	0·154	1·66	0·474	58·0	13·9
156·8	279·6	19800	43·0	0·212	0·156	1·56	0·462	72·5	13·5
166·0	295·0	22200	43·0	0·214	0·151	1·54	0·462	77·0	13·5
51·4	50·5	1610	19·2	0·1064	0·098	3·1	0·24	12·3	7·0
69·5	68·3	3000	19·0	0·112	0·0975	2·95	0·24	16·7	7·0
95·4	94·5	5730	18·3	0·1155	0·111	2·85	0·25	23·8	7·3
109·2	105·8	7500	17·8	0·1163	0·104	2·85	0·24	26·2	7·0
123·2	118·3	9630	17·3	0·1194	0·105	2·75	0·24	29·6	7·0

* "Single Phase Electric Traction," C. F. Jenkin, *Proc. I. C. E.* 1906—1907.

Tests on railway signalling instruments with alternating currents.*

(*a*) **Polarised relay.** Alternating voltages applied.

Volts	Frequency	Effect	
1·96	50	No vibration continuous, slight vibration on closing circuit.	
3·92	„	No vibration continuous, slight vibration on closing circuit.	
5·24	„	No vibration continuous, just makes contact on closing.	
19·2	„	No vibration continuous, makes contact regularly on closing.	
1·78	25	Very slight vibration continuously, slight vibration on closing circuit.	
2·85 3·56	„	Slight continuous vibration, contact made generally on closing circuit.	
4·76	„	Continuous vibration, contact made every time on closing circuit.	
6·54	„	Continuous vibration making contact.	
8·75	„	Continuous vibration without contact	2nd needle.
10·5	„	Continuous vibration making contact	
1·53	16	Continuous vibration, not making contact.	
1·83	„	Continuous vibration, making contact.	
2·97	„	Continuous vibration, not making contact	2nd needle.
3·47	„	Continuous vibration, making contact	

[Contacts for 2nd needle set further apart than for 1st needle.]

(*b*) **Track relay.** Alternating voltages applied.

Volts	Frequency	Effect
·9	50	No vibration.
1·35	„	Continuous vibration, making contact.
·6	25	No vibration.
·8	„	Continuous vibration, making contact.
·44	16	No vibration.
·55	„	Continuous vibration, making contact.

(*c*) **Block instrument.** Alternating voltages applied.

Volts	Frequency	Effect
1·79	50	$\frac{1}{64}''$ deflection on closing circuit. No permanent deflection. No permanent vibration.
2·38	„	$\frac{1}{16}''$ deflection on closing circuit only.
4·0	„	$\frac{1}{8}''$ „ „ „
7·16	„	Permanent deflection gradually increasing to $\frac{1}{4}''$. No permanent vibration.
8·5	„	Permanent vibration against stops.

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* These tests were made at Stafford, and are published with the kind permission of Messrs Siemens Bros.

Volts	Frequency	Effect
1·79	25	$\frac{1}{32}''$ deflection on closing circuit. No permanent deflection. Slight permanent vibration.
2·38	„	$\frac{1}{8}''$ deflection on closing circuit only.
4·0	„	$\frac{3}{16}''$ deflection on closing circuit. No permanent deflection. Permanent vibration.
7·16	„	Permanent deflection gradually increasing to $\frac{1}{4}''$. Permanent vibration.
7·5	„	Permanent deflection gradually increasing to $\frac{3}{8}''$.
10·5	„	Permanent deflection vibrating against stop.
·6	16	$\frac{1}{64}''$ deflection on closing circuit. No permanent deflection. Very slight permanent vibration.
1·11	„	$\frac{1}{32}''$ deflection on closing circuit. No permanent deflection. Very slight permanent vibration.
2·36	„	$\frac{1}{8}''$ deflection on closing. No permanent deflection. Permanent vibration.
4·55	„	$\frac{1}{4}''$ deflection on closing. No permanent deflection. Permanent vibration.
6·55	„	Vibration between stops for some seconds on closing. No permanent deflection. Bad permanent vibration.
8·66	„	Permanent vibration against both stops.

[NOTE. Full deflection is $\frac{1}{2}''$ on each side. Resistance of instrument 200 ohms.]

CHAPTER 9.

CALCULATION OF ENERGY CONSUMPTION, &c. ON SINGLE PHASE RAILWAYS.

Much that has been written in previous chapters with regard to the calculations of energy consumption on electric railways, holds good for railways operating on the single phase alternating current system. Details of tractive resistance, weights of cars, etc. are equally applicable, and the general methods of calculation are also the same.

Weights of single phase equipments. Much discussion has recently taken place as to the relative weights of direct current and single phase equipments. Thus, for instance, it is claimed* for the Westinghouse single phase equipment, that it is only 15 per cent. heavier than the corresponding direct current apparatus; on the other hand, Carter†, in his paper before the Institution of Electrical Engineers, stated that the increase was more like 150 per cent. No doubt the uncertainty is partly due to different methods of rating; but besides that, designs of the various sizes and types of apparatus are still somewhat experimental.

Some information as to the weights of existing motors and controllers, etc. has already been given, and from what has been said above it will be obvious that there would be no advantage in attempting to lay down any precise rules. For the purposes of the calculations in this chapter the following figures may be taken in connection with motors rated nominally at 100 H.P., the designs and performances of which are given in Chapter 5 of this volume.

Each motor would weigh about 2·5 tons (a direct current motor rated at 100 H.P. would weigh about 2·1 tons, but its speed would be approximately 30 per cent. lower). A variable ratio step-down transformer for two such motors would weigh from 1·5 to 2 tons. The rest allowances that should be made for the heating of the single phase motors. In order to emphasise this point, and also to obtain an idea as to the distribution of the motor losses, an analysis has been made of

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* *Street Railway Journal*, March 26, 1904, p. 483.

† *Journal of the Inst. Elec. Engineers*, Vol. 36, p. 254.

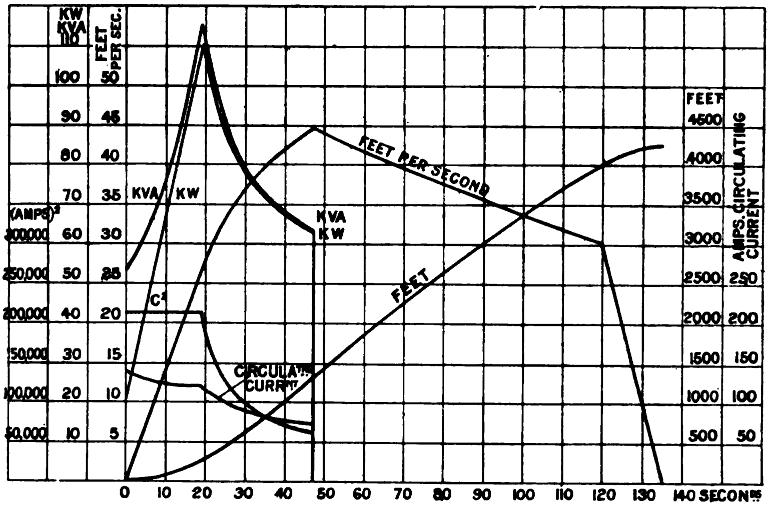

FIG. 181. Speed-time curves for typical run with single phase series motor.

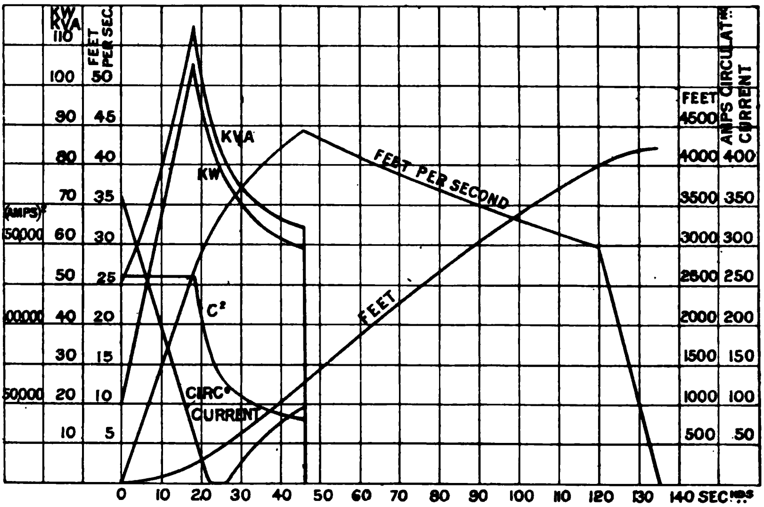

FIG. 182. Speed-time curves for typical run with single phase series motor with commutation poles.

the losses occurring in all four cases. In examining the figures in the table, attention should be fixed chiefly on the watts per square inch on the armature and commutator.

As will be seen, in the direct current motor the allowance for heating on the armature is ·415 watts per square inch (C^2R only) which with a totally enclosed unventilated motor should correspond to

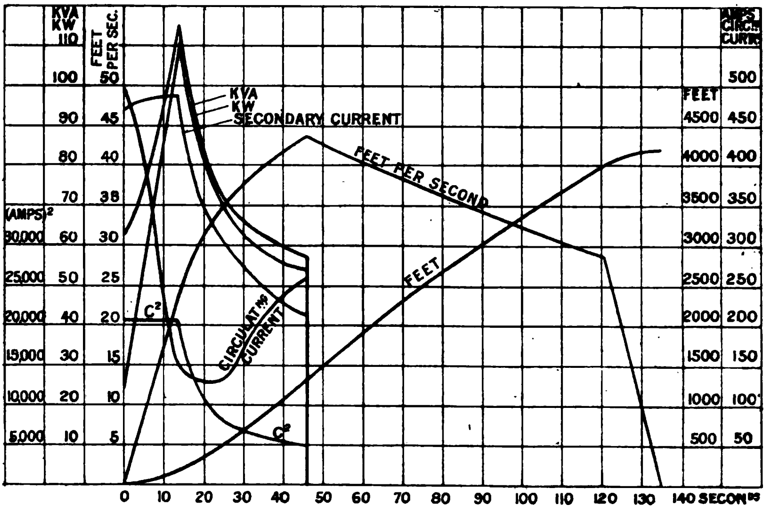

FIG. 183. Speed-time curves for typical run with single phase compensated repulsion motor.

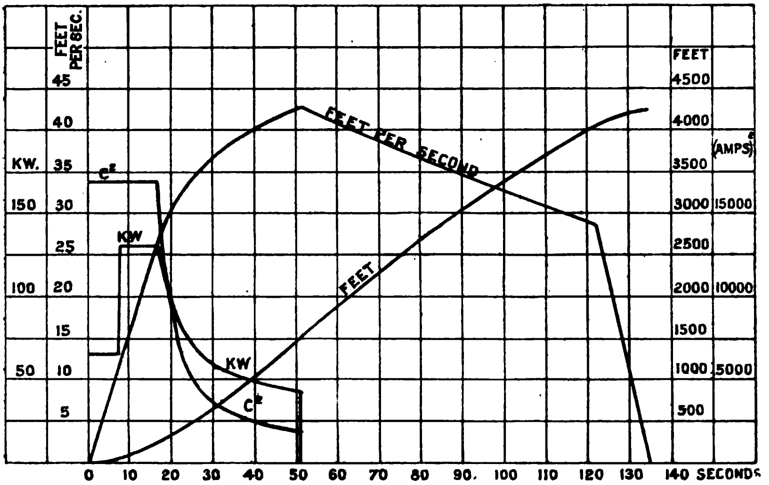

FIG. 184. Speed-time curves for typical run with 500-volt direct current motor.

a temperature rise of about 70° to 75° C. in continuous operation. The corresponding figures for the three single phase motors are considerably higher, but reasons have already been given* for supposing that higher figures are permissible.

Particulars of run, and analysis of motor losses.

	250 volt series motor, no com. poles	320 volt series motor with com. poles	800 volt compens. repulsion motor	500 volt direct current motor
Distance run, feet	4220	4220	4220	4220
Duration of run, seconds	135	135	135	135
Average speed incl. 30 sec. stop, m.p.h....	17·5	17·5	17·5	17·5
Maximum demand, K.V.A.	920	920	920	780
Watt-hours per ton mile (motors) ...	63	57·3	59	51·5
Watt-hours per ton mile (transformers)...	2	2	2	8·2 (rheostat)
Watt-hours per ton mile (total)	65	59·3	61	59·7
Watt-hours per seat mile	33·6	30·6	31·6	28
Effective power factor $\left(=\frac{\text{watt secs.}}{\text{volt. amp. secs.}}\right)$	·915	·865	·92	1·0
Motor losses—				
R.M.S. current per motor	186	140	52·5 prim. 175 secy.	93
Average current per motor	88	67	24·3 prim. 80·3 secy.	44
R.M.S. circulating current	51†	73	120	
Average circulating current	28·5†	25·3	52·6	
C²R losses due to main currents	1950	1460	1790	1230
C²R losses due to circulating currents ...	1250	390	720	say 60
Average iron losses, about...	150	170	160	420
Average commutator friction	520	300	650	120
Average bearing friction	1340	1340	1340	850
Average total losses, kw.	5·21	3·66	4·66	2·68
Average input, kw.	16·83	15·3	15·75	16·7
Average motor efficiency (excl. gear), %...	69	76	70·5	84 (nett) 72·5 (incl. rheo.)
Armature losses (excl. iron loss) ...	1800	890	1410	530
Armature surface, sq. ins.	1550	1400	1660	1270
Watts per square inch	1·16	·635	·85	·415
Commutator losses (incl. friction) ...	1040	624	1300	290
Commutator surface, sq. ins.	610	480	644	345
Watts per square inch	1·71	1·3	2·02	·84
C²R in stator	700	640	450	590

The same remarks, however, cannot apply to the commutator, and it will be seen that in this respect the single phase motors are in a worse case than the direct current motor. There seems no choice but to accept this; but there is no doubt that improvements in the mechanical design of the commutator will be evolved to meet this difficulty, and that this will not be allowed to bar the progress of the single phase system of electric traction.

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* See page 182.

† It should be noticed that this figure is for a 4-pole multiple circuit armature; to make it comparable with the other motors which have two circuit double windings, it must be doubled.

General remarks as to energy consumption. In discussions as to the energy consumption on hypothetical single phase systems, it is usual to institute a comparison with direct current systems. The latter have become standardised to such an extent that they form a convenient and well known basis from which to examine the performance of other systems.

A general comparison cannot be made of any great value; but it is of interest in particular cases to investigate the difference between the two systems.

Consider first the case already worked out in the foregoing pages. As has been shewn, the energy consumption per seat-mile differs by about 15 per cent. in favour of the direct current system. Assuming the same drop in the distribution in the two cases, there will probably be a slight saving with the single phase system, since the drop is partly inductive. Further the losses in the rotary converter will be eliminated entirely, amounting to say seven per cent.; also possibly the sub-station step-down transformers may be dispensed with, since the range of transmission is much greater with high tension alternating current than with low tension direct current. Estimating this saving at three per cent. and the economy in distribution at one per cent. the comparison is as follows:

Excess in single phase system	15 per cent.
Economy in single phase system	1 ,, 7 ,, 3 ,,
Nett excess in single phase system	4 ,,

This small value of nett excess is chiefly due to the elimination of the rheostatic losses, which in the case under consideration amount to nearly 14 per cent. of the input to the train. In other cases, especially where long runs are made with few intermediate stops, the rheostatic losses are much less important, and then the single phase system is at a greater disadvantage. It is, however, in just such cases that other considerations would have a preponderating influence, chiefly the comparison between the two distribution systems, and the first cost of sub-stations, etc.

Then, again, the 15 per cent. excess due to increased weight of train is by no means a fixed figure. For instance, in the case of a locomotive which is required to exert a specified draw-bar pull, it is quite possible that the weight of the complete locomotive may be fixed by considerations of adhesion, regardless of the weight of the equipment.

On the whole, therefore, it seems highly probable that in all cases the choice between the single phase and the direct current system for any particular scheme of railway electrification will depend on considerations of first cost, facility of working and non-interference with the track, and not upon comparisons of energy consumption.

CHAPTER 10.

FIRST COSTS OF ELECTRIC RAILWAYS.

Very little reliable information has been made public with regard to the first cost of electric railways; the following, however, may be of use to engineers, but must be used with discretion.

Direct current railways. 1. The contract* for 480 electric equipments for motor cars for the Underground Electric Railways of London was awarded to the British Thomson-Houston Co., the price being £1400000.

Thus in this case a two 200 H.P. motor equipment cost £2920.

2. For the Baker Street and Waterloo Railway, one of the Underground Electric Railways of London, 100 car bodies were purchased† from the American Car Co. of New York, U.S.A., at a price of £1000 each.

3. In the arbitration proceedings between the Metropolitan and Metropolitan District Railways, the following were stated to be the prices put forward by the British Thomson-Houston Co. to the Metropolitan District Railway:

20 motor cars complete with four 240 H.P. motors, each	£5941
65 miles of 80 lb. conductor rail, installed complete ...	£71502
Installation for 6 telephone circuits connecting generating stations with sub-stations	£1681

Alternating current railways. 1. New York, New Haven and Hartford Railroad‡.

Power house (including real estate)...	$1130000
Overhead construction (21·45 miles of quadruple track)	$570000
35 single phase direct current locomotives	$1050000

(each loco. is equipped with 4 motors each of which is rated at 400 H.P. when tested for one hour with direct current).

2. The following comparison has been drawn by Mr G. Westinghouse§ between the single phase and the direct current systems.

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* *Electrician*, March 18, 1904.

† *Electrician*, September 23, 1904.

‡ *Street Railway Journal*, Feb. 17, 1906, page 276.

§ *Street Railway Journal*, Dec. 23, 1905, page 1123.

Comparative cost per mile of four track line.

	Single phase	Direct current
Sub-stations	$1714	$16150
Contact line	12436	18872
Transmission line	1815	2181
Track bonding	308	308
	$16273	$37511

Comparative cost per mile of double track.

	Single phase	Direct current
Sub-stations	$1542	$13840
Contact line	6750	9436
Transmission line	1815	2181
Track bonding	154	154

The comparison is based on the employment of 6000 volts single phase alternating current, and 600 volts direct current with rotary converter sub-stations.

3. The following information is published* by Herr Wolfgang Adolf Müller in connection with electric railways in Switzerland.

Railway	Gauge	Length, km.	Cost per km. of railway (Marks)		
			Track	Over-head system	Electric feeders, etc.
1. Burgdorf-Thun	4′ 8½″	40·214	31769	23283	14546
2. Freiburg-Murten-Ins	4′ 8½″	32·301	22147	18346	11458
3. Orbe-Chavornay	4′ 8½″	3·899	34370	22026	5015
4. Aarau-Schöftland	metre	11·049	5825	18166	8188
5. Aigle-Leysin	,,	6·785	68762	38132	10872
6. Bex-Gryon-Villars	,,	12·444	31442	29101	6937
7. Bremgarten-Dietikon	,,	10·836	7796	15194	6881
8. Châtel-Bulle-Montbovon	,,	32·296	25615	21191	10931
9. Châtel-Palézieux	,,	6·807	16511	16169	10028
10. Genève-Veyrier	,,	5·548	24118	16446	20111
11. Grütschalp-Mürren	,,	4·279	55877	13938	5090
12. Lausanne-Moudon et Savigny	,,	26·947	15581	25425	9741
13. Montreux-Berner Oberland	,,	22·133	109517	30854	11128
14. St Gallen-Speicher-Trogen	,,	9·980	17135	19631	14274
15. Sissach-Gelterkinden	,,	3·148	23952	15143	8581
16. Stansstad-Engelberg	,,	22·518	17975	19742	8185
17. Vevey-Chamby	,,	8·532	36565	19979	8549
18. Wetzikon-Meilen	,,	22·476	10987	15353	6273
19. Gornergratbahn	,,	9·022	126414	46099	9684
20. Jungfraubahn	,,	4·339	303866	52697	35227

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* *Elektrische Bahnen und Betrieb*, May 14, 1906.

CHAPTER 11.

WORKING EXPENSES, ACCOUNTS, &c. FOR ELECTRIC RAILWAYS.

General. The working expenses for electric railways are best considered by comparison with well-known results for steam railways. So far instances of electrification in this country have been primarily on tube railways designed from the start for electrical working; subsequently, however, steam lines or branches of large steam railways have been converted. In both cases, nevertheless, the greater part of the working expenses are practically unaffected by the use of electricity. It is, therefore, most convenient to consider only those items which are affected by the change in motive power.

Locomotive power, fuel. Naturally the most important item from this point of view is the cost of power. From a purely theoretical point of view, a simple comparison may be made thus :

A passenger steam locomotive requires about 3·5 lbs. of coal per I.H.P. hour.

A freight steam locomotive requires about 5 lbs. of coal per I.H.P. hour.

Suppose cost of coal 15*s.* per ton; now the cost of electrical energy from a generating station varies over somewhat wide limits say from ¼*d.* to 1½*d.* per unit; for the sake of comparison assume ¾*d.* per unit; further, allow say 85 per cent. efficiency of the steam loco., and say 75 per cent. efficiency for the electric loco. (including transmission and distribution losses). The comparison will then be as follows :

Passenger steam loco. ...	·33 pence per B.H.P. hour.
Freight steam loco. ...	·47 pence per B.H.P. hour.
Electric loco.	·75 pence per B.H.P. hour.

According to this simple comparison, therefore, electrical energy at ¾*d.* per unit is too expensive, and the price would have to be reduced to ½*d.* or ¼*d.* per unit if the two systems are to work on equal terms.

Strictly speaking there should be included in the cost of electrical energy the interest and maintenance charges on the transmission and distribution system, which will vary considerably for different cases.

This comparison is too simple to be of much value, since it assumes that the same B.H.P. would be required on a steam and on an electric service. This is not necessarily the case, as the introduction of electricity may easily effect a diminution in the weight of the train. Thus, for instance*, on the New York Central and Hudson River Railroad a steam loco. weighs 150 tons with its tender, whereas an electric loco. for the same duty weighs only 95 tons. (In the first case the weight available for adhesion is only 47 tons, against 69 tons in the second case.) Thus there is a saving of 55 tons dead weight in a total of about 550 to 600 tons in favour of the electrical system.

Further, this comparison would not hold good in cases in which the trains are frequently stopping and starting, as the result of electrification may be a marked diminution of energy consumption, amounting in a favourable case to as much as 50 per cent. Thus the question of cost of electrical energy as against cost of fuel must be examined for each individual case.

A good example of this occurs on the Electrified Inner Circle. Before electrification the consumption of coal was stated to be 36 lbs. of best steam coal per train mile (each train weighed about 155 tons). This would be equivalent to an expenditure of about 12 B.H.P. hours per train mile; whereas if the same average speed were maintained after electrification the energy consumption could be reduced to about 45 watt hours per ton mile or 6·2 kw. hours per train mile, or say 7·3 kw. hours at the generating station. In this case the comparison would be:

With steam locos. and coal at 16*s.* per ton...	3·1 pence per train mile.
With electrical driving and energy at ¾*d.* per unit	6·2 pence per train mile.

As a matter of fact, the average speed has been considerably increased, and the energy consumption is approximately 70 watt hours per ton mile, in which case the cost on the same basis as above would be 9·7 pence per train mile.

In the balance sheet of a railway company under the heading locomotive power are included other items besides cost of fuel. These items are (1) salaries, office expenses and general superintendence, (2) wages connected with working the locomotives, (3) gas and water, (4) oil, tallow and other stores, and (5) repairs and renewals.

Of these items, the first may be assumed to be the same for both steam and electrical working; the second in many cases may be reduced by the introduction of electricity; the third is likewise reduced as the cost of water is included in the operating costs of the electric generating station ; some slight reduction may be made in the fourth item ; and probably a very considerable reduction in the fifth.

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* *Street Railway Journal*, November 19, 1904.

Before discussing these probable economies in detail, it will be instructive to give a typical example* from a large railway company shewing the amounts of the various items.

Wages ...	...	3·81	pence per train mile
Fuel ...	...	4·50	„ „
Water, oil, etc.	...	·77	„ „
Repairs	Wages ...	1·34	„ „
	Material	1·05	„ „
	Total ...	11·47	pence per train mile.

From this example it may be seen that the locomotive wages and the cost of fuel are the most important items, after which comes the maintenance, and the least important is the cost of water, oil and stores.

Locomotive wages. This item may be considerably reduced by the introduction of electric traction. In the first place less skill is required to drive an electric train than to manage a steam locomotive, and in the second place it is possible to reduce the number of men. The Board of Trade requires that there shall always be two men on a steam locomotive, the object being to provide a safeguard in the case of an accident to the driver. Apart from this, the amount of work involved in attending to the locomotive and in driving is more than one man could manage. In an electric train, however, the driving of the train is quite within the powers of one man, and the provision of a dead man's handle† or some similar device is quite sufficient as a safeguard.

On the Metropolitan and Metropolitan District Railways only one man is employed in the driver's cab ; and it has been stated‡ that the wages of drivers have been reduced from 42 to 56 shillings per week to 35 to 42 shillings per week.

Similarly on the Liverpool to Southport branch of the Lancashire and Yorkshire Railway§, on the express electric trains there are only two men, one driver and one conductor, and on the local trains one driver and two conductors. And on the electric railway from Les Invalides-Versailles§, the locomotive is in charge of one man whose wages amount to 2200 francs per annum.

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* Quoted by Prof. Carus-Wilson at the International Railway Congress, 1901.

† See Vol. I. p. 308.

‡ See *Elektrische Bahnen und Betrieb*, July 24, 1905, p. 395.

§ *Street Railway Journal*, May 20, 1905. International Railway Congress.

In addition to this modification, a further reduction is effected due to the fact that the number of train miles in any given time is generally much greater than before the electrification. The original speed of the trains on the Inner Circle was 11·2 miles per hour including speeds, and it was intended to accelerate the service up to 15·7 miles per hour. That is to say, apart from any other change, the wages per train mile would be reduced about 29 per cent.

Still one more source of economy is introduced with electricity in that the cleaning of locomotives and the wages involved therein would be largely reduced.

Thus in a favourable case, very considerable economies can be effected in the item for locomotive wages.

Repairs and renewals. For steam railways the item for repairs and renewals is fairly well known and more or less constant; for electric railways, on the other hand, the information on this point is very scanty and conflicting, as is evident from the following short list:

Electric locomotives and motor coaches. **Maintenance per mile.**

	Pence
[*]**Baltimore and Ohio Railway** (800 H.P. 625 volts direct current). Mechanical parts 1·4*d.*, electrical 1·6*d.*, total ...	3
[†]**Valtellina Railway.** Mechanical and electrical	·7
[‡]**City and South London Railway.** Including say ·2*d.* for the maintenance of the generating plant per train mile ...	·78
[§]**Bloomington, Pontiac and Jolliett single phase Railway.** Electric equipments only (for 50000 car miles) ...	·34
[‖]**New York Central and Hudson River Railroad.** New electric loco. (after covering 25000 miles)...	·82
[‖]**Buffalo and Lockport.** (Two 40 ton locos. each of which has run 35000 miles per annum for the last four years.) Average	·24
[¶]**Orleans Railway.** Maintenance and repairs of locos. per train mile	1·15

(Train consists of electric loco. and load of about 150 tons.)

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[*] Muhlfield, *Street Railway Journal*, Feb. 24, 1906, p. 327.
[†] *Street Railway Journal*, Aug. 26, 1905, p. 303.
[‡] For half-year ending Dec. 1902.
[§] *Street Railway Journal*, June 16, 1906.
[‖] Armstrong, *Street Railway Journal*, Dec. 16, 1906.
[¶] International Railway Congress, *Street Railway Journal*, May 20, 1905.

***Les Invalides-Versailles Railway.** Maintenance and repairs of locos. and motor cars per train mile ... ... 2·46

(Train consists of electric loco. and load of about 76 tons.)

***Paris Metropolitan Railway.** Maintenance of rolling stock per car mile ... ... ·57

In addition to the above a few miscellaneous detailed maintenance costs may be given.

†Valtellina Railway. *Collector* (consisting of a steel roller covered with electrolytic copper). Will run 12000 to 25000 miles before requiring recoating.

Trolley wire. No wear in 3½ years.

Motor bearings. Most of the motor cars have run over 100000 miles without bearings being renewed.

Overhead line. Including trolley wire, poles, and transformer stations, per mile per annum £21. 10*s*.

‡Bloomington, Pontiac and Jolliett Railway. Maintenance per car mile (for about 50000 car miles):

Track and roadway ...	·31	cents.
Electric line	·09	„
Power plant	·001	„
Cars	·195	„
Equipments	·7	„
Miscellaneous	·016	„
Total ...	1·312	cents.

§Les Invalides-Versailles Railway. Maintenance of third rail per mile per annum £44.

In comparing these maintenance costs with the figures given in the balance sheets of steam railways, it is necessary to bear in mind that the expenditure in the latter case is not simply for repairs; the cost of renewals is an important component. In the locomotive shops of any of the large railways a number of locomotives are built every year, the cost of which is not put down to the capital account but to revenue. These locomotives are necessary for the purpose of replacing worn-out or obsolete stock, and consequently the cost is included in the item "repairs and renewals" in the table for "locomotive power." Further than this, these new locomotives do not only replace the old stock but raise the level of the stock as a whole. The cost of the new units is, in general, considerably above the original cost of the old units. It is, therefore, necessary to exercise caution in instituting a comparison between published figures for electric rolling stock, and the items for repairs and renewals abstracted from the balance sheets of the steam railways.

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* International Railway Congress, *Street Railway Journal*, May 20, 1905.

† *Street Railway Journal*, Aug. 26, 1905, and Aug. 5, 1905.

‡ *Street Railway Journal*, June 16, 1906.

§ *Street Railway Journal*, May 20, 1905.

Considerations other than locomotive expenses. Having touched upon the chief items of expenditure under the heading of locomotive expenses, it remains to consider other ways in which electric traction may influence the working expenses, or rather the relation between the working expenses and the receipts.

So far as electrification of railways has at present been developed, the only expenditure apart from those mentioned above which may be affected is that for the maintenance of the track. How far this item may be affected cannot be stated.

The other side of the balance sheet, however, must be taken into account; more particularly since the reason for electrification is frequently the necessity of increasing the accommodation for passengers. In some cases lines have been electrified to avoid the use of steam locomotives in tunnels; but even this procedure is due entirely to the requirements of the passengers.

For several reasons engineers are of the opinion that the introduction of electrical traction provides a means of increasing the train miles that can be worked by a given rolling stock on a given track. Chief among these reasons are (1) increase of terminal facilities, (2) increase of average speed, due to higher acceleration, (3) possibility of splitting up trains to suit the convenience of passengers.

In considering this point it will be best to abstract the results that have already been accomplished as set out in the annual meetings of the various railway companies.

North-Eastern Railway—Tyneside lines (about 20 miles of double track).

For the last half year of ...	1903	1904	1905
Passengers carried ...	2844000 (steam)	3519000 (electric)	3548000 (electric)
For the whole of year ...	1903	1904	1905
Gross receipts	£129000	£146000	£151000
Working expenses ...	£42761		£47779
Running cost per train mile	... electric 0*s*. 6¾*d*.		
„ „ „	... steam 1*s*. 2½*d*.		

Lancashire and Yorkshire Railway. Liverpool-Southport line (about 23 miles of double track).

In the first half of 1905, 600000 additional passengers with ordinary tickets were carried.

The increase of traffic on this line has been such that two more tracks are being prepared for electric running.

The working expenses per train mile are rather greater with electricity than with steam.

Metropolitan District Railway—London.

	1904 last half	1905 first half	1905 last half	1906 first half
Receipts	£197417	£206602	£203509	£221055
Working expenses...	£114213	£114880	£144076	£166912
Passengers carried...		27669075	increase of 2220631 over 1904	32759662

In this instance the process of electrification has been gradual, the first half of 1906 representing the final stage. The increase of passengers and receipts is very marked, but the working expenses have gone up in a still greater proportion. This is stated to be chiefly due to the wages of the train crew. On a steam train, this consisted of driver, stoker and two guards; on an electric train seven men are employed.

Metropolitan Railway—London. At the half-yearly meeting in July, 1906, the following results of electrification were reported:

	Receipts	Expenses
For the half year to June 30, 1906	£389014	£213032
For the half year to June 30, 1905	£430330	£216685

The directors consider that the falling off in the receipts is partly due to diversion of traffic by the Great Central Railway, and partly due to the competition of motor omnibuses. The electrification had not been completed, and the full advantage therefrom had not yet been realised.

Mersey Railway.

	1902*	1903		1904		1905		1906
1	2844000	about 3200000	4153777	4499147	4657876	4862378	4937840	about 5360000
2	£29470	£26136	£37358	£40918	£41789	£43537	£44060	£47129
3	£28082†	£32061†	£32442†	£33591†	£32883†	£34728†	£30471‡	
4	155039	218208	401046	411683	415625	412215	417683	

1 Passengers carried exclusive of season ticket-holders, per half year.
2 Gross receipts, per half year. 3 Working expenses, per half year.
4 Train mileage, per half year.

* Last half year of steam working.
† Including extra charges for pumping, ventilation and lifts.
‡ Excluding extra charges for pumping, ventilation and lifts.

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